Розподіл густини в ядрах

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План

1. Вступ
2. Макро-мікро підход
3. Функціонали густини енергії Skyrme, ETF
4. Енергії зв'язку, розподіл щільності тощо
5. Властивості ядер, далеких від лінії бета-стабільності
6. Ядерно-ядерні потенціали та ETF
7. Ядерно-ядерні потенціали та розподіл густини
8. Висновки

Експериментальні наближення

HO Harmonic-oscillator model:

$$\rho(r) = \rho_0(1 + \alpha(r/a)^2)\exp(-(r/a)^2)$$

$$\alpha = \alpha_0 a_0^2/(a^2 + \tfrac{3}{2}\,\alpha_0(a^2 - a_0^2))$$

$$a_0^2 = (a^2 - a_p^2)A/(A-1)$$

$$\alpha_0 = (Z-2)/3; \quad a_p^2 = \tfrac{2}{3}\langle r^2\rangle_{\text{proton}}$$

$$\rho(r) = \begin{cases} \sum_v a_v j_0(v\pi r/R) & \text{for} \quad r \leqslant R \\ 0 & \text{for} \quad r \geqslant R, \end{cases}$$

$j_0(qr)$ denotes the Bessel function of order zero

MHO Modified harmonic-oscillator model, with the same expression for $\rho(r)$ as in HO but with α as an additional free parameter

2pF Two-parameter Fermi model

$$\rho(r) = \rho_0/(1 + \exp((r-c)/z))$$

3pF Three-parameter Fermi model

$$\rho(r) = \rho_0(1 + wr^2/c^2)/(1 + \exp((r-c)/z))$$

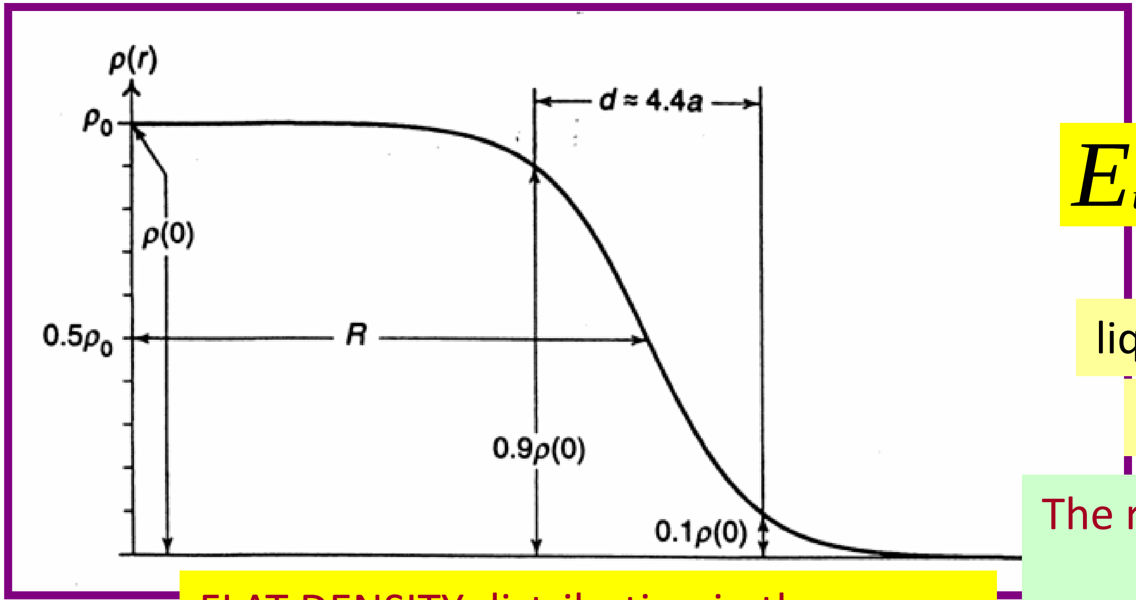
3pG Three-parameter Gaussian model

$$\rho(r) = \rho_0(1 + wr^2/c^2)/(1 + \exp((r^2-c^2)/z^2))$$

UG Uniform Gaussian model

$$\rho(r) = \rho_0 \int \exp(-(r-x)^2)/g^2)x^2 dx$$

Макроскопічний + мікроскопічний підхід передбачає Z=114 та N=184



$$E_{tot} = E_{LD} + \delta E_{shell} + E_{pair}$$

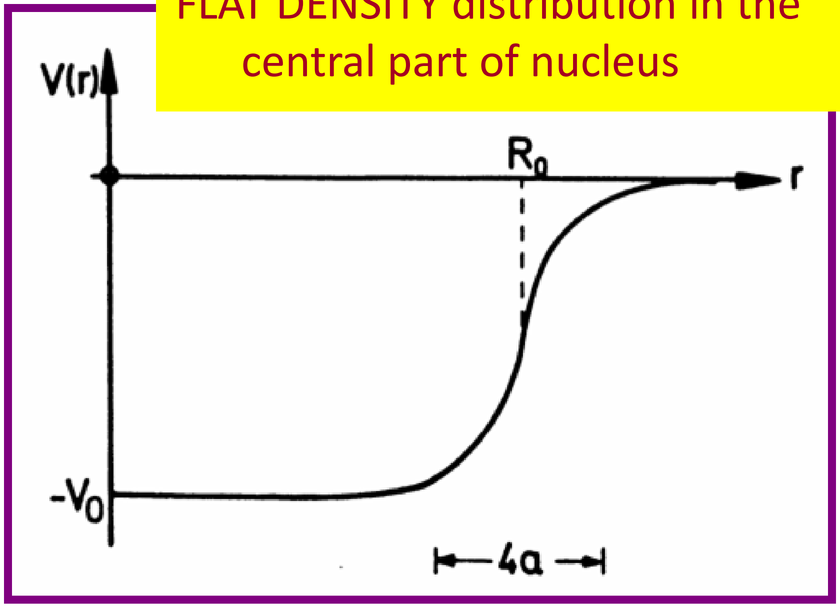
liquid drop

quantum (shell) correction

pairing

The radial density dependence used in liquid drop model

FLAT DENSITY distribution in the central part of nucleus



Woods-Saxon potential

$$V_{WS}(r) = \frac{-V_0}{1 + exp\left(\frac{r-R_0}{a}\right)},$$

$R_0 = r_0 A^{1/3}$ $r_0 \sim 1.2$ fm

$V_0 \sim 50$ MeV $a \sim 0.5$ fm

Extended Thomas-Fermi model

Equations of Extended Thomas-Fermi (ETF) approach

$$\frac{\delta\varepsilon(\rho_n,\rho_p)}{\delta\rho_p}-\lambda_p=0 \qquad \frac{\delta\varepsilon(\rho_n,\rho_p)}{\delta\rho_n}-\lambda_n=0$$

$\varepsilon(\rho_n,\rho_p)$ - Energy-density functional depended on neutron and proton densities

$$\frac{\delta}{\delta\rho_q}=\frac{\partial}{\partial\rho_q}-\nabla\frac{\partial}{\partial(\nabla\rho_q)}+\Delta\frac{\partial}{\partial(\Delta\rho_q)}$$

- Variational derivative

$$\int d\vec{r}\,\rho_{n(p)}(\vec{r})=N(Z)$$

Conservation of proton and neutron number leads to extremum with conditions and to the Lagrange coefficients

The ground-state energy of the system can be evaluated by using energy density functional, which depends on the density and density derivative (*Hohenberg P., Kohn W. Theorem*)

$$E[\rho_n,\rho_p]=\int d\vec{r}\,\varepsilon[\rho_n(\vec{r}),\rho_p(\vec{r})]$$

Skyrme energy-density functional

$$\varepsilon = \varepsilon_{kin} + \varepsilon_{pot} + \varepsilon_{coul} = \frac{\hbar^2}{2m}\tau_p + \frac{\hbar^2}{2m}\tau_n$$

$$+\frac{1}{2}t_0\left[\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(x_0+\frac{1}{2}\right)\times\left(\rho_n^2+\rho_p^2\right)\right] +$$

$$\frac{1}{12}t_3\rho^\alpha\left[\left(1+\frac{1}{2}x_3\right)\rho^2 - \left(x_3+\frac{1}{2}\right)\times\left(\rho_n^2+\rho_p^2\right)\right] +$$

$$\frac{1}{4}\left[t_1\left(1+\frac{1}{2}x_1\right)+t_2\left(1+\frac{1}{2}x_2\right)\right]\tau\rho + \frac{1}{4}\left[t_2\left(x_2+\frac{1}{2}\right)-t_1\left(x_1+\frac{1}{2}\right)\right]\times$$

$$(\tau_n\rho_n+\tau_p\rho_p) + \frac{1}{16}\left[3\,t_1\left(1+\frac{1}{2}x_1\right)-t_2\left(1+\frac{1}{2}x_2\right)\right](\nabla\rho)^2 -$$

$$\frac{1}{16}\left[3t_1\left(x_1+\frac{1}{2}\right)+t_2\left(x_2+\frac{1}{2}\right)\right]\times\left((\nabla\rho_n)^2+(\nabla\rho_p)^2\right)+$$

$$\frac{1}{2}W_0\left[J\,\nabla\rho + J_n\nabla\rho_n + J_p\nabla\rho_p\right] + \varepsilon_{coul}\ .$$

Another form EDF, when terms with kinetic energy are merged with the single kinetic energy term. The effective mass is introduced.

$$\varepsilon = \frac{\hbar^2}{2m_n^*}\tau_n + \frac{\hbar^2}{2m_p^*}\tau_p + \frac{1}{2}t_0\left[\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(x_0+\frac{1}{2}\right)\times\left(\rho_n^2+\rho_p^2\right)\right] +$$

$$\frac{1}{12}t_3\rho^\alpha\left[\left(1+\frac{1}{2}x_3\right)\rho^2 - \left(x_3+\frac{1}{2}\right)\times\left(\rho_n^2+\rho_p^2\right)\right]$$

$$+\frac{1}{16}\left[3\ t_1\left(1+\frac{1}{2}x_1\right) - t_2\left(1+\frac{1}{2}x_2\right)\right](\nabla\rho)^2 -$$

$$\frac{1}{16}\left[3t_1\left(x_1+\frac{1}{2}\right) + t_2\left(x_2+\frac{1}{2}\right)\right]\times\left((\nabla\rho_n)^2 + (\nabla\rho_p)^2\right) +$$

$$\frac{1}{2}W_0\left[J\ \nabla\rho + J_n\nabla\rho_n + J_p\nabla\rho_p\right] + \varepsilon_{coul}$$

$$f_q = \frac{m}{m_q^*},\ f_q = 1 + \frac{2m}{\hbar^2}\left[\frac{1}{4}\left[t_1\left(1+\frac{1}{2}x_1\right) + t_2\left(1+\frac{1}{2}x_2\right)\right]\rho + \frac{1}{4}\left[t_2\left(x_2+\frac{1}{2}\right) - t_1\left(x_1+\frac{1}{2}\right)\right]\rho_q\right]$$

If x_1=x_2=0, x_3=1, α=1, [SIII set]

$$\varepsilon_{skyrme} = \frac{\hbar^2}{2m}\tau(\vec{r}) + \frac{1}{4}(t_1+t_2)\rho\tau + \frac{1}{8}(t_1-t_2)(\rho_n\tau_n - \rho_p\tau_p)$$

$$+\frac{1}{2}t_0\left[(1+\frac{1}{2}x_0)\rho^2 - (x_0+\frac{1}{2})(\rho_n^2+\rho_p^2)\right] + \frac{1}{4}t_3\rho_n\rho_p\rho$$

$$+\frac{1}{16}(t_2-3t_1)\rho\nabla^2\rho + \frac{1}{32}(3t_1+t_2)(\rho_n\nabla^2\rho_n + \rho_p\nabla^2\rho_p)$$

$$+\frac{1}{16}(t_1-t_2)(J_n^2+J_p^2) - \frac{1}{2}W_0(\rho\nabla J + \rho_n\nabla J_n + \rho_p\nabla J_p)$$

$$+\varepsilon_{coul} \;,$$

Kinetic energy

- Thomas-Fermi $\tau_{TF,n(p)} = k\rho_{n(p)}^{5/3}$

$$k = \frac{5}{3}(3\pi^2)^{2/3}$$

- Ћ² derivative correction terms

$$\tau_{2q} = b_1 \frac{(\nabla\rho_q)^2}{\rho_q} + b_2\nabla^2\rho_q + b_3\frac{(\nabla f_q \nabla\rho_q)}{f_q} + b_4\rho_q\frac{\nabla^2 f_q}{f_q} + b_5\rho_q\left(\frac{\nabla f_q}{f_q}\right)^2 + b_6 h_m^2\rho_q\left(\frac{\vec{W}_q}{f_q}\right)^2$$

$$b_1 = 1/36 \quad b_2 = 1/3 \quad b_3 = 1/6 \quad b_4 = 1/6 \quad b_5 = -1/12 \quad b_6 = 1/2 \quad h_m = \hbar^2/2m$$

Spin-orbit contribution

$$\vec{W}_q = \frac{\delta\varepsilon(r)}{\delta\vec{J}_q(r)} = \frac{W_0}{2}\nabla(\rho + \rho_q)$$

