ПОВЕРХНЕВІ КОЛИВАННЯ, ГІГАНТСЬКІ МУЛЬТИПОЛЬНІ РЕЗОНАНСИ

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План

1. Вступ
2. Поверхневі вібрації
3. Ізоскалярний ГМР
4. Ізовекторний ГМР
5. Висновки

Вібрації в ядрах

Поверхневі осциляції мультипольности ℓ ($\ell \neq 0$):

$$R(t)=R_0[1+\beta_\ell(t)\, Y_{\ell 0}(\theta)]$$

$$\beta_\ell(t)= \beta_\ell \cos(\omega t)$$

Об'єм, який займають нуклоні в ядрі, постійний:

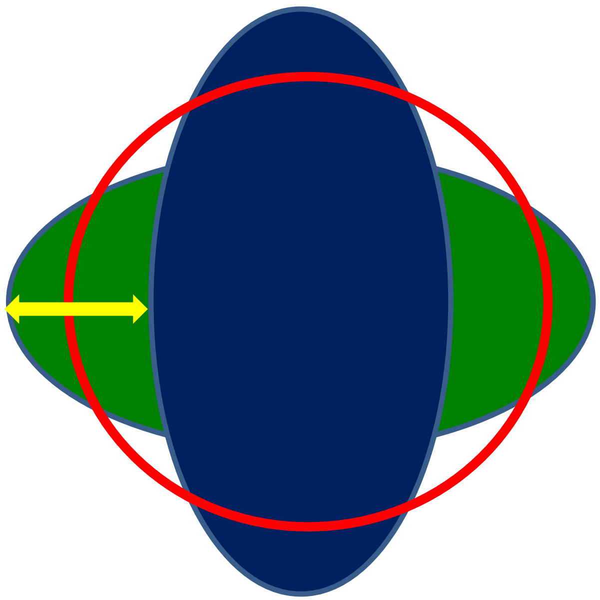
V_p=const, V_n=const;

Густини протонів або нейтронів постійні в будь-якій точці ядерного об'єму :

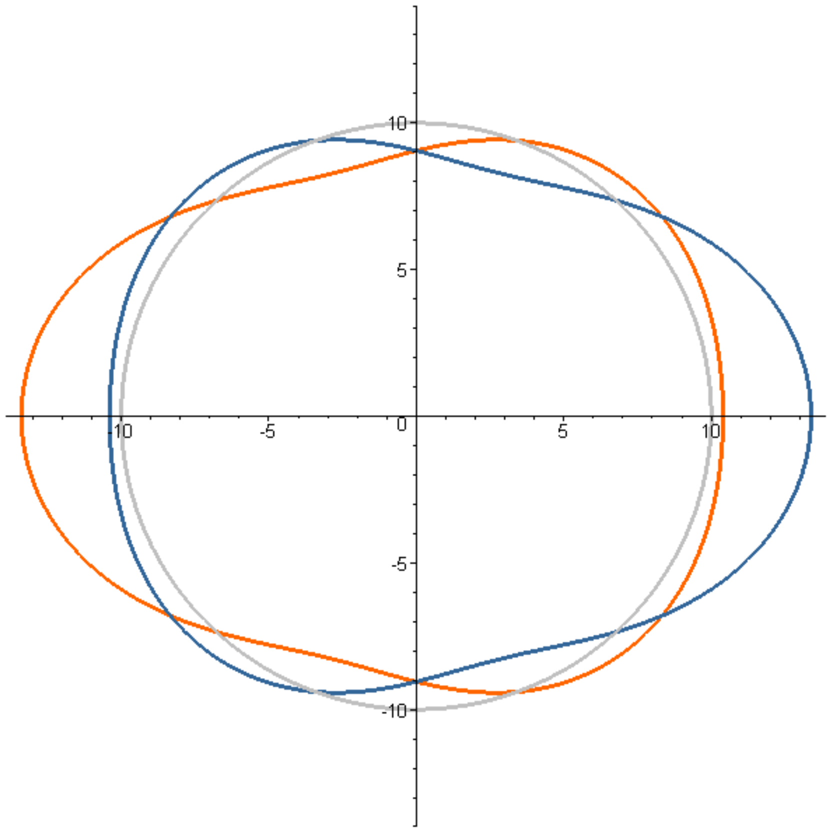
ρ_p=const, ρ_n=const.

Поверхневі коливання - це низькоенергетичні вібраційні стани

ℓ=2: E~1-4 MeV



ℓ=3: E~2-6 MeV



Поверхневі коливання

- $E_\ell = \hbar\omega_\ell$
- $\omega_\ell = (C_\ell / B_\ell)^{1/2}$
- $C_\ell = 1/(4\pi)[b_{surf}(\ell-1)(\ell+2)A^{2/3}$
  $-6e^2(\ell-1)/(r_0(2\ell+1))Z^2A^{-1/3}]$
- b_{surf}=16—21 MeV;
- e^2/r_0 =1.2MeV
- $D_\ell = 3Mr_0{}^2A^{5/3}/(4\pi\ell)$
- $E=T+V=(1/2)\Sigma_{\ell\mu} D_\ell\, |d/dt(\alpha_{\ell\mu})|^2$
  $+(1/2)\Sigma_{\ell\mu} C_\ell\, |\alpha_{\ell\mu}|^2$