Coulomb EDF

with direct and exchange term in Slater form

$$\varepsilon_{col} = \frac{1}{2} e^2 \rho_p(r) \int d\vec{r}\,' \; \frac{\rho_p(r')}{|\vec{r} - \vec{r}\,'|}$$

$$- \frac{3}{4} e^2 \left(\frac{3}{\pi} \right)^{1/3} \rho_p^{4/3}(r)$$

For Spherical Systems

$$\varepsilon_{col} = \tfrac{1}{2} e^2 \rho_p(r) \left[\int_0^r \rho_p(r') r' dr' + \frac{1}{r} \int_r^\infty r'^2 \rho_p(r') dr' \right] -$$

$$\frac{3}{4} e^2 \left(\frac{3}{\pi} \right)^{1/3} \rho_p^{4/3}(r).$$

Equations of Extended Thomas-Fermi (ETF) approach

$$\frac{\delta\varepsilon(\rho_n,\rho_p)}{\delta\rho_p}-\lambda_p=0 \qquad \frac{\delta\varepsilon(\rho_n,\rho_p)}{\delta\rho_n}-\lambda_n=0$$

$$\begin{cases} A_{nn}\Delta\rho_n + A_{np}\Delta\rho_p + B_{nn}(\nabla\rho_n)^2 + B_{np}(\nabla\rho_p)^2 + \\ D_{np}\ (\nabla\rho_n\cdot\nabla\rho_p) + F_n + \lambda_n = 0 \\ A_{pp}\Delta\rho_p + A_{pn}\Delta\rho_n + B_{pp}(\nabla\rho_p)^2 + B_{pn}(\nabla\rho_n)^2 + \\ D_{pn}(\nabla\rho_p\cdot\nabla\rho_n) + F_p + C_p + \lambda_p = 0 \end{cases}$$

Boundary conditions

At *r=0:* density finite

At *r=∞:*

$$\rho_{n,\infty} = \frac{e^{-\chi_n r}}{r^2}$$

$$\rho_{p,\infty} = \frac{e^{-\chi_p r}}{r^2}$$

$$\chi_{n(p)} = \sqrt{|\lambda_{n(p)}| / b_1 h_m}$$

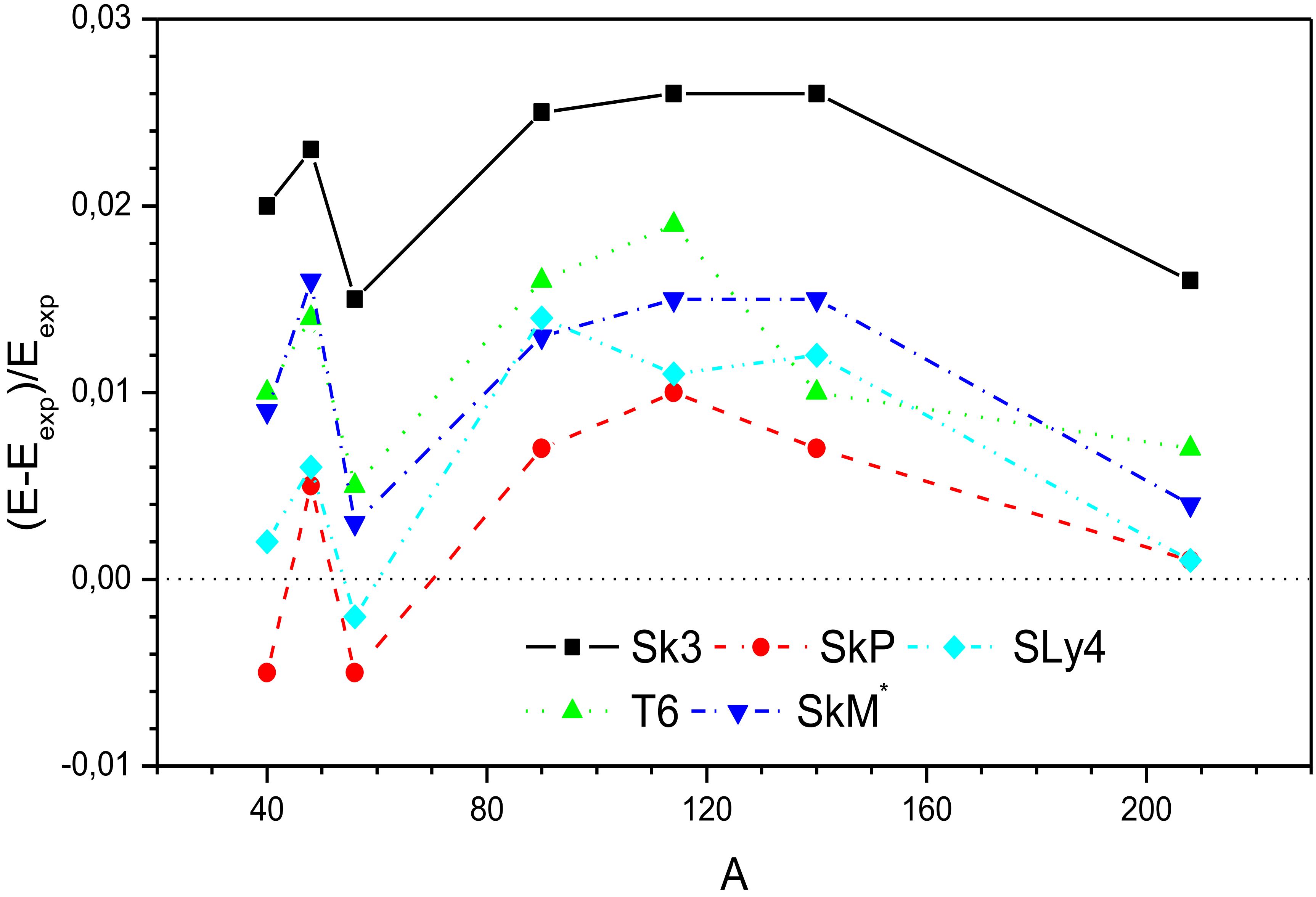
Parameters sets for various Skyrme force parametrizations

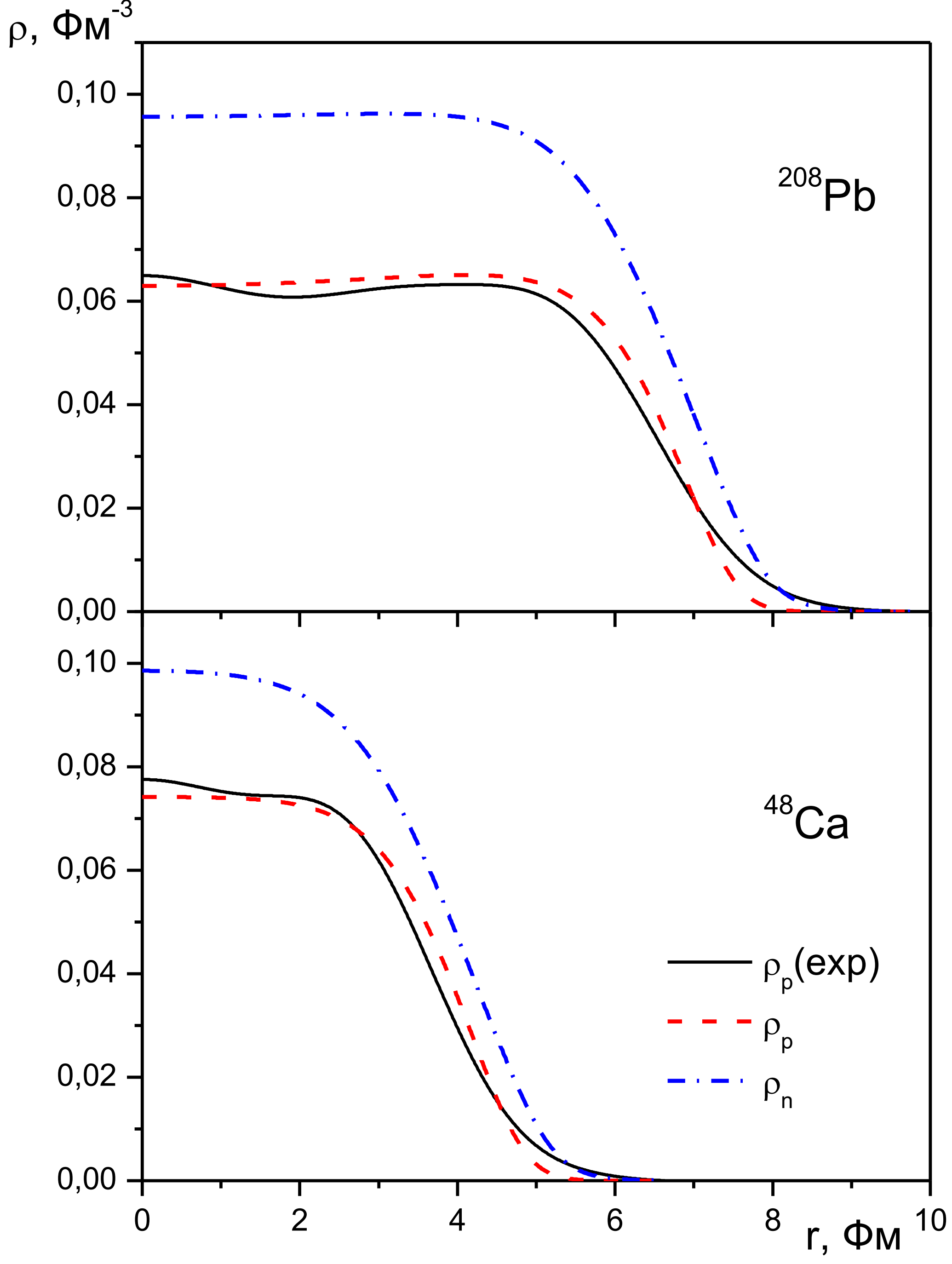
	SIII	SkM*	SLy4*	SkP	T6
t_0	-1128.75	-2645.0	-2488.913	-2931.70	-1794.20
t_1	395.0	410.0	486.818	320.62	294.0
t_2	-95.0	-135.0	-546.395	-337.41	-294.0
t_3	14000.0	15595.0	13777.0	18708.97	12817.0
x_0	0.45	0.09	0.8340	0.29215	0.392
x_1	0.0	0.0	-0.3438	0.65318	-0.5
x_2	0.0	0.0	-1.0	-0.53732	-0.5
x_3	1.0	0.0	1.3540	0.18103	0.5
α	1.0	1.0 /6.0	1.0 /6.0	1.0 /6.0	1.0 /3.0
W_0	120.0	130.0	123.0	100.0	107.0

Binding energy, root mean squared radii and chemical potentials for stable nuclei

Nucleus	$E_{експ}$ [MeV]	E [MeV]	$<r^2_p>^{1/2}_{exp}$ [fm]	$<r^2_p>^{1/2}$ [fm]	$<r^2_n>^{1/2}$ [fm]	λ_n [MeV]	λ_p [MeV]
^{40}Ca	342.1	340.4	3.450	3.231	3.209	-12.14	-10.61
^{48}Ca	416.1	418.0	3.451	3.327	3.504	-6.25	-18.93
^{58}Ni	506.5	505.8	3.769	3.562	3.618	-11.25	-11.54
^{90}Zr	783.9	789.9	4.258	4.070	4.171	-8.65	-14.54
^{114}Sn	971.6	981.8	4.602	4.391	3.492	-8.36	-14.41
^{140}Ce	1172.7	1181.8	4.880	4.682	4.819	-6.60	-16.62
^{208}Pb	1636.5	1638.4	5.503	5.330	5.486	-5.29	-17.44

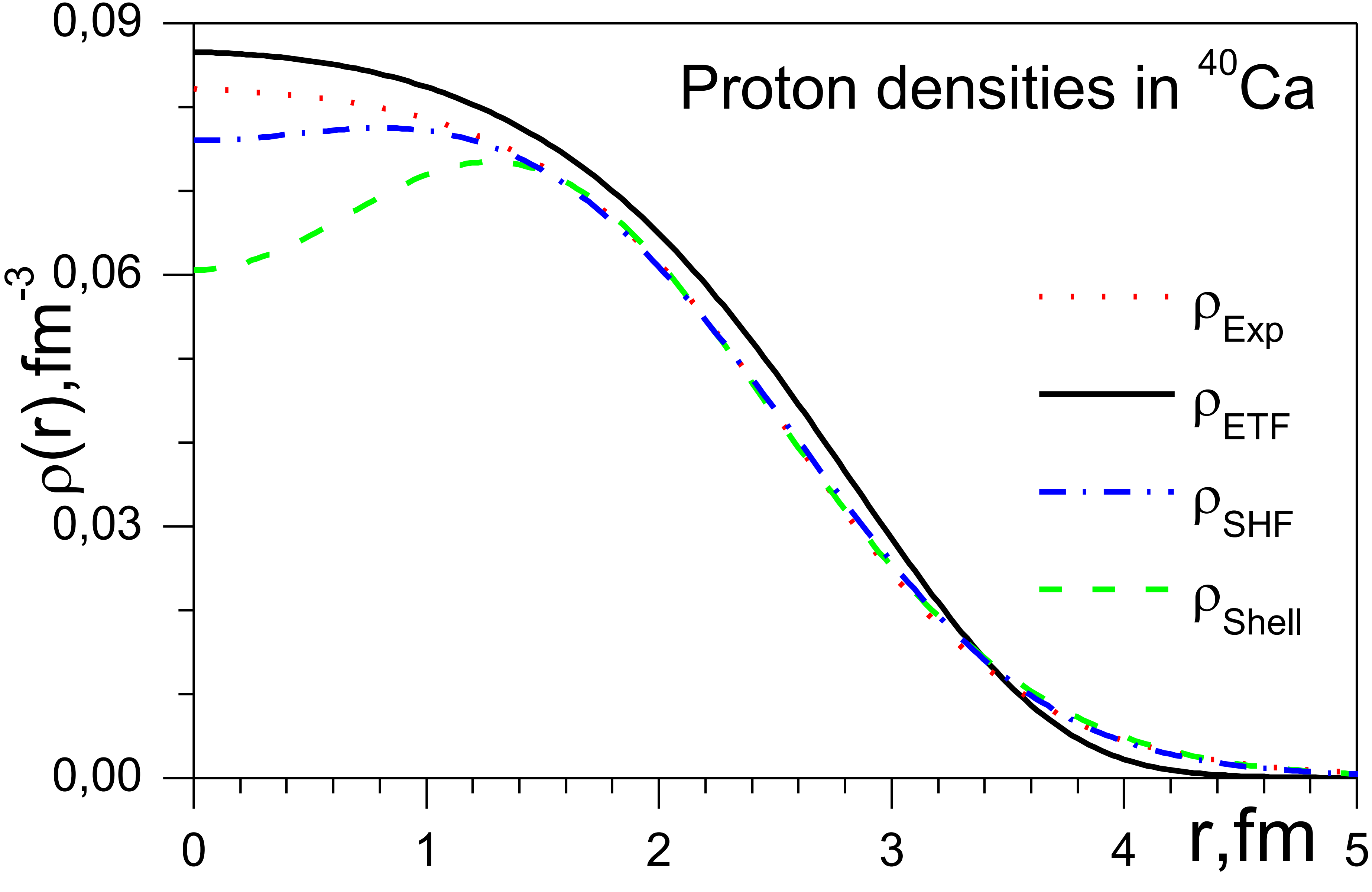
Relative deviations of binding energy



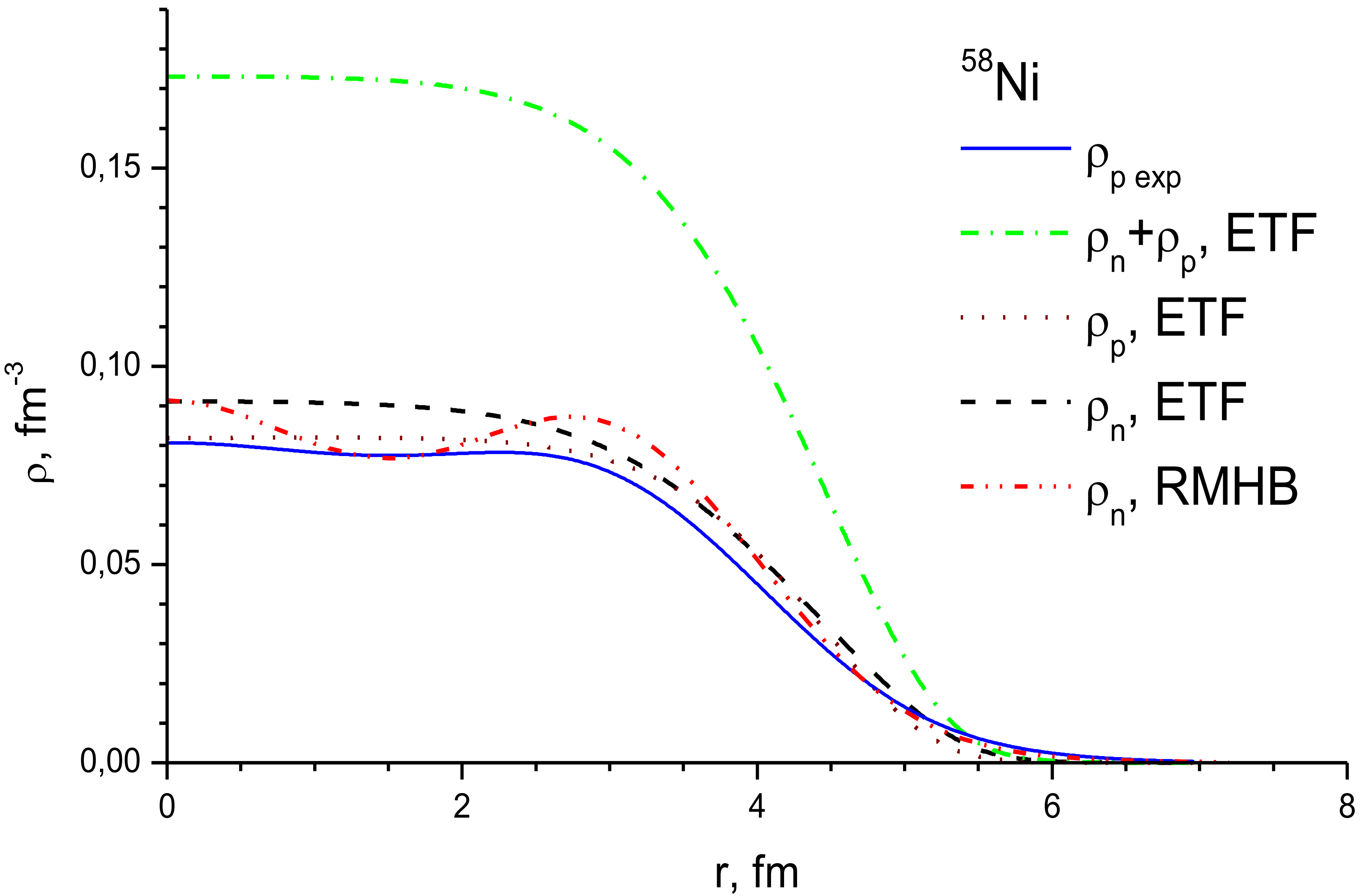


Розподіл густини протонів та нейтронів

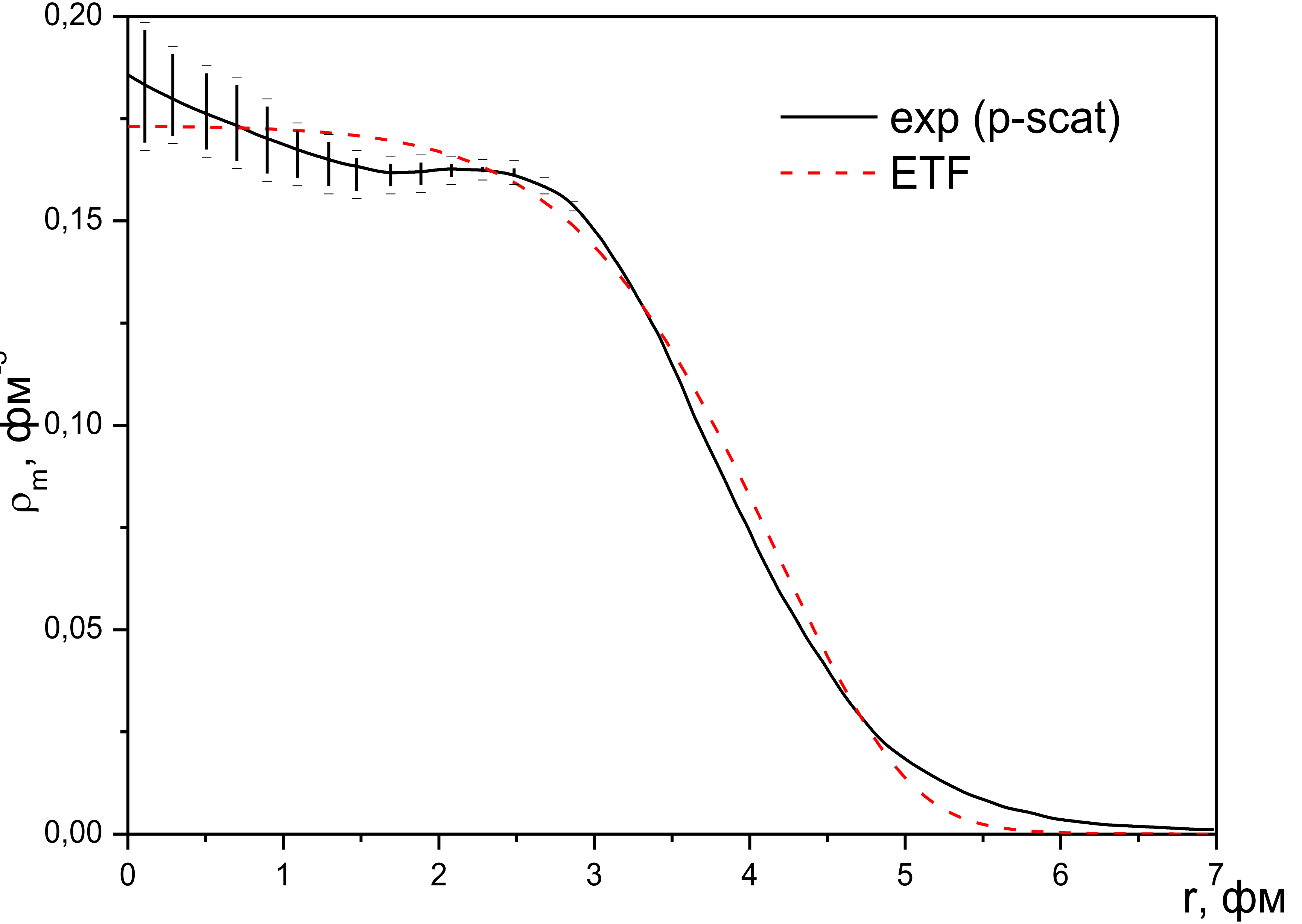
Density distribution in various approaches



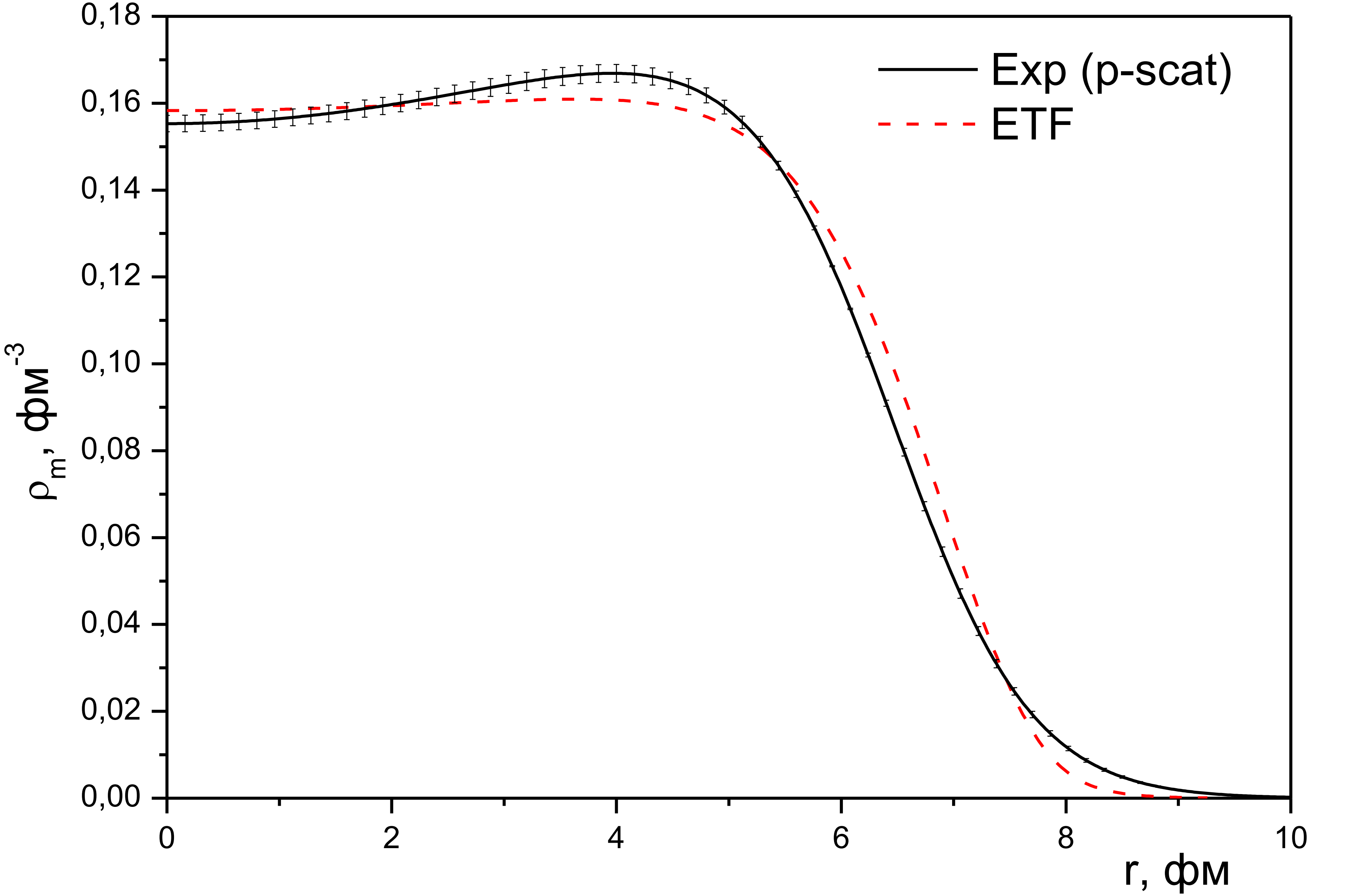
Density distribution in various approaches