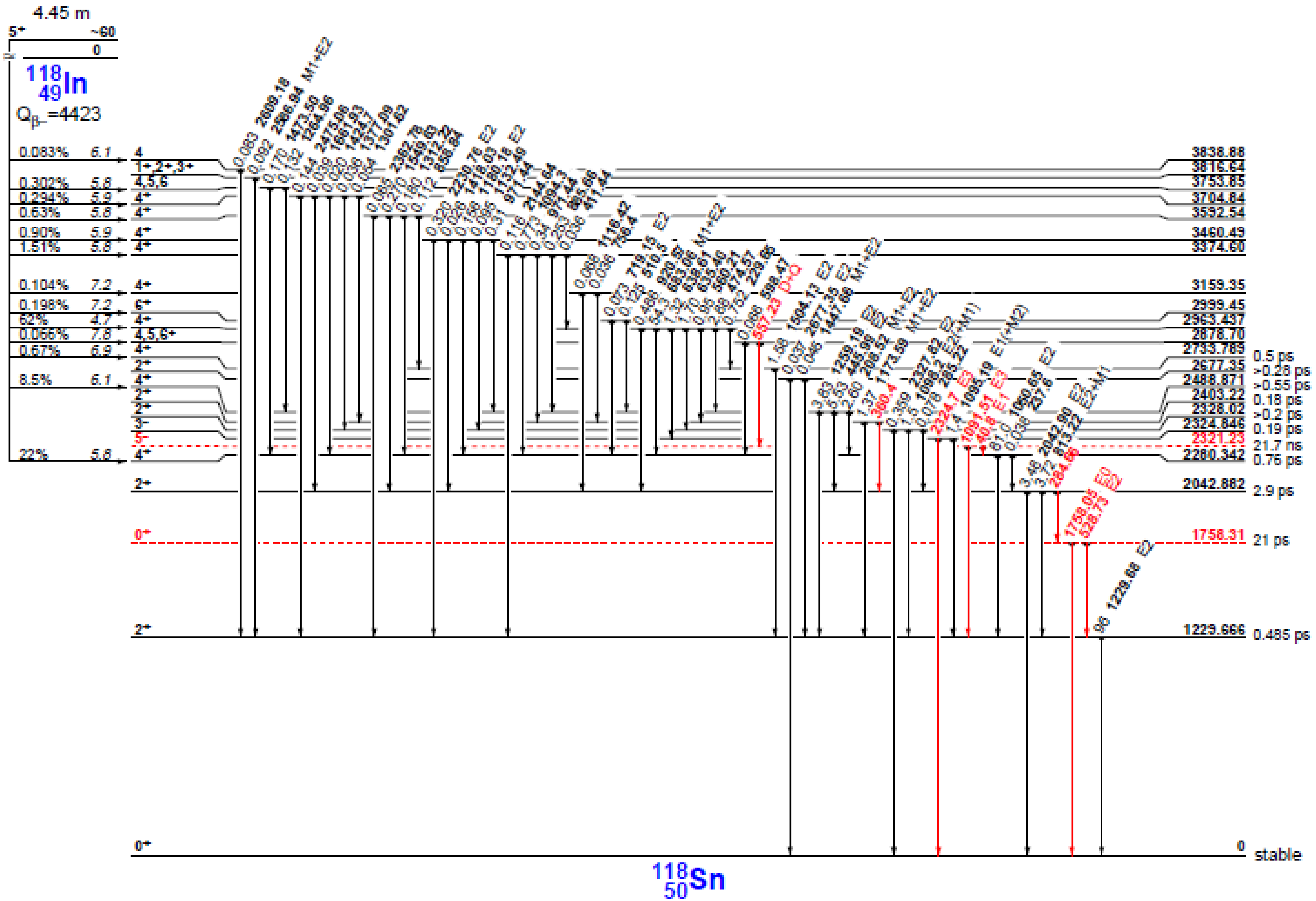

4.45 m
5+
~60
0
118 49 In
Qβ-=4423
0.083% 6.1
0.302% 5.8
0.294% 5.9
0.63% 5.8
0.90% 5.9
1.51% 5.8
0.104% 7.2
0.198% 7.2
62% 4.7
0.066% 7.8
0.67% 6.9
8.5% 6.1
22% 5.8
3838.88
3816.64
3753.85
3704.84
3592.54
3460.49
3374.60
3159.35
2999.45
2963.437
2878.70
2733.789
2677.35
2488.871
2403.22
2328.02
2324.846
2321.23
2280.342
2042.882
1758.31
1229.666
0
0.5 ps
>0.28 ps
>0.55 ps
0.18 ps
>0.2 ps
0.19 ps
21.7 ns
0.76 ps
2.9 ps
21 ps
0.485 ps
stable
118 50 Sn