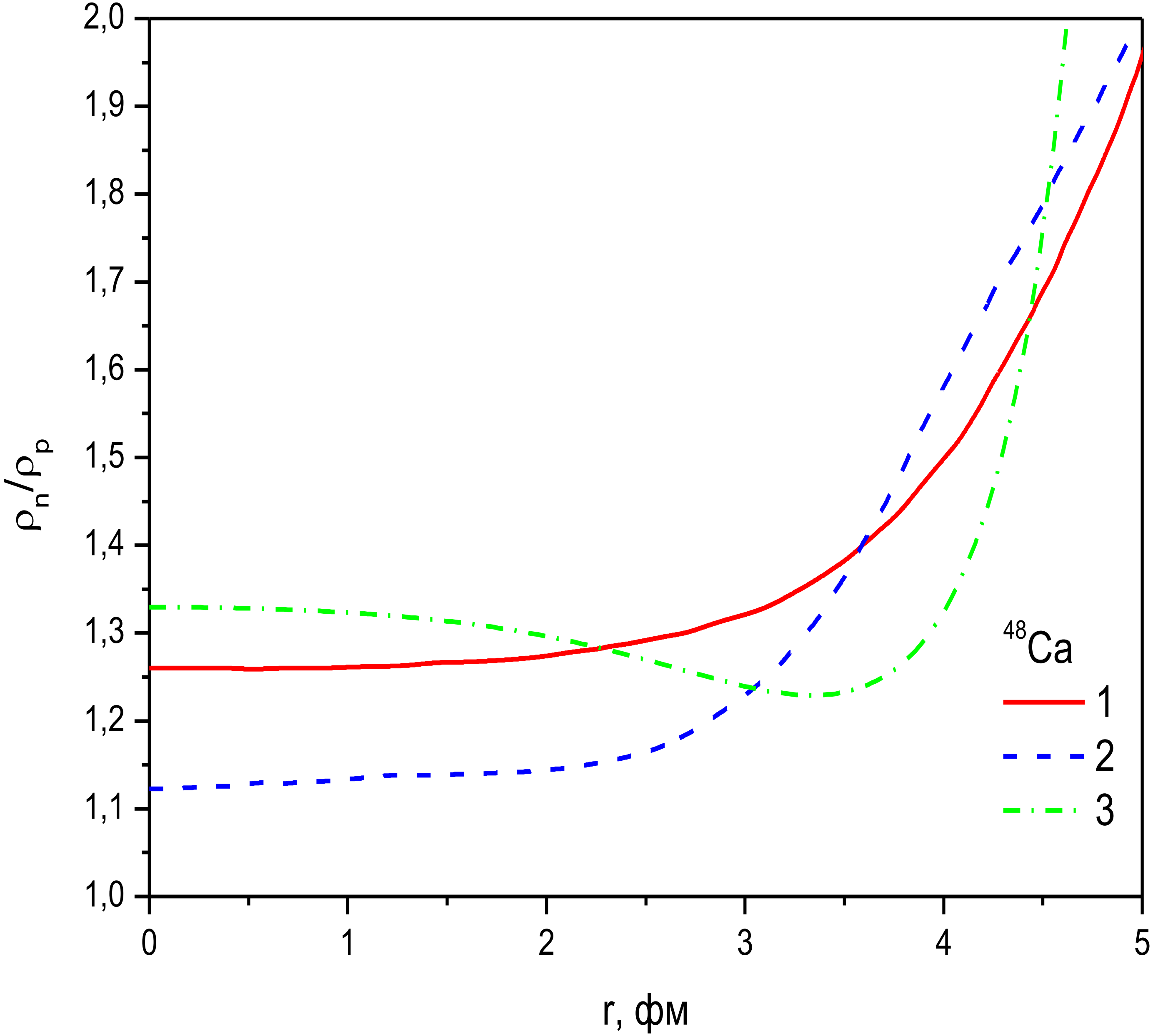
Total density distribution ^{48}Ca



Total density distribution ^{208}Pb

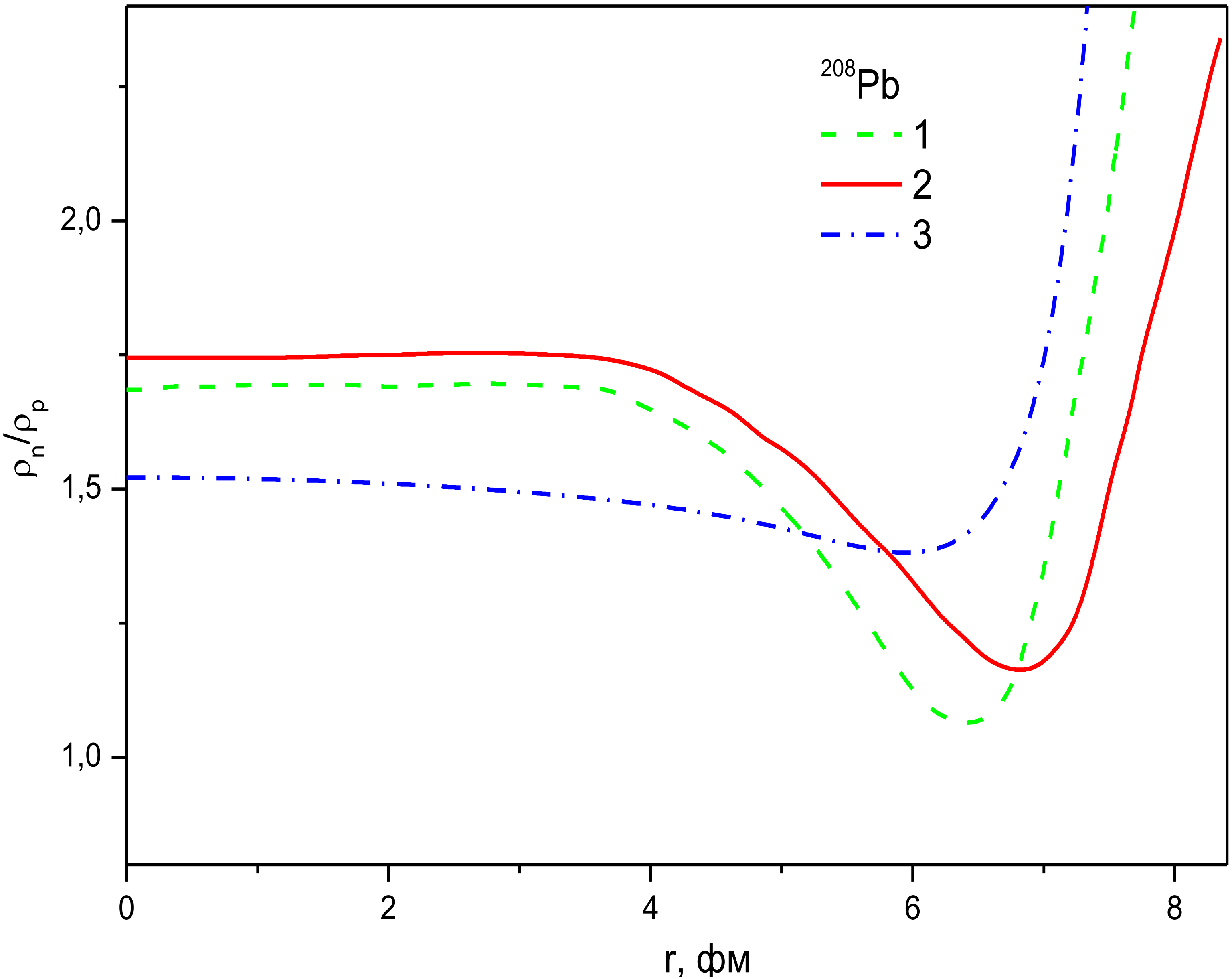


Neutron distribution

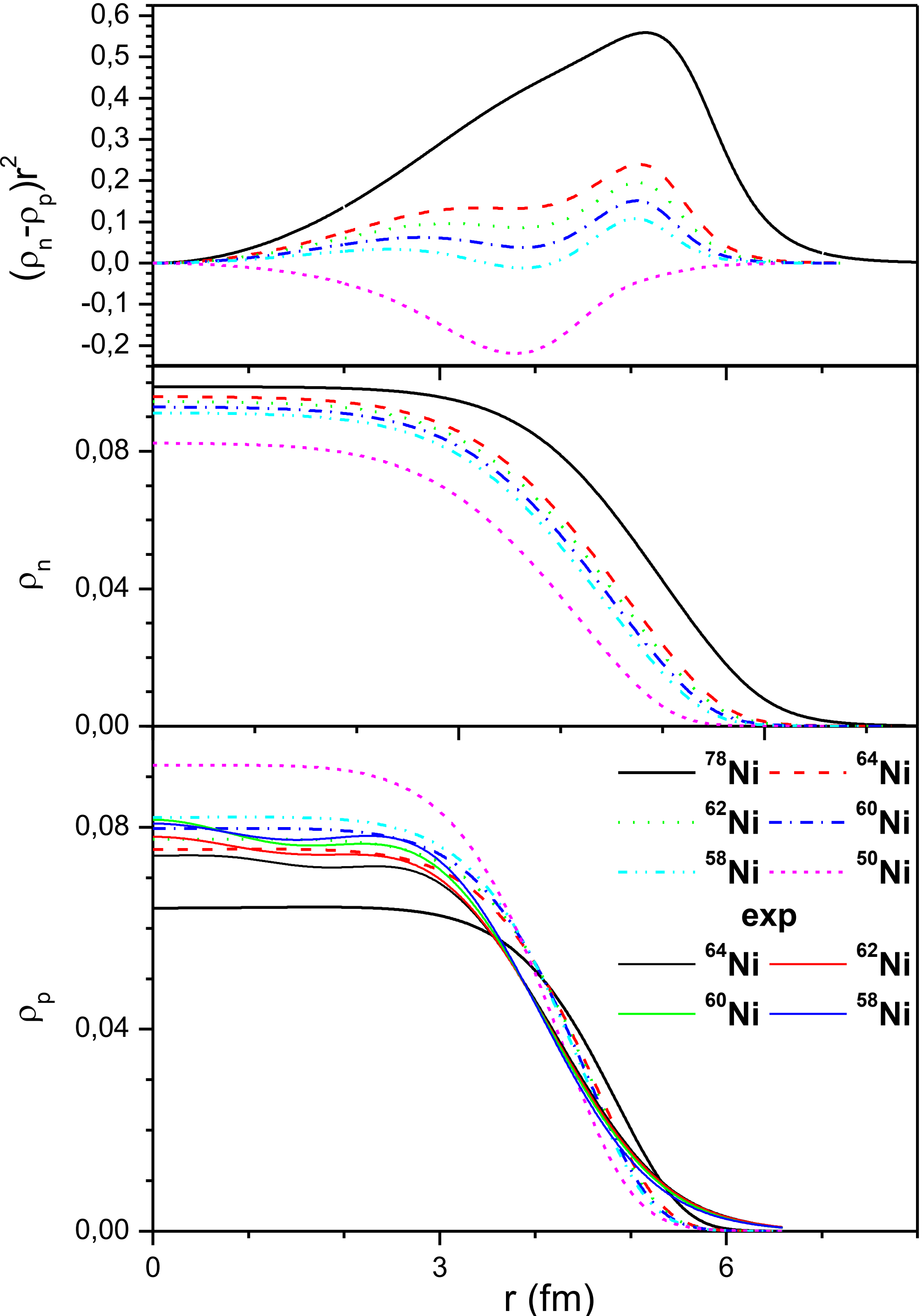


EXP
1-Gatchina/Sacle
2-Los Alamos
3-ETF

Neutron distribution



EXP
1-Gatchina/Sacle
2-Los Alamos
3-ETF

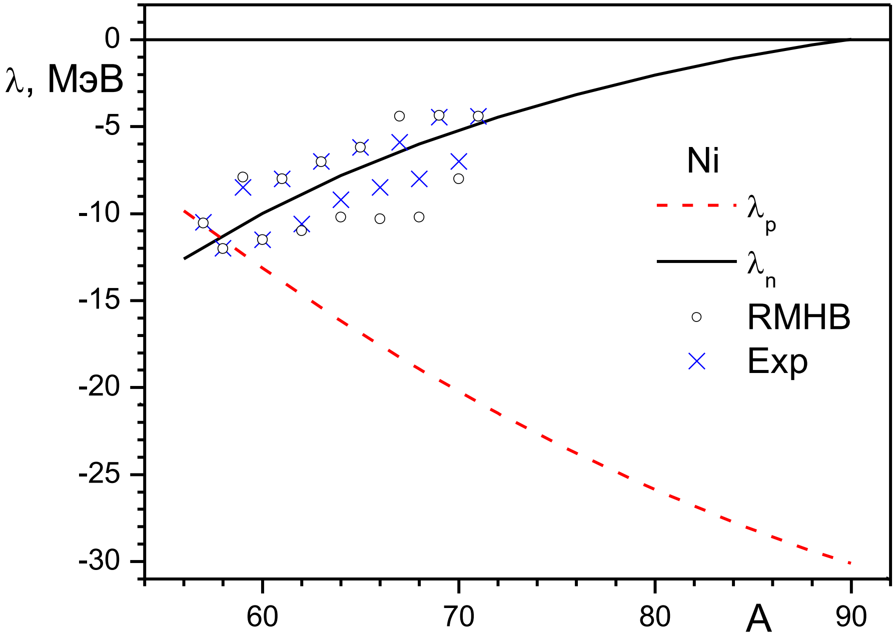


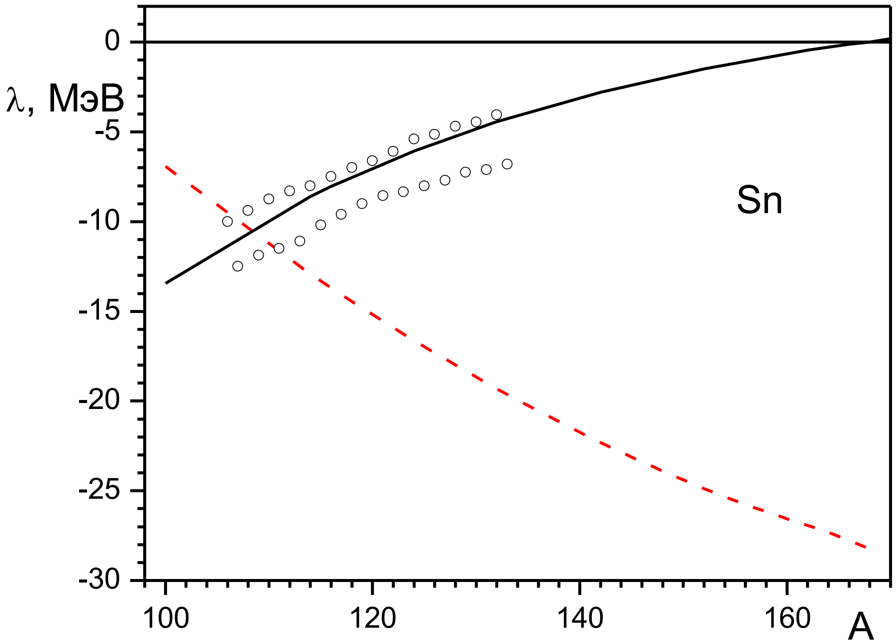
Нейтронно збагачені ядра та нейтронна кожа

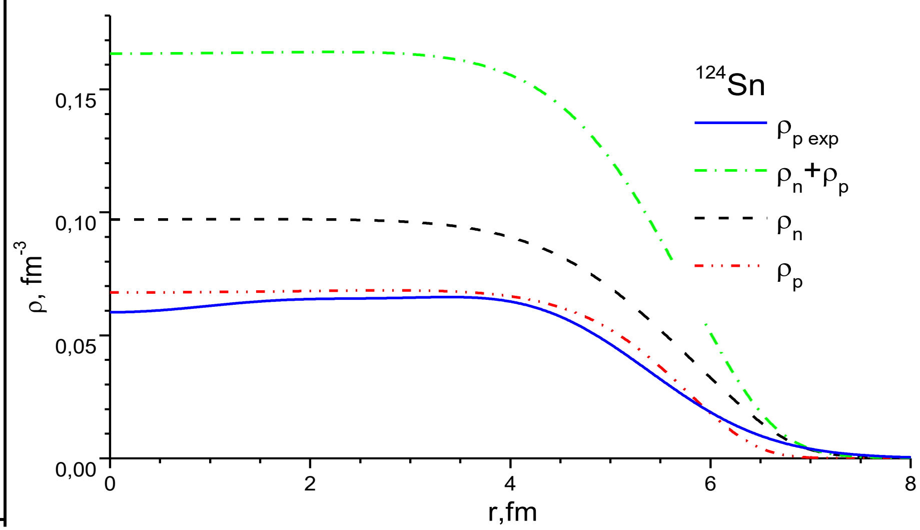
Neutron-reach and proton reach nuclei

Nucleus	E_{exp} [MeV]	E [MeV]	$\langle r^2_p \rangle^{1/2}$ [fm]	$\langle r^2_n \rangle^{1/2}$ [fm]	λ_n [MeV]	λ_p [MeV]
^{32}Ca	--	203.3	3.101	2.922	-22.491	-0.609
^{56}Ca	449.6	455.6	3.448	3.754	-2.443	-25.243
^{48}Ni	--	346.3	3.433	3.328	-19.707	-2.510
^{50}Ni	385.5	385.3	4.664	4.790	-17.643	-4.414
^{60}Ni	526.9	528.7	3.591	3.672	-10.012	-13.157
^{62}Ni	545.3	549.1	3.620	3.725	-8.879	-14.704
^{64}Ni	561.8	567.3	3.648	3.776	-7.845	-16.185
^{78}Ni	641.4	655.6	3.822	4.125	-3.157	-23.388
^{100}Sn	825.8	818.8	4.248	4.243	-13.38	-7.84
^{124}Sn	1049.4	1059.5	4.490	4.654	-5.699	-18.460
^{132}Sn	1102.7	1103.8	4.568	4.798	-3.98	-21.35
^{142}Sn	--	1142.6	4.658	4.964	-2.216	-24.581
^{152}Sn	--	1164.8	4.751	5.134	-0.806	-27.375

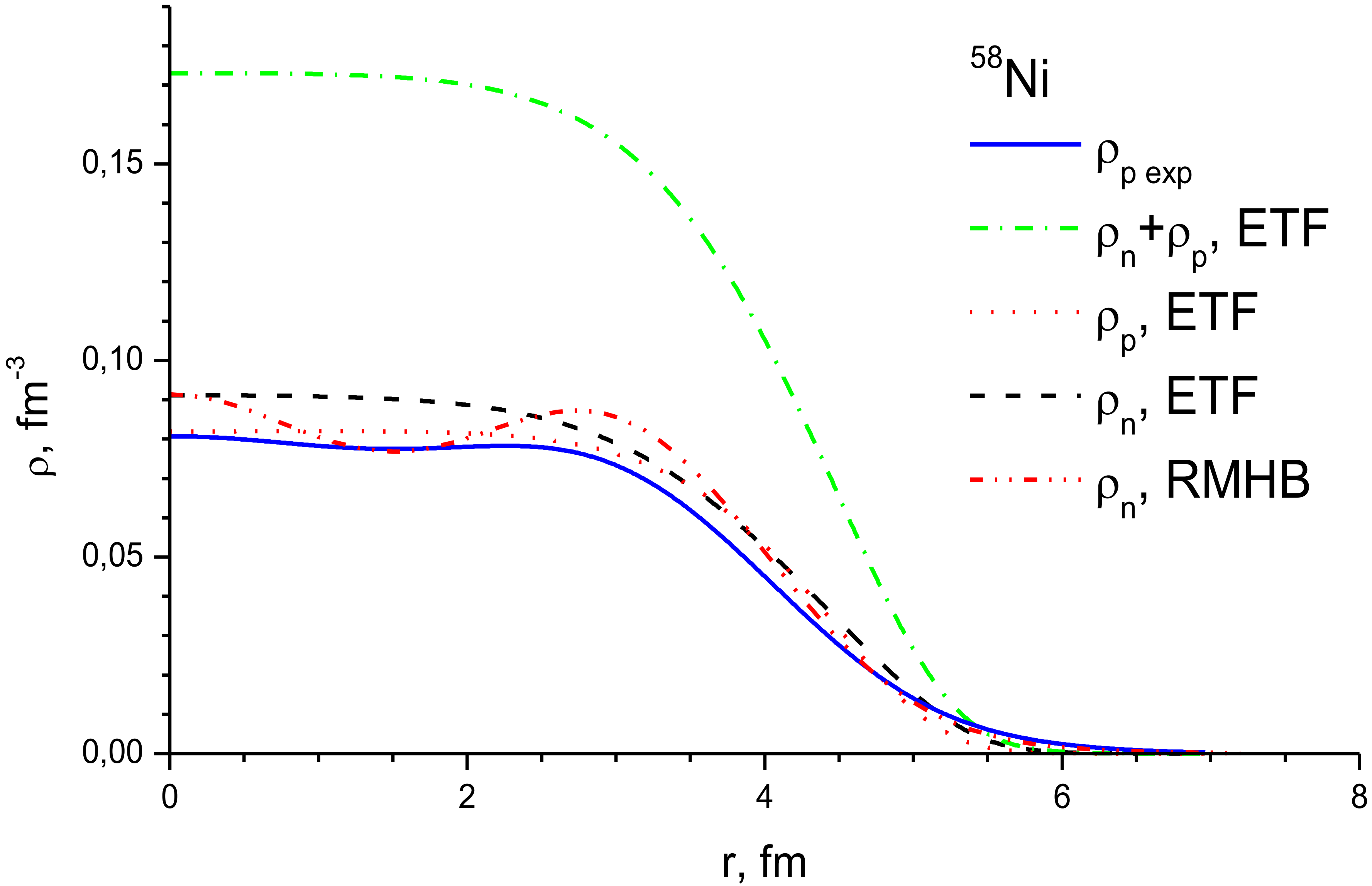
Chemical potential (Fermi energy) in **Ni** and **Sn** isotopes



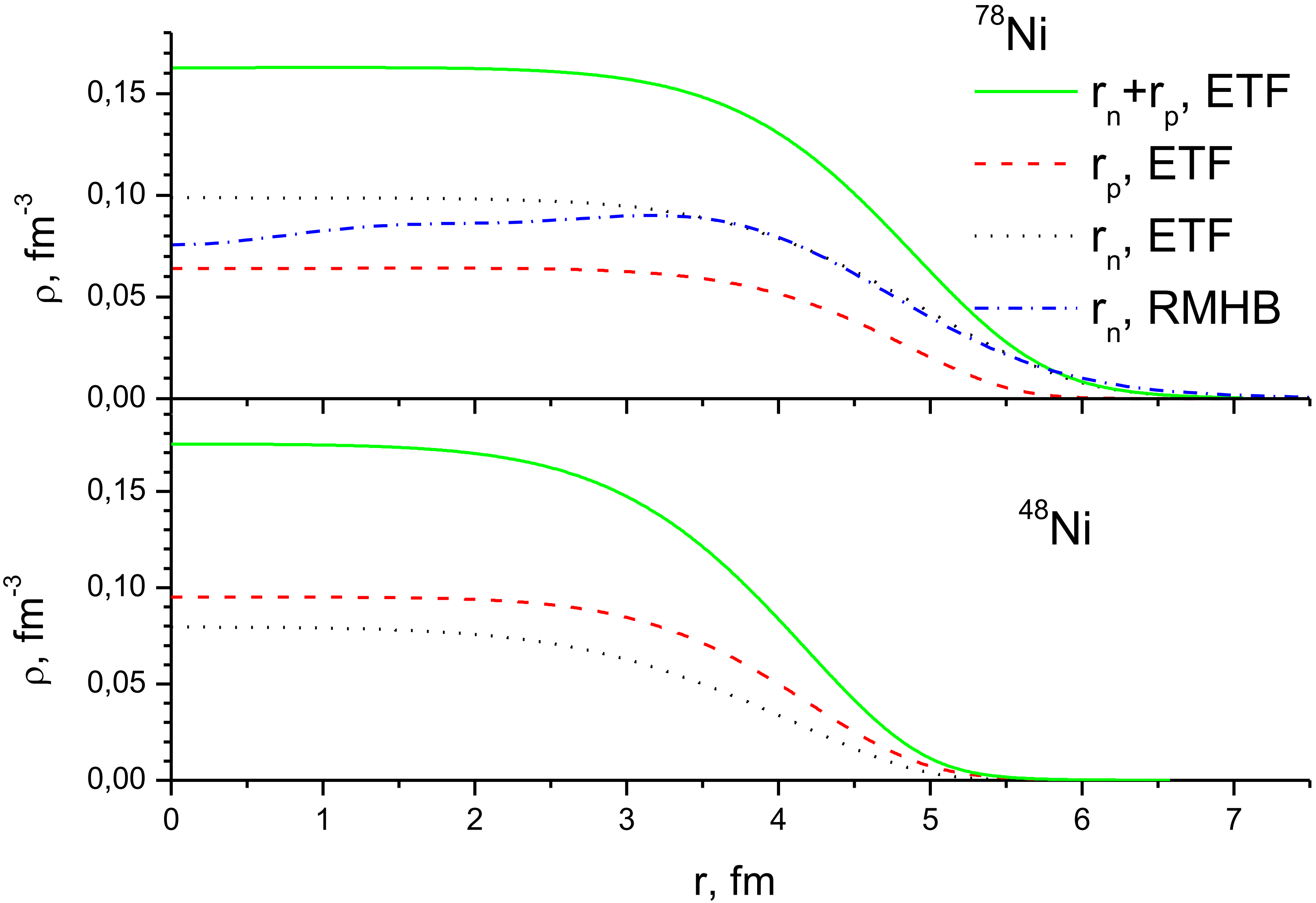




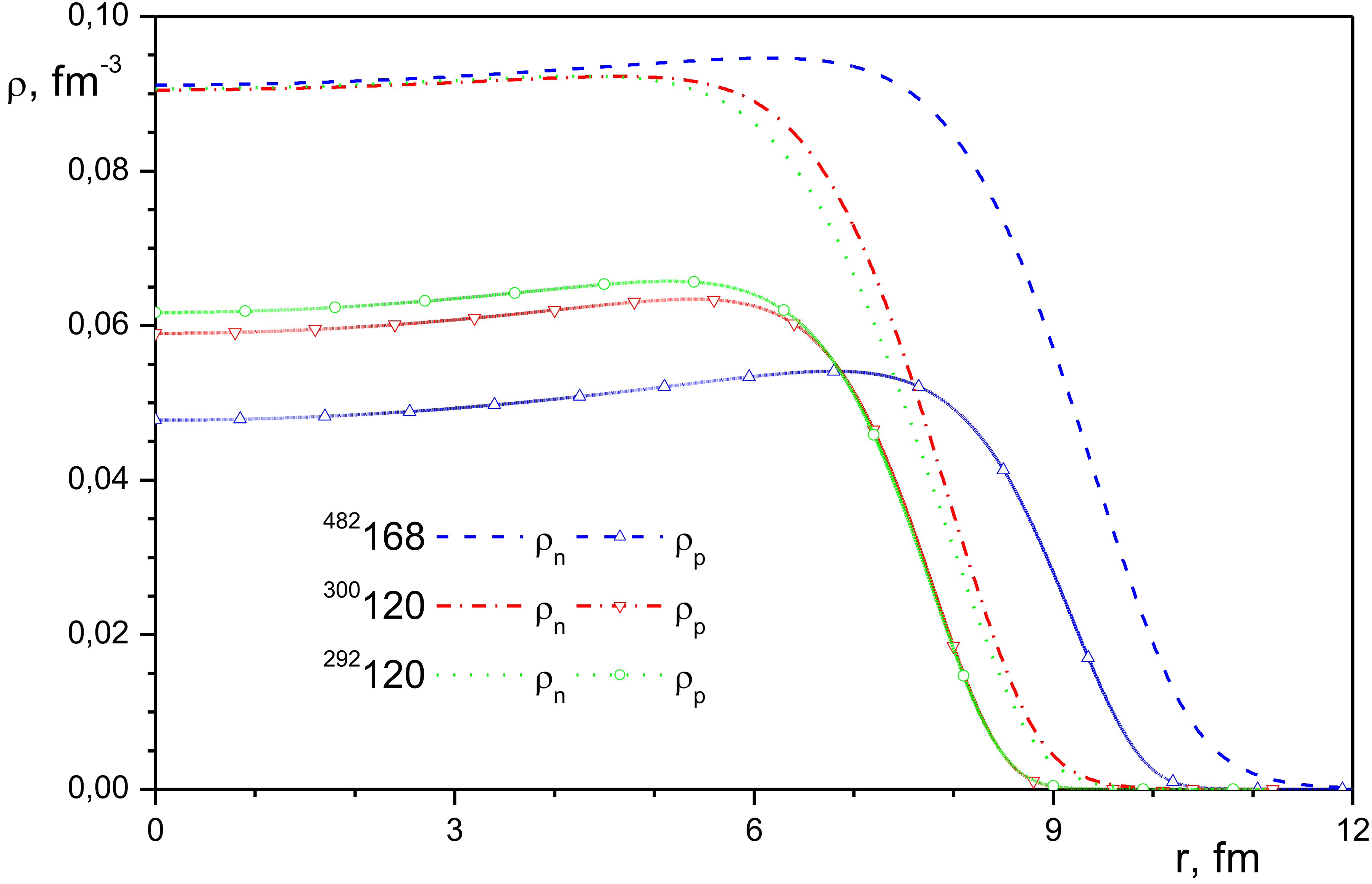
Density distribution in various approaches