FIGURE I. Summary of Various Adopted Quantities as a Function of Mass Number A

See page 13 for Explanation of Figures

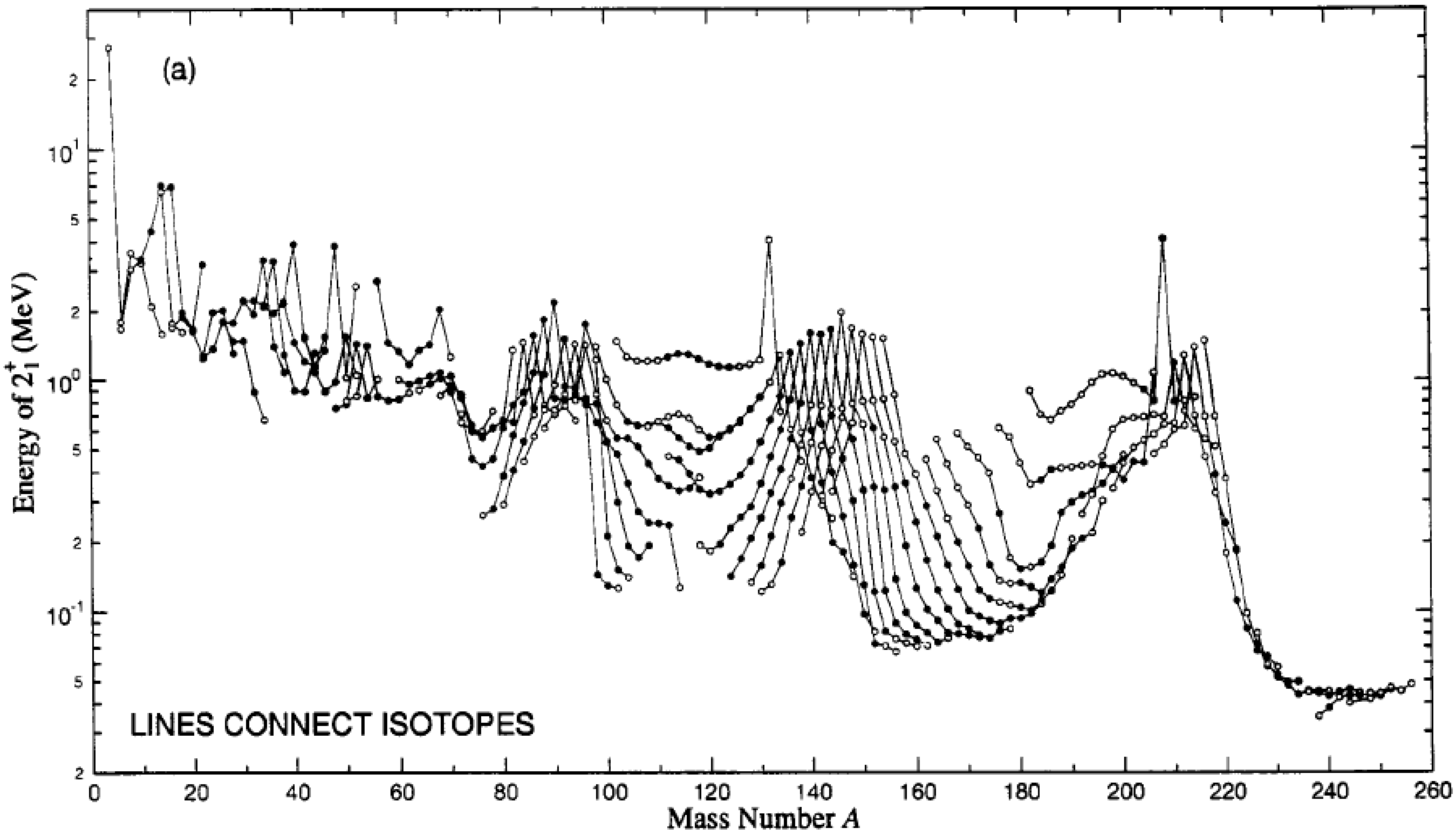
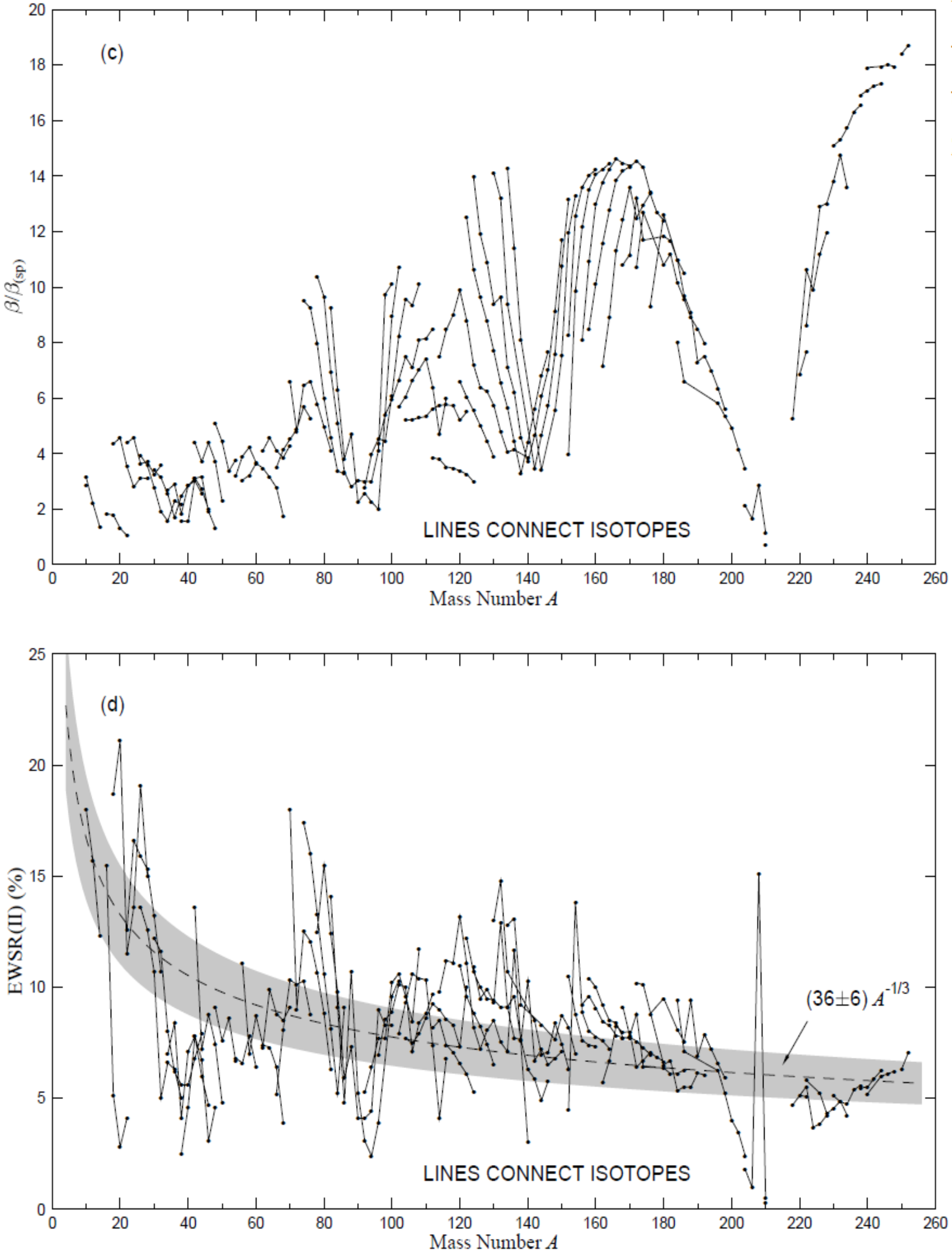


FIGURE I. Summary of Various Adopted Quantities as a Function of Mass Number A

See page 13 for Explanation of Figures



The $B(E2)\uparrow$ values are basic experimental quantities that do not depend on nuclear models. A quantity that, though model dependent, is quite useful because of its easy visualization is the deformation parameter, β. Assuming a uniform charge distribution out to the distance $R(\theta, \phi)$ and zero charge beyond, β is related to $B(E2)\uparrow$ by the formula

$$\beta = \left(4\pi/3ZR_0^2\right)[B(E2)\uparrow/e^2]^{1/2}. \quad (2)$$

as an indication of collective quadrupole motion in nuclei, we have calculated the ratio $\beta/\beta_{(sp)}$. The quantity $\beta_{(sp)}$ is assumed to be $1.59/Z$, which follows from using the single-particle value $B(E2)\uparrow_{(sp)}$ in Eq. (2). This (Weisskopf) single-particle $B(E2)\uparrow$ value is given by