Розподіли густини у протонно та нейтронно збагачених ядрах

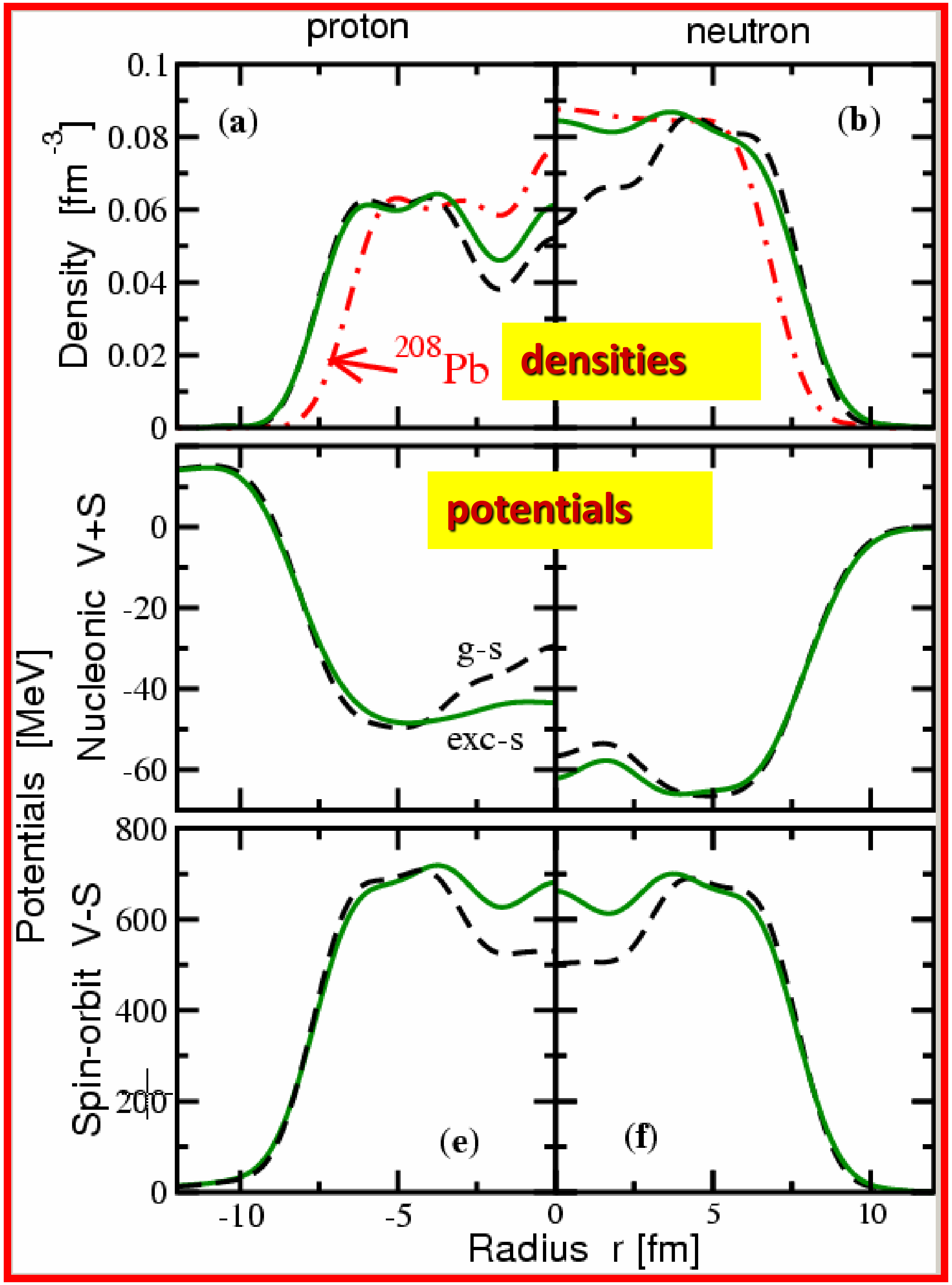


Density distribution in superheavy nuclei



Binding energy, root mean squared radii and chemical potentials for stable nuclei

Z	N	E [MeV]	E_{TF} [MeV]	$\langle r_p \rangle$ [fm]	$\langle r_n \rangle$ [fm]	λ_n [MeV]	λ_p [MeV]
114	182	2113.1	2099.83	6.019	6.198	-5.218	-15.604
118	182	2124.1	2109.91	6.045	6.204	-5.761	-14.318
120	182	2130.8	2112.70	6.061	6.210	-6.064	-13.711
126	182	2134.9	2112.32	6.109	6.230	-6.973	-11.934
126	184	2146.5	2127.50	6.122	6.248	-6.773	-12.253
164	272	2665.9	-	6.871	7.016	-4.336	-15.012
164	318	2848.4	-	7.084	7.316	-1.735	-19.920



1. Large density depression in the central part of nucleus: **shell gaps at Z=120, N=172**

2. Flat density distribution in the central part of nucleus: **Z=126 appears, N=184 becomes larger and Z=120 (N=172) shrink**

Skyrme SkP [m*/m=1]
double shell closure at Z=126, N=184
(SkM*, **????**

Skyrme SkI3 [m*/m=0.57]
gaps at Z=120, N=184
no double shell closure,
SLy6

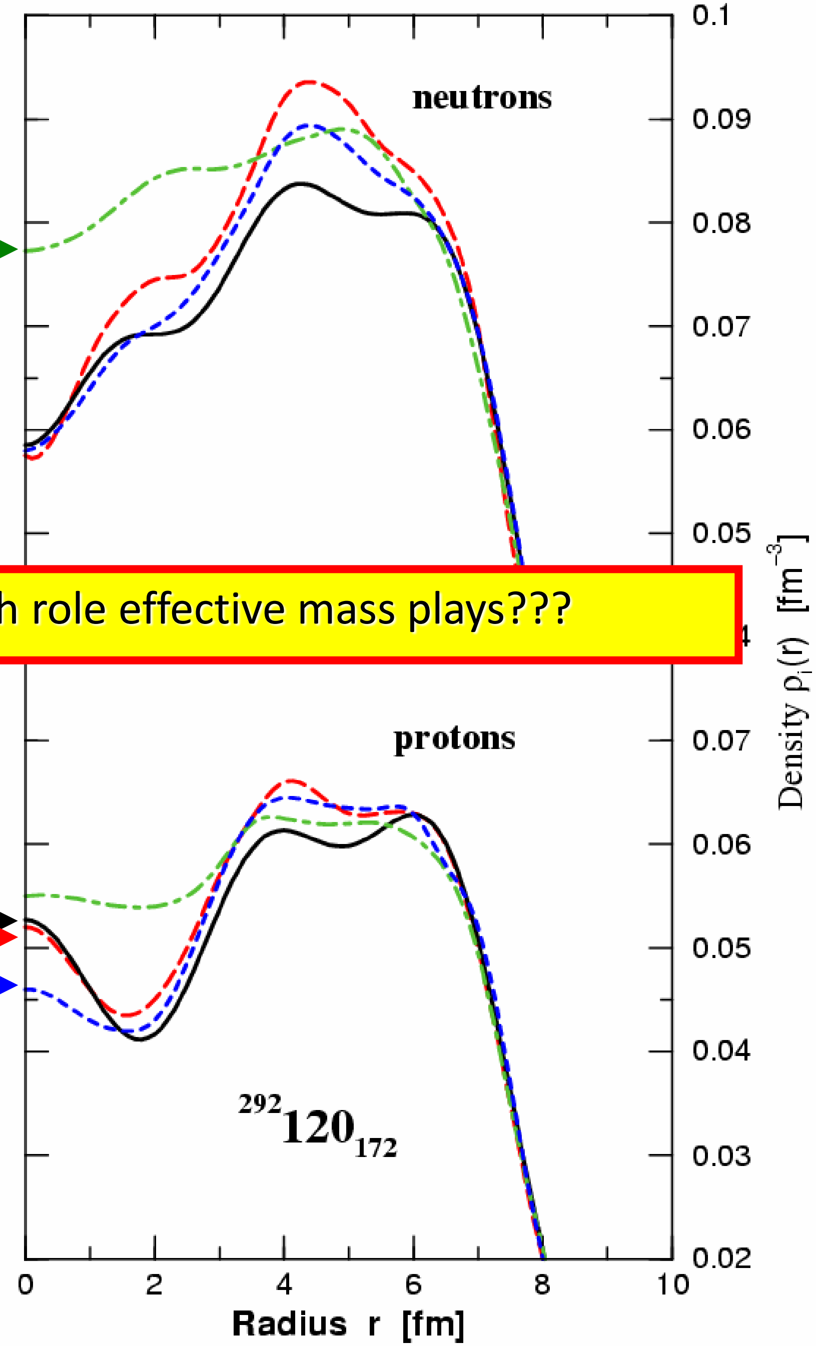
Gogny D1S
Z=120, N=172(?)
Z=126, N=184

RMF
double shell closure at Z=120,N=172

Large effective mass m*/m~0.8-1.0

Low effective mass m*/m ~ 0.65

Which role effective mass plays???



Density distributions in nuclei and properties of nucleus-nucleus potentials

$$V(R) = V_{coul}(R) + V_N(R) + V_{rot}(R),$$

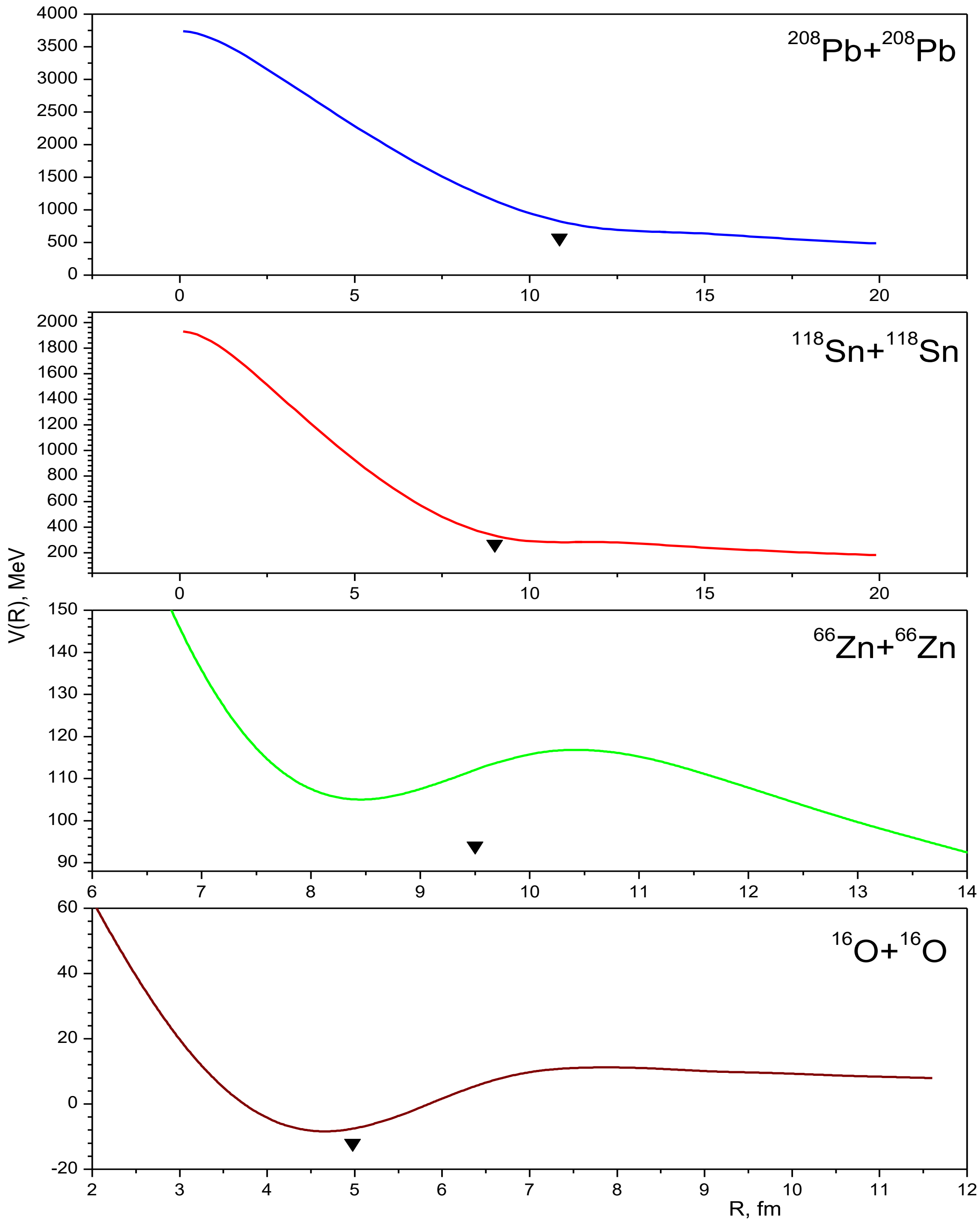
$$V_{rot}(R) = \frac{\hbar^2 L(L+1)}{2\mu R^2}$$

$$V(R) = E_{12}(R) - (E_1 + E_2)$$

$$E_{12} = \int \varepsilon \left[\rho_{1p}(\vec{r}) + \rho_{2p}(\vec{r}, R),\ \ \rho_{1n}(\vec{r}) + \rho_{2n}(\vec{r}, R)\right] d\vec{r}$$