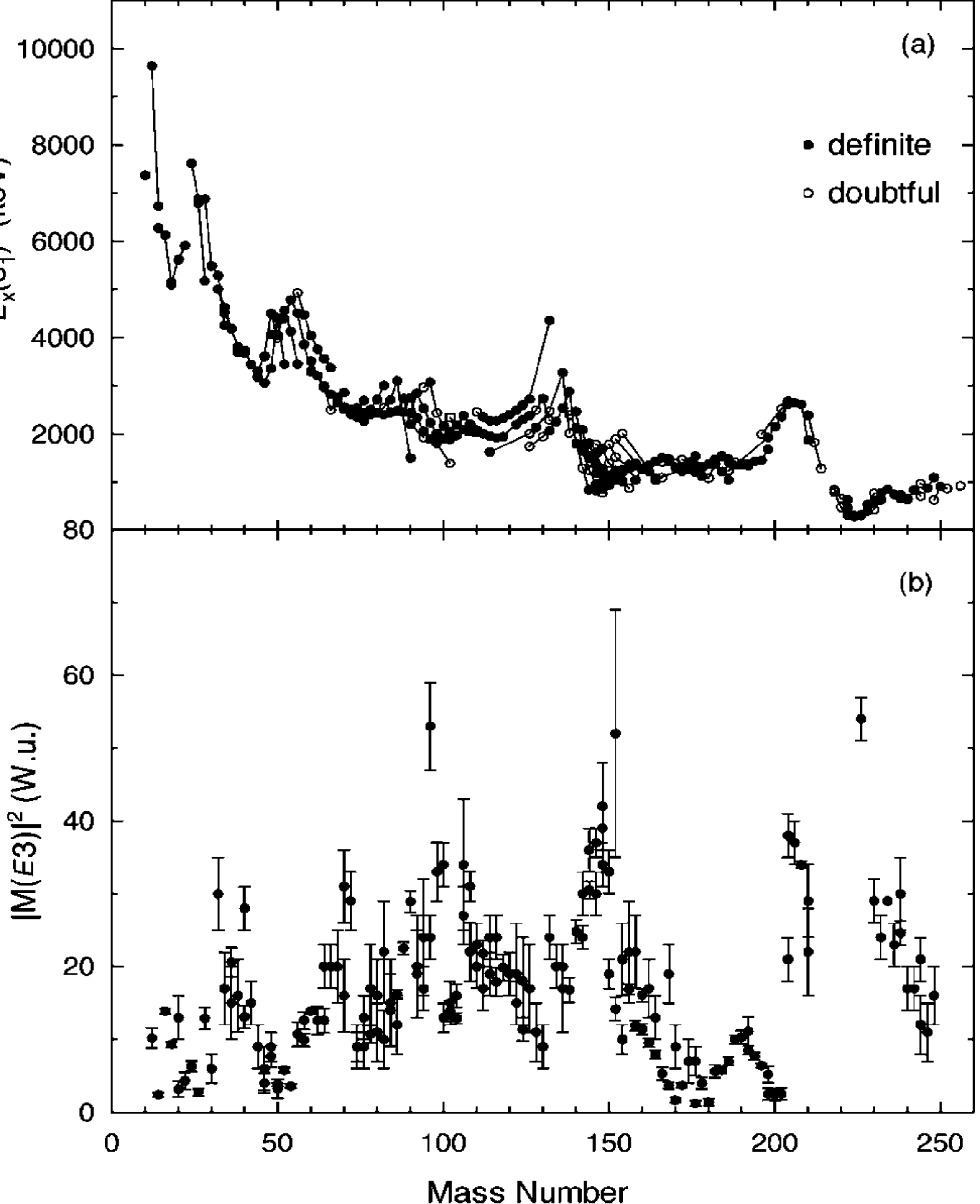
$$B(E2)\uparrow_{(sp)} = 2.97 \times 10^{-5} A^{4/3}\ e^2b^2. \quad (3)$$

The energy-weighted sum-rule (EWSR) strength, on the other hand, tells us how much total transition strength we can expect in a particular nucleus. It is given by [3]

$$S(\mathrm{I}) = \sum E \times B(E2)\uparrow\ = 30\, e^2(\hbar^2/8\pi m)AR_0^2, \quad (4)$$

where m is the nucleon mass and $(3/5)R_0^2$ is used for the single-particle, mean-square radius. The isoscalar part of the full sum is given by [4]

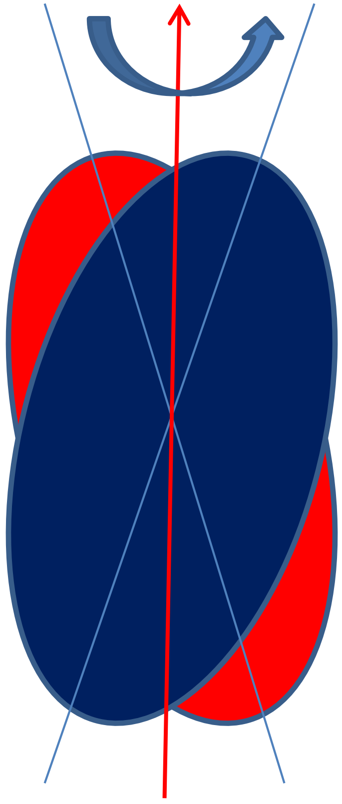
$$S(\mathrm{II}) = S(\mathrm{I})(Z/A)^2. \quad (5)$$



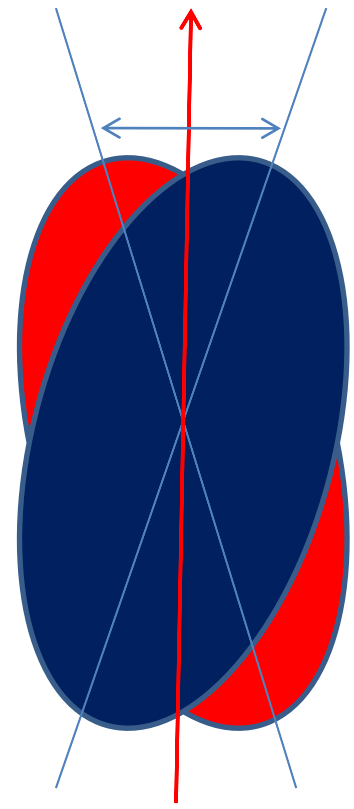
Octupole Surface Vibrations

Ізовекторні поверхневі коливання

M1, rotations ω



E2, vibrations ω



Гігантські мультипольні резонанси - це високоенергетичні стани

GMR: Поверхневі коливання протонної та нейтронної поверхні мультипольности ℓ

$$R_p(t)=R_{p0}[1+\beta_{p\ell}(t)\, Y_{\ell 0}(\theta)]$$

$$R_n(t)=R_{n0}[1+\beta_{n\ell}(t)\, Y_{\ell 0}(\theta)]$$

які связані з коливаннями густини протонів та нейтронів в об'ємній частині ядер

$$\rho_p(t)= \alpha_{p\ell}(t)\, \rho_0\, f(r)\, Y_{\ell 0}(\theta),$$

$$\rho_n(t)= \alpha_{n\ell}(t)\, \rho_0\, f(r)\, Y_{\ell 0}(\theta).$$

Амплитуда коливань

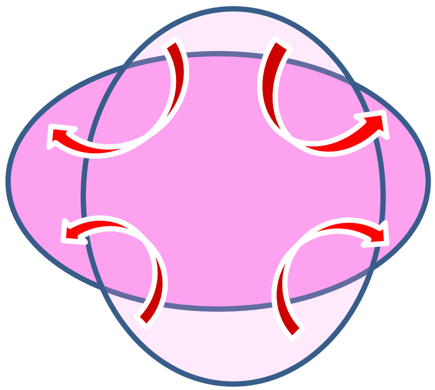
$$\alpha_\ell(t) \sim \beta_\ell(t) \sim \cos(\omega t).$$