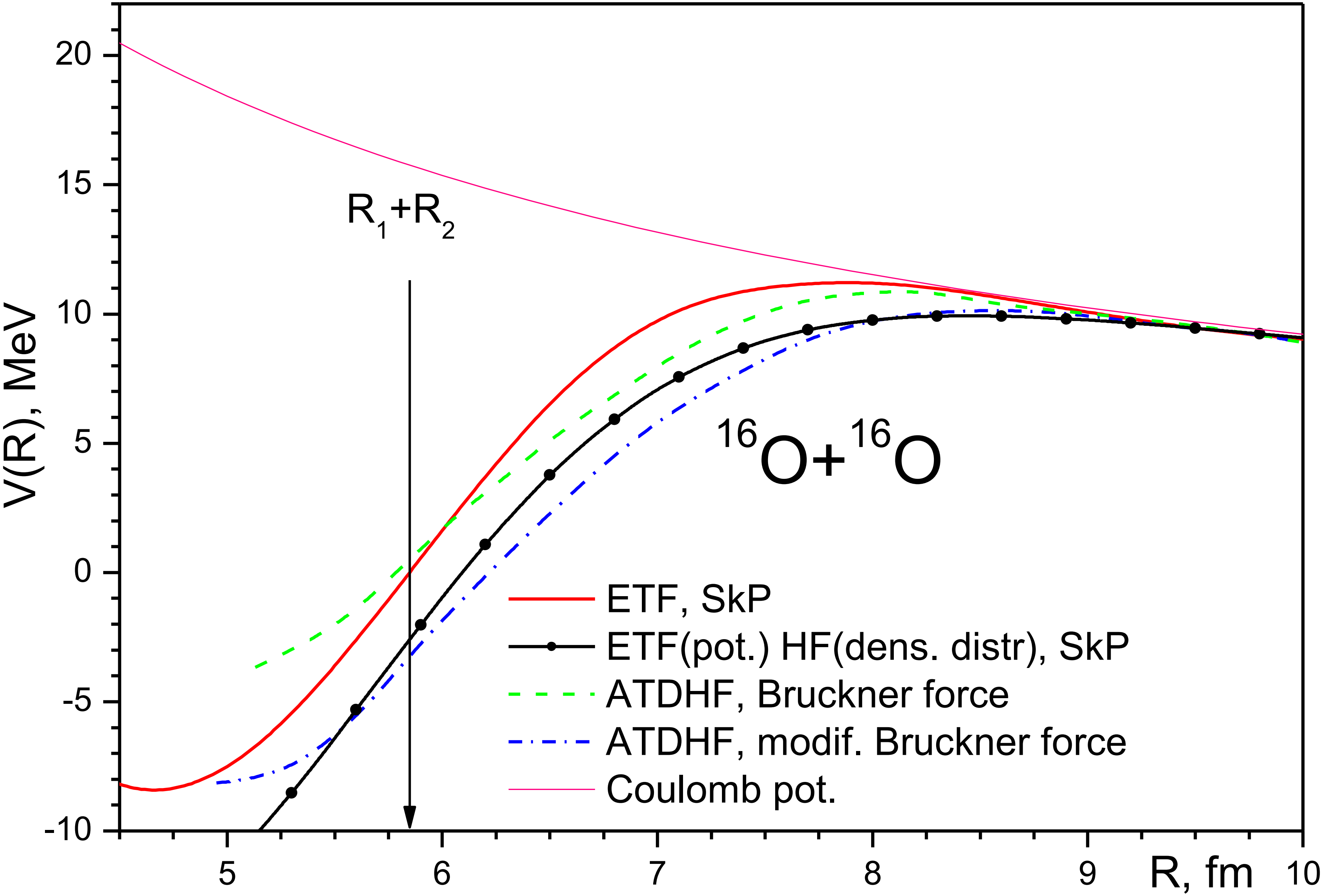
$$E_1 = \int \varepsilon \left[\rho_{1p}(\vec{r}),\ \ \rho_{1n}(\vec{r})\right] d\vec{r}$$

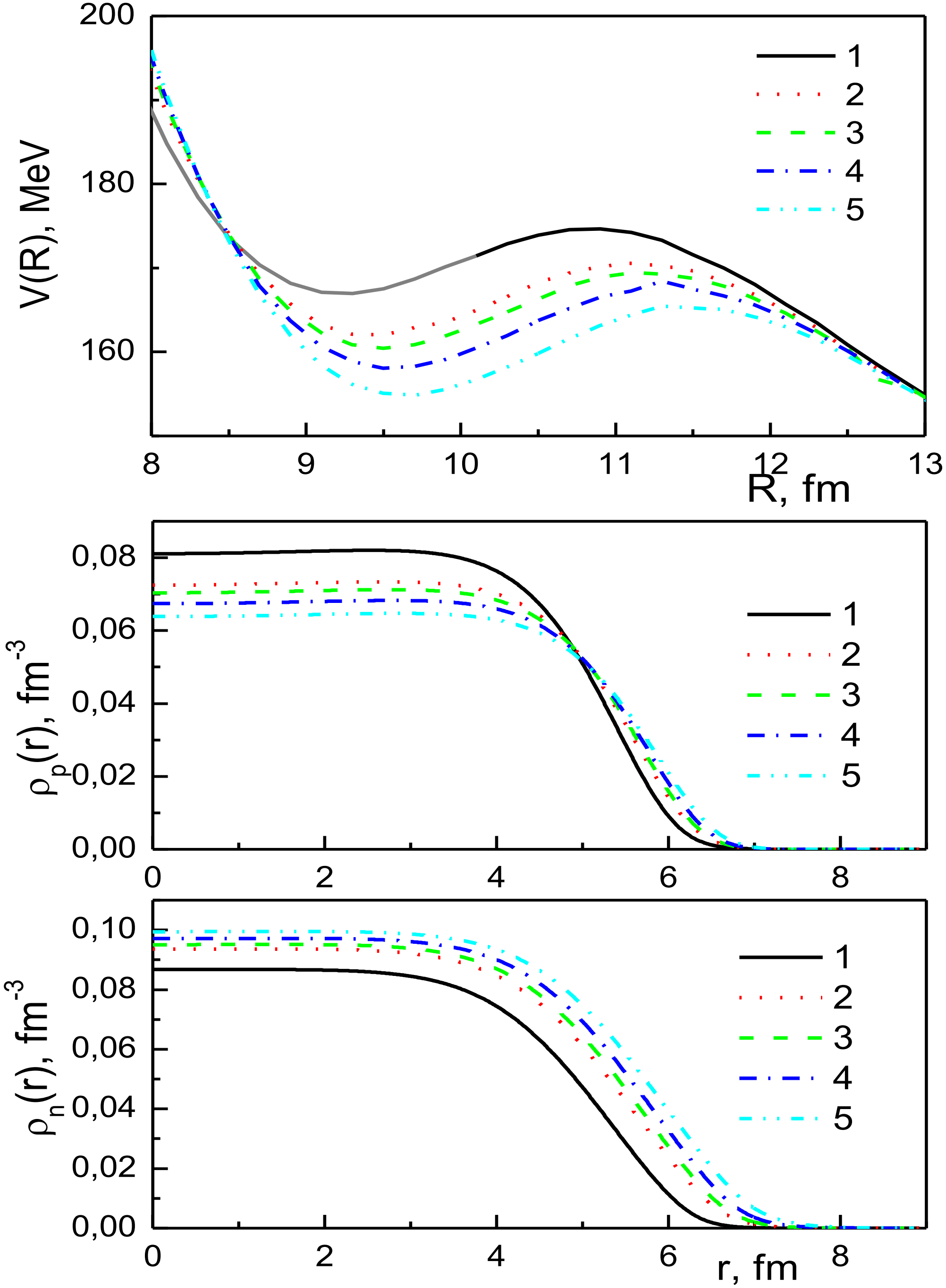
$$E_2 = \int \varepsilon \left[\rho_{2p}(\vec{r}),\ \ \rho_{2n}(\vec{r})\right] d\vec{r}$$



^{208}Pb+^{208}Pb
^{118}Sn+^{118}Sn
^{66}Zn+^{66}Zn
^{16}O+^{16}O
V(R), MeV
R, fm

Potentials in various approaches





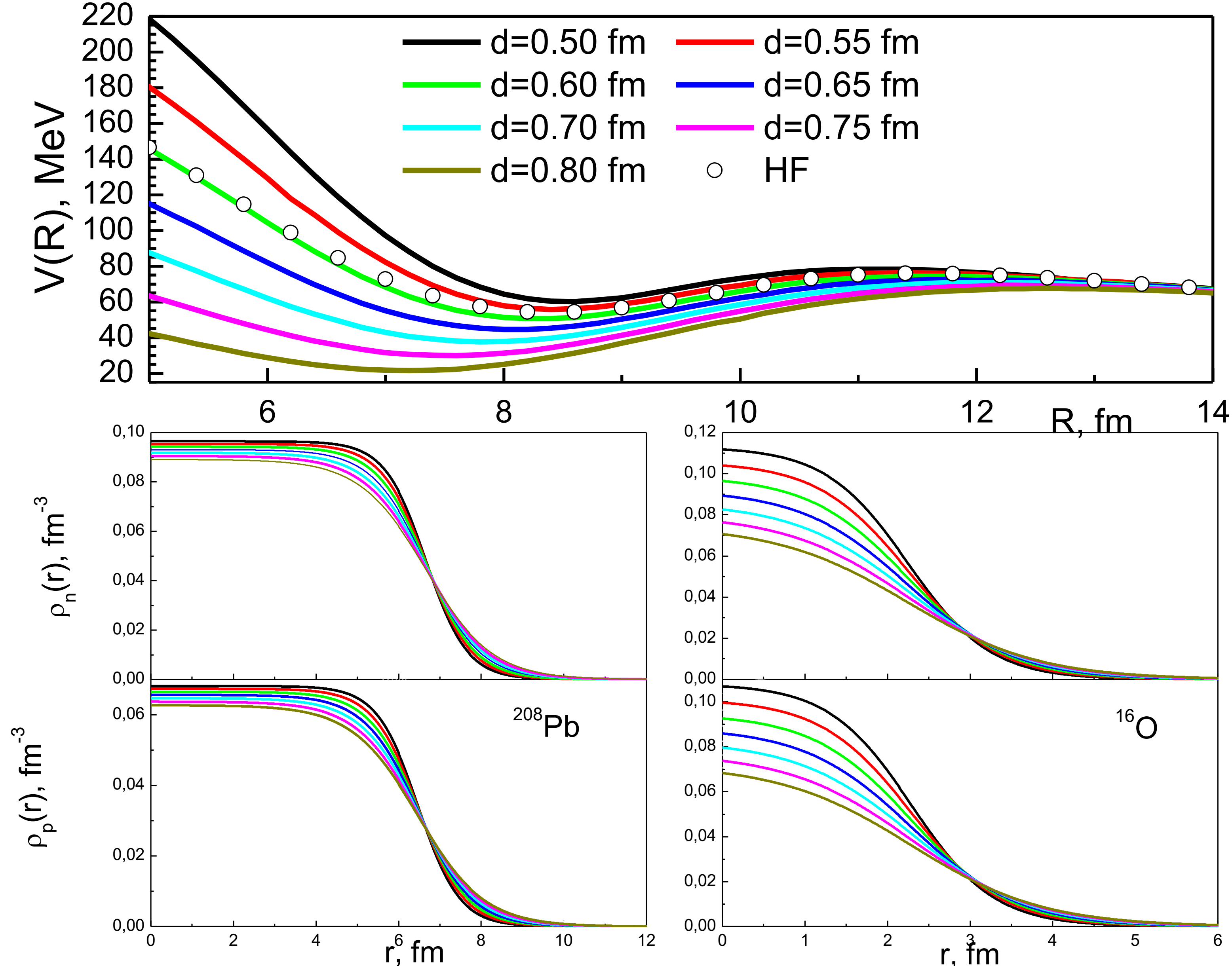
Potentials for
^{64}Ni + ^{100}Sn (1),
+^{114}Sn (2),
+^{118}Sn (3),
+^{124}Sn (4),
+ ^{132}Sn (5)

and

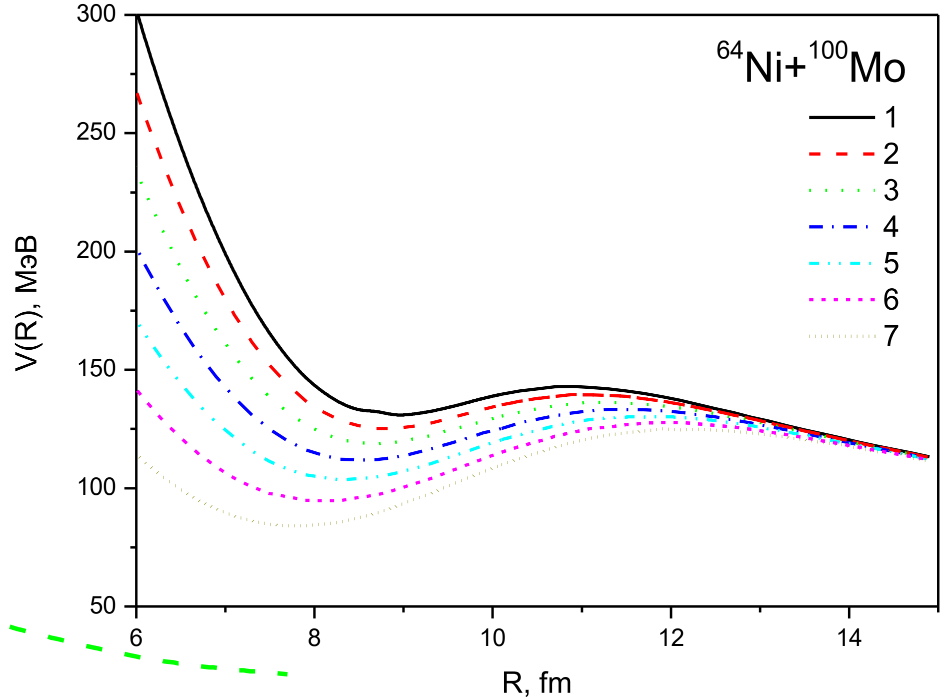
density distributions for
^{100}Sn (1),
^{114}Sn (2),
^{118}Sn (3),
^{124}Sn (4),
^{132}Sn (5).