Кількість протонів або нейтронів є постійною (зберігається):

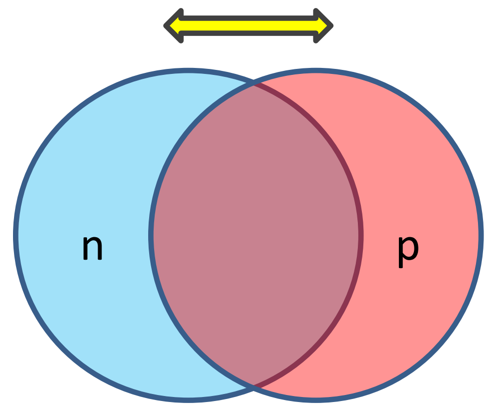
$$Z=const,\ N=const.$$

Різни моди збуджень є у ядрах (Spin x Isospin x Multipolarity)

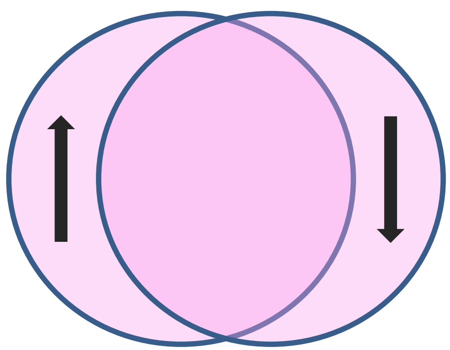
IS mode $r^2 Y_{20}(\hat{r})$



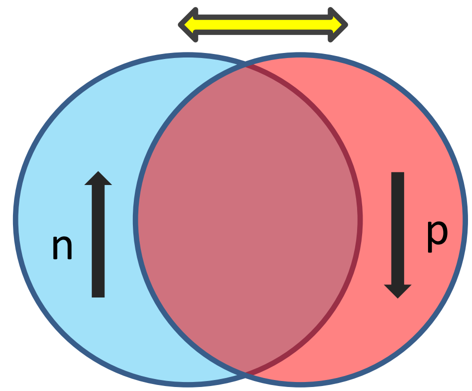
IV(Isospin) mode τ

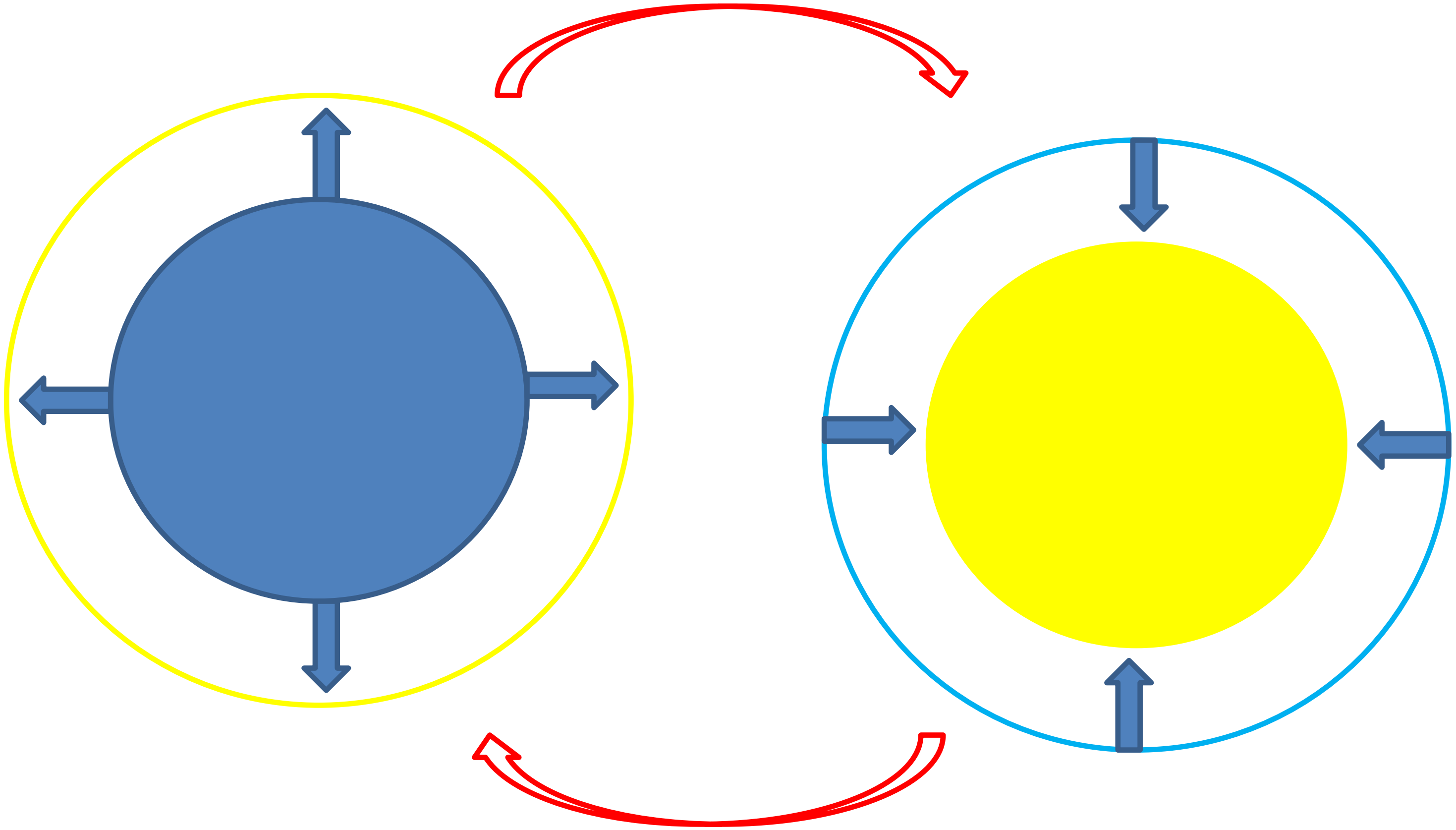


Spin mode σ



Spin-Isospin mode $\sigma\tau$



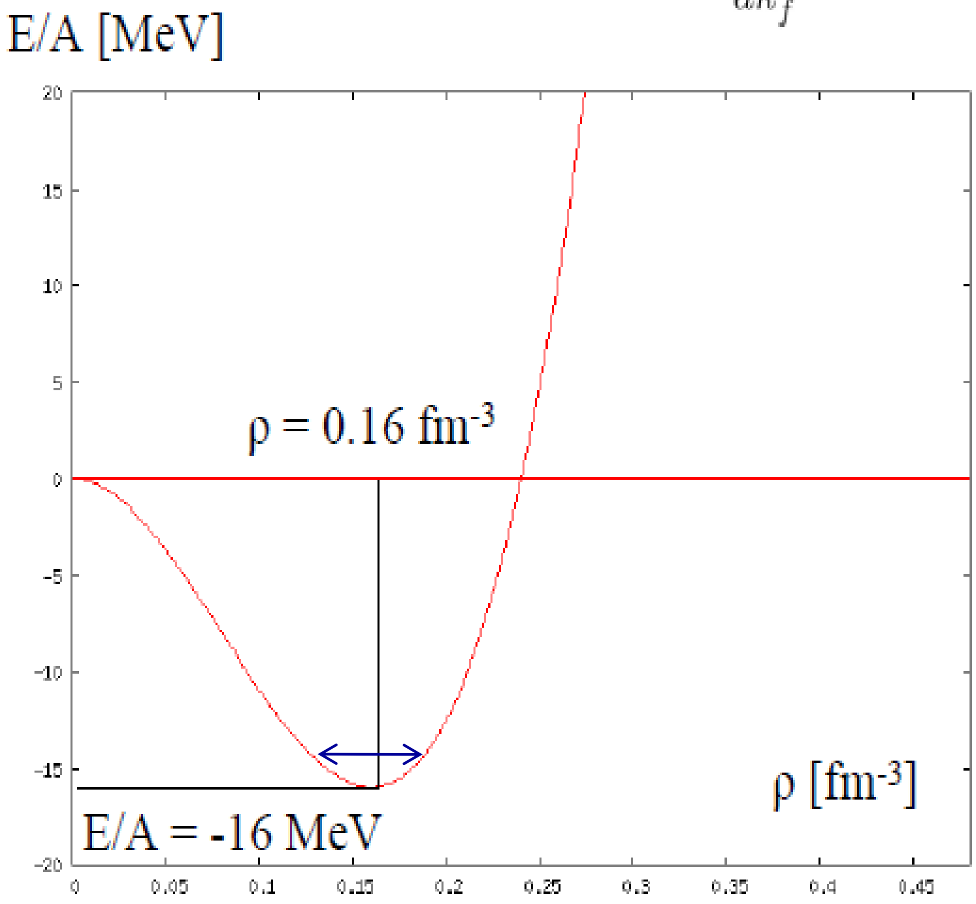


Ізоскалярна монопольна мода

Історія

- А.Б. Мігдал (1944/45) оцінів енергію GDR.
- Baldwin and Klaiber (1947/48) виміряли GDR
- Isoscalar Giant Quadrupole Res. (1971)
- Isoscalar Giant Monopole Res. 1977
  (E~13.5 MeV, 208Pb)
- Isoscalar Giant Dipole Res. 1980
  (E~19-21 MeV, 208Pb)

$$K_\infty = k_f^2 \frac{d^2(E/A)}{dk_f^2}|_{k_{f0}} = 9\rho^2 \frac{d^2(E/A)}{d\rho^2}|_{\rho_0}$$

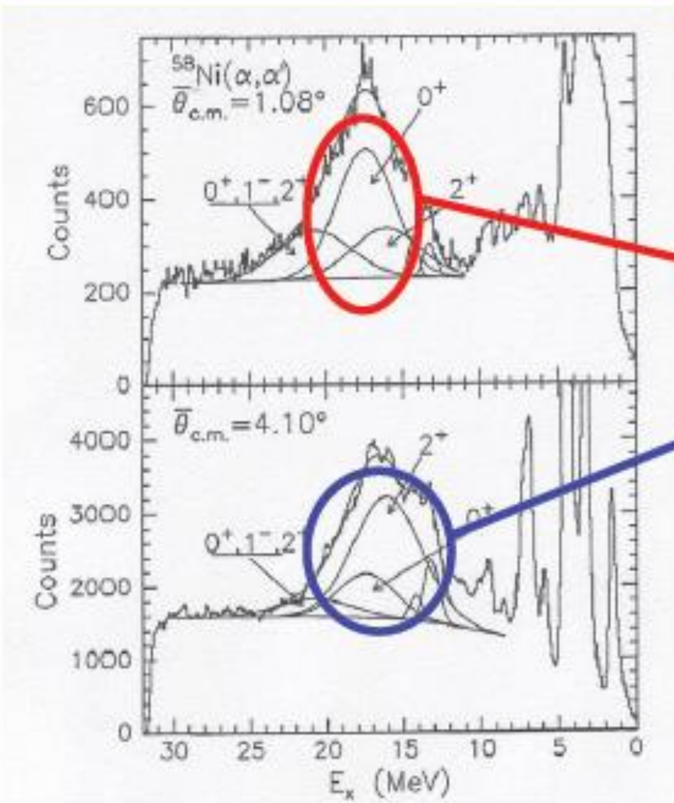


$$E[\rho] = E[\rho_0] + \frac{1}{18} K_\infty (\frac{\rho - \rho_0}{\rho_0})^2$$

Experimental probes for isoscalar giant resonances

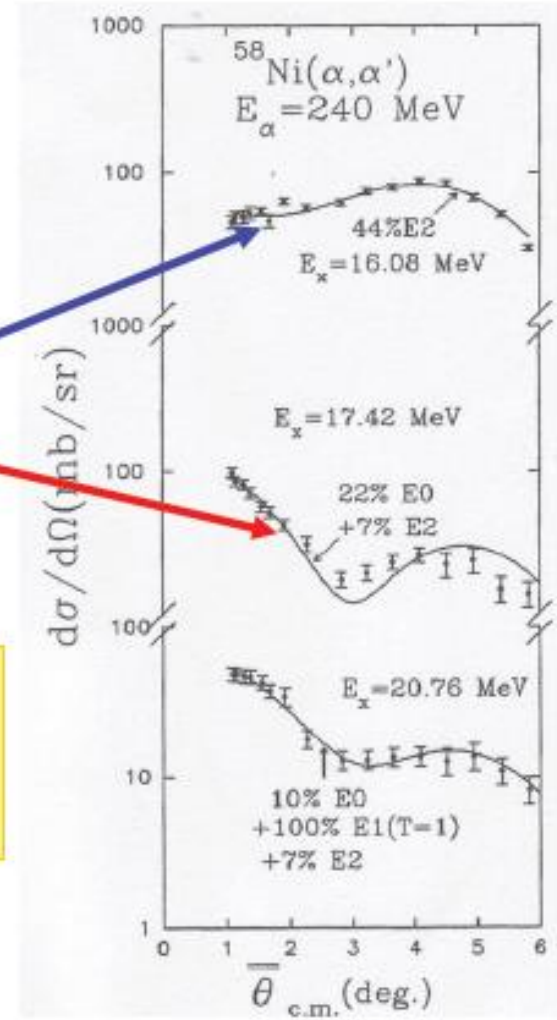
Inelastic scattering : (d,d') (α,α') @ E ≥ 25 A.MeV