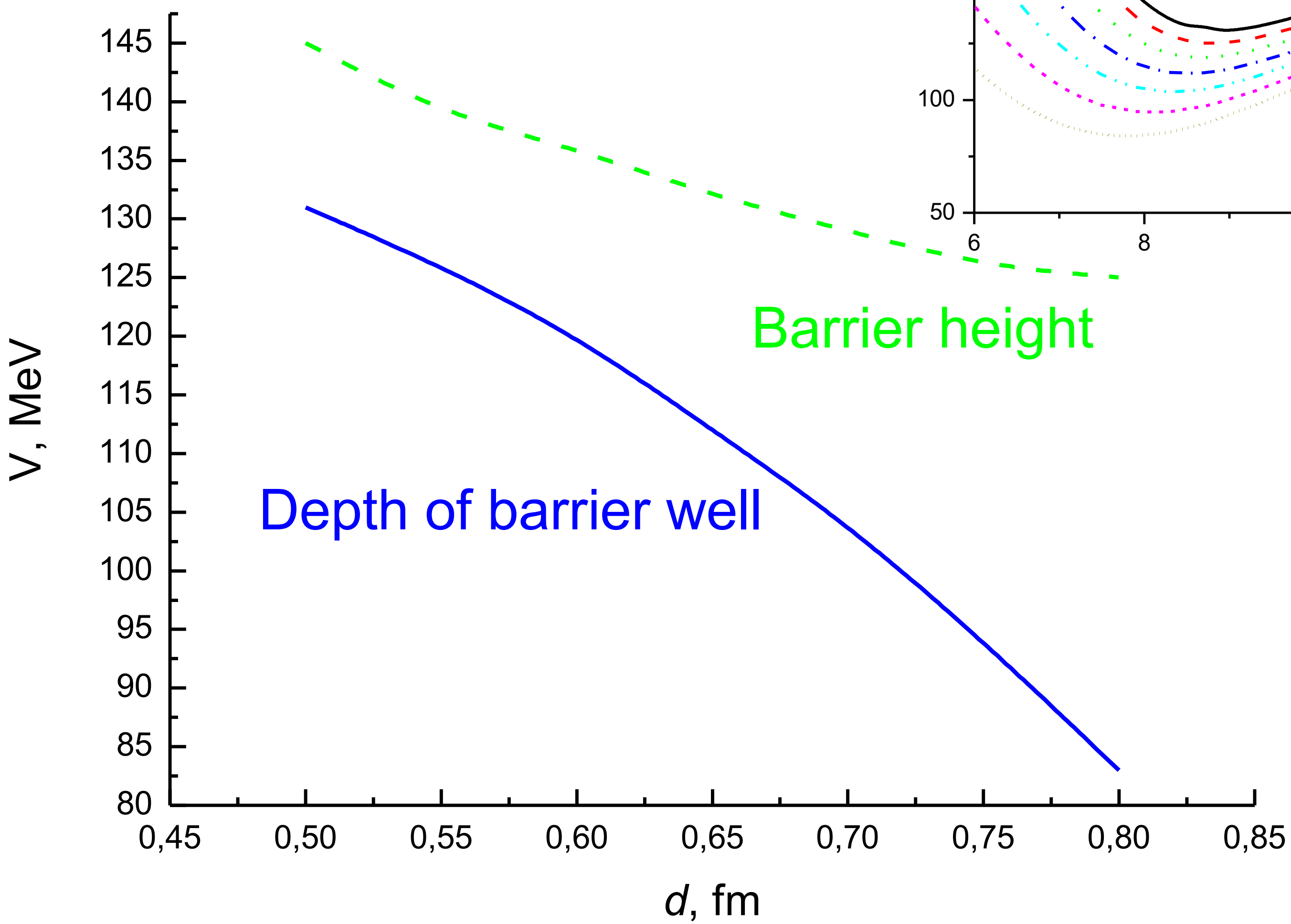
Density distributions and potential

$$\rho_{n(p)}(r) = \rho_{0n(p)} / \{1 + \exp[(r - R_{n(p)}) / d\}$$



Dependences of barrier height and of depth of barrier well on diffuseness

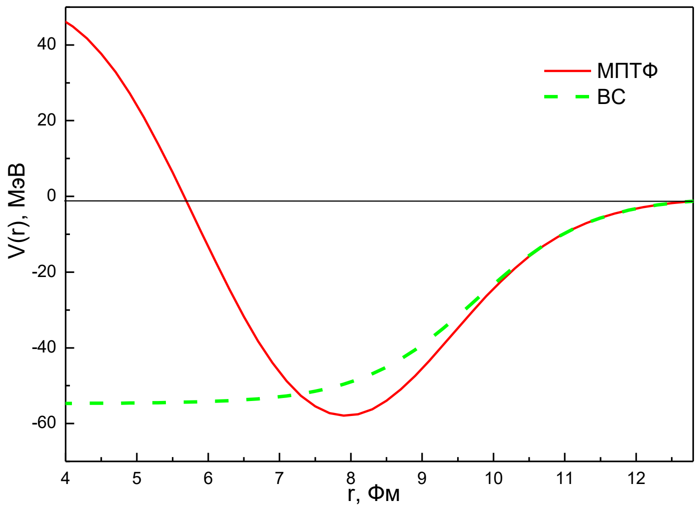


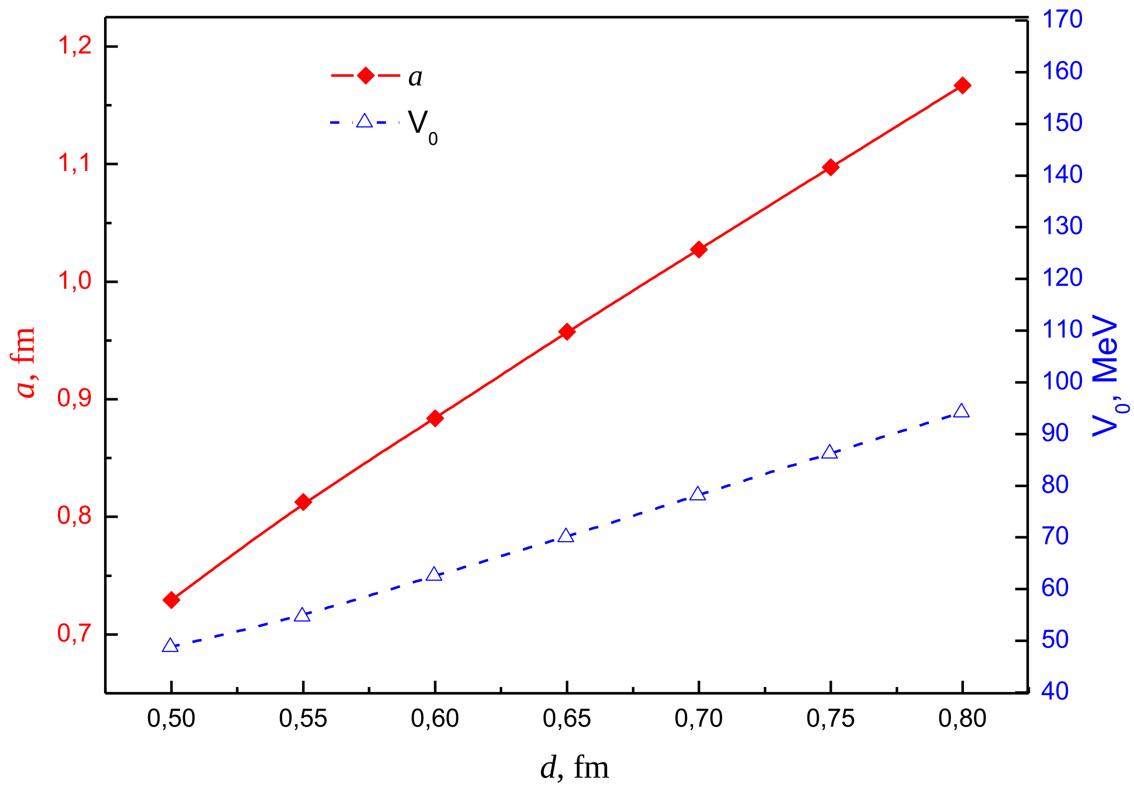


The ETF potential is evaluated for densities with various values of diffuseness *d*

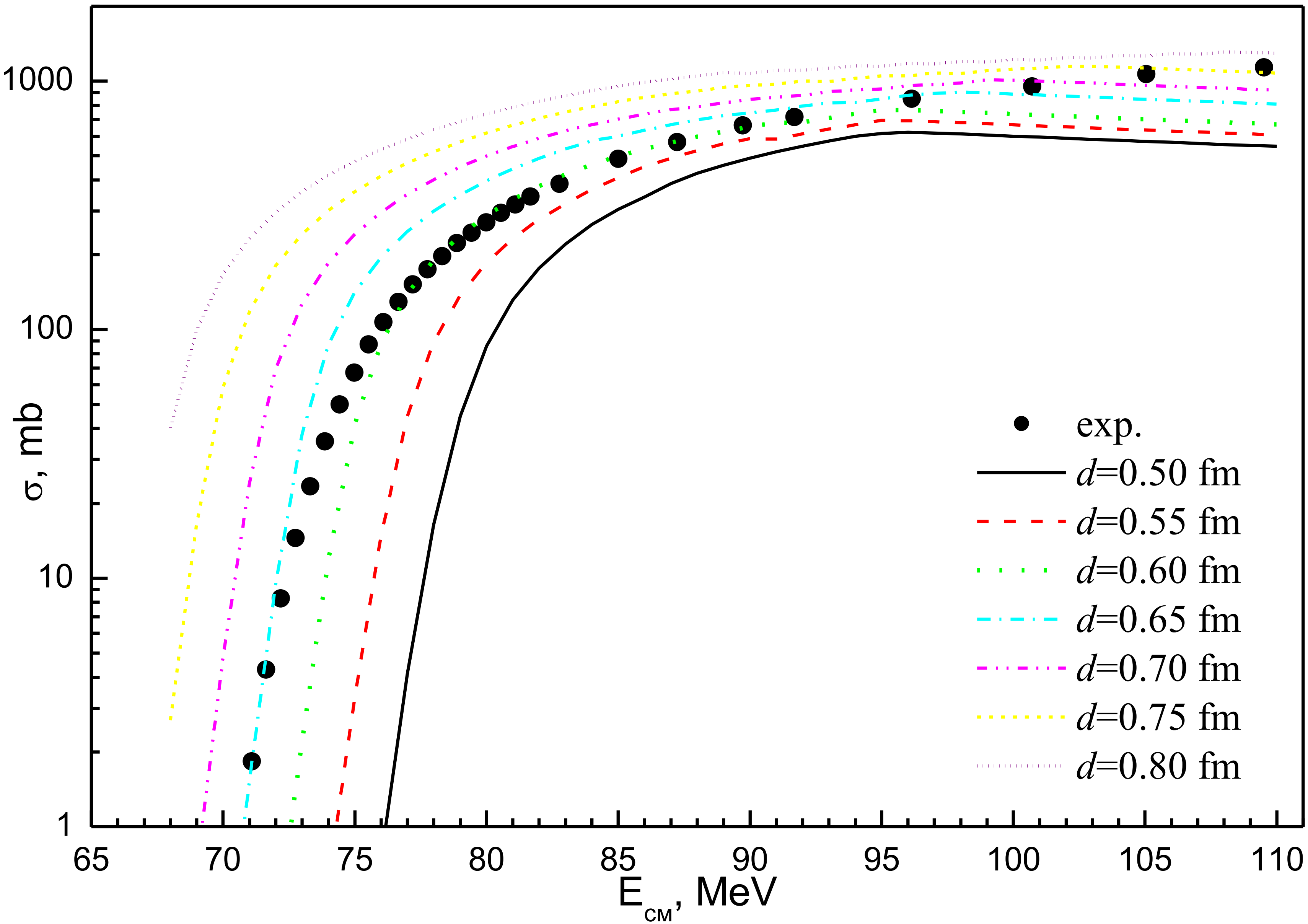
$$\rho_{n(p)}(r) = \rho_{0n(p)} / \{1 + \exp[(r - R_{n(p)}) / d\}$$

Obtained ETF potentials are approximated by
$V(R) = -V_0/\{1+exp[(R-R_{pot})/a]\}$ at large distances





$^{16}O+^{208}Pb$ fusion cross sections



Висновки

ETF із Skyrme EDF - це прості та потужні інструменти для вивчення розподілів нуклонів

Енергії зв'язування добре описані в ETF разом із Skyrme EDF.

Ядерно-ядерний потенціал можна оцінити в рамках ETF за допомогою Skyrme EDF.

Ядерно-ядерний потенціал залежить від розподілу густини в ядрах і параметра дифузності.

Поперечний переріз злиття ядер при підбар'єрних енергіях залежить від розподілу густини в ядрах і параметра дифузії.

Thanks for attention!!!