^{58}Ni (α,α') E_α=240 MeV



GMR

GQR



GR in ^{58}Ni : analysis mixing 0^+ and 2^+

D.H. Younblood *et al*, Phys. Rev. Lett **76**, 1429 (1996)

Isoscalar monopole and quadrupole resonances

GDR (isovector), Photoabsorption

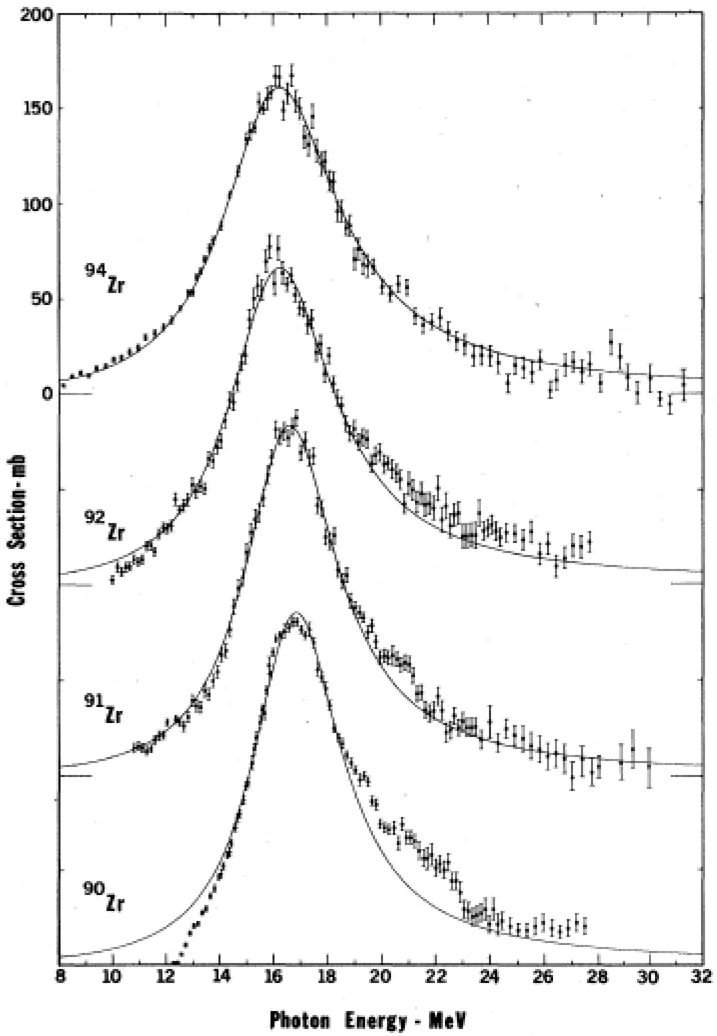


FIG. 24. Total photoneutron cross sections for the zirconium isotopes, showing the broadening of the giant resonance as one adds neutrons to a nucleus having a closed neutron shell (Livermore).

Rev. Mod. Phys.. Vol. 47. No. 3. July 1975

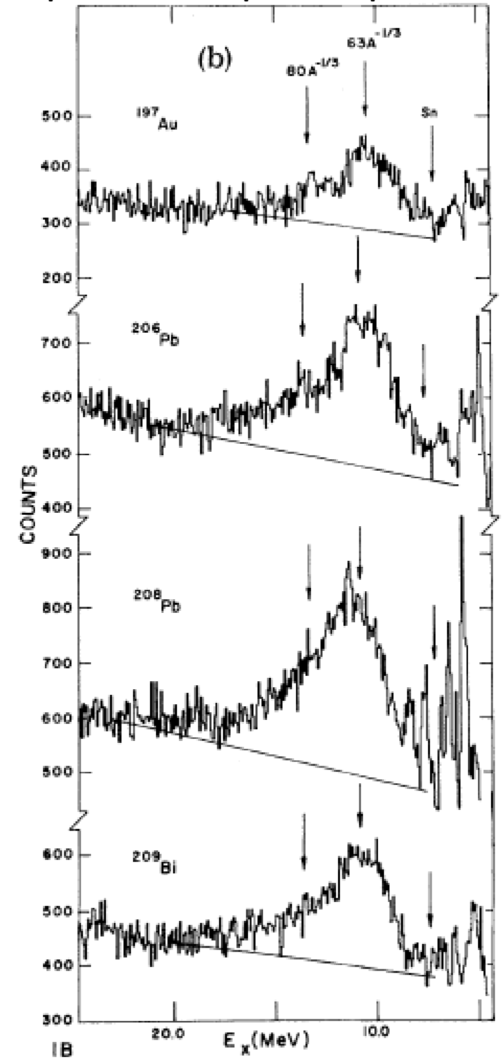
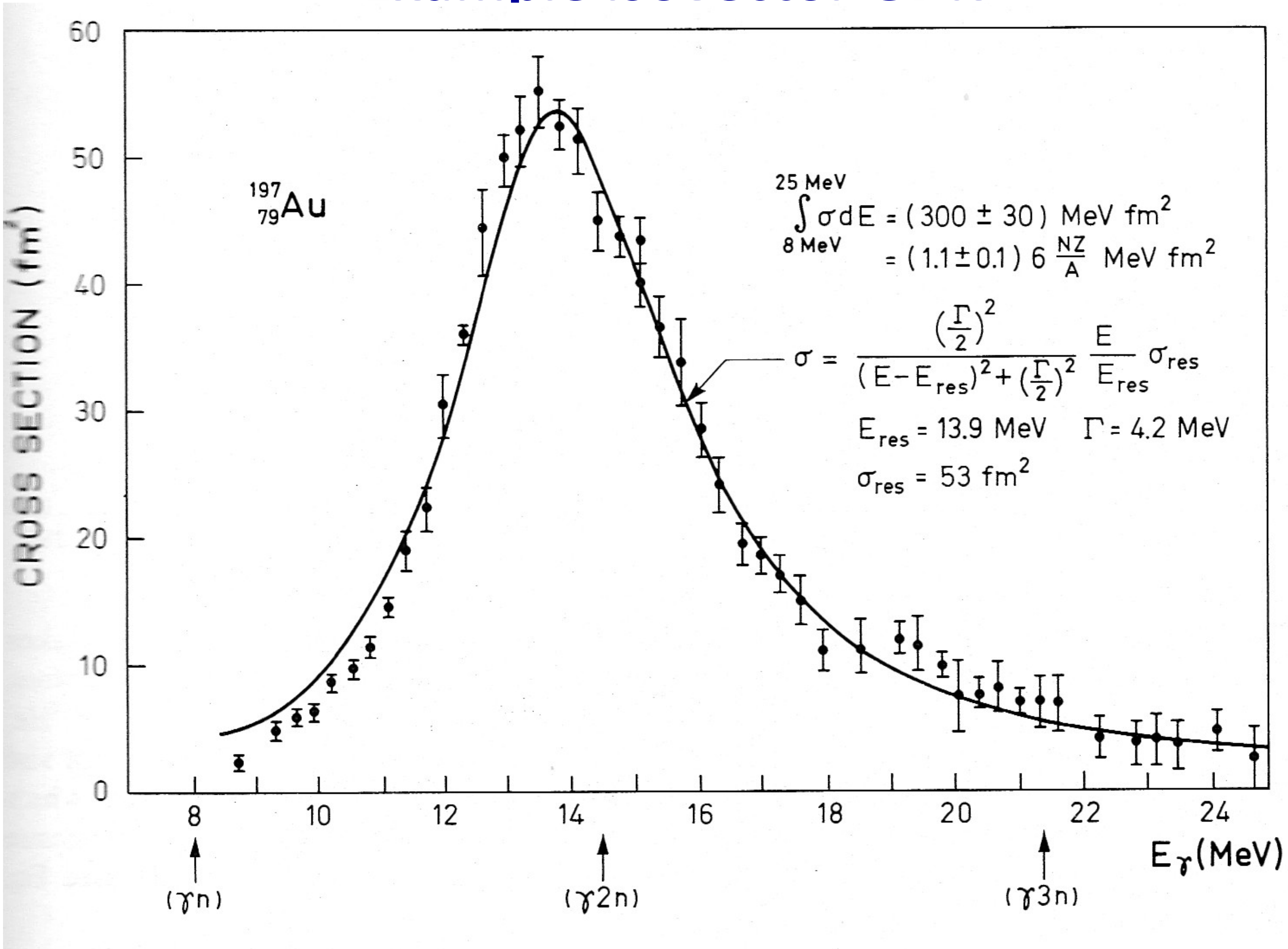


FIG. 1. (a) ^{208}Pb spectra taken at 14° and 17°. The two Gaussian peaks and the background fitted to the data are indicated. (b) Spectra taken at 13° for ^{197}Au, ^{206}Pb, ^{208}Pb, and ^{209}Bi are shown. The neutron separation energies as well as excitation energies corresponding to $63A^{-1/3}$ and $80A^{-1/3}$ MeV are indicated by arrows. The straight lines drawn are only to guide the eye.

Experimental description of GMR
Example isovector GDR



Модель рідкої краплі

У моделі рідкої краплі IVGDR описується як коливання нейтронної рідини проти протонної рідини з повертючою силою, пов'язаною з нейтронно-протонною взаємодією. Густині протонів або нейтронів та їх радіуси задаються

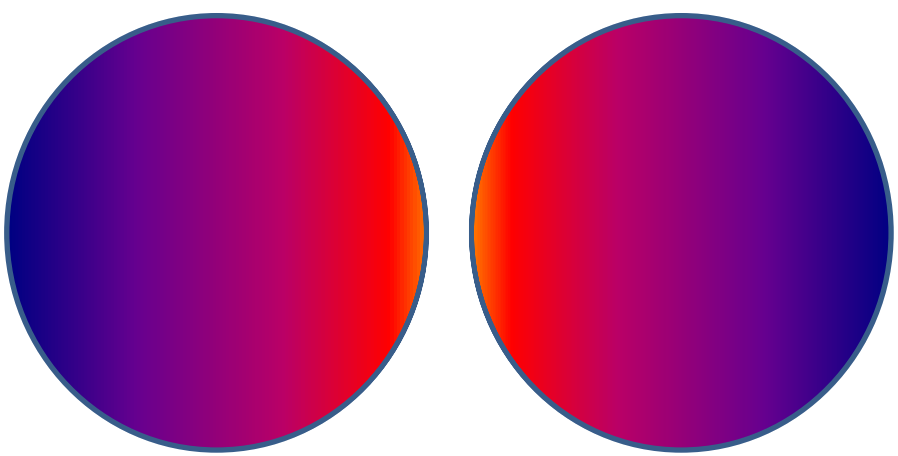
$$\rho_p(t) = \underline{\rho}_p + \alpha_{p\ell}(t)\, \rho_0\, f(r)\, Y_{\ell 0}(\theta), \qquad R_p(t) = \underline{R}_p + \alpha_{p\ell}(t)\, R_0\, f(r)\, Y_{\ell 0}(\theta),$$

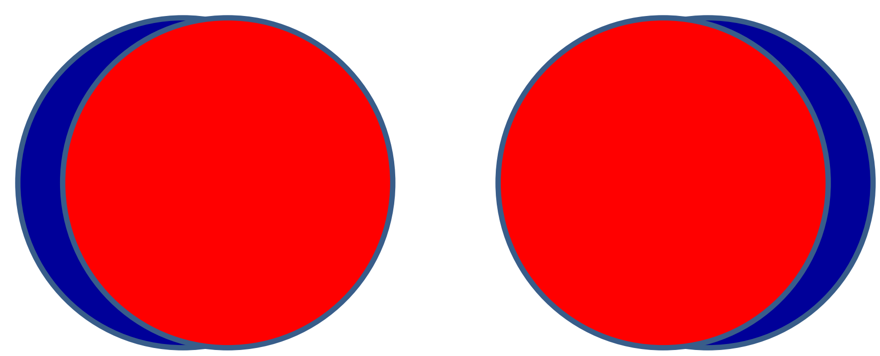
$$\rho_n(t) = \underline{\rho}_n + \alpha_{n\ell}(t)\, \rho_0\, f(r)\, Y_{\ell 0}(\theta), \qquad R_n(t) = \underline{R}_n + \alpha_{n\ell}(t)\, R_0\, f(r)\, Y_{\ell 0}(\theta).$$

Steinwedel-Jensen model — Goldhaber-Teller model

об'ємні коливання — поверхневі коливання

об’ємна повертюча сила — поверхнева повертюча сила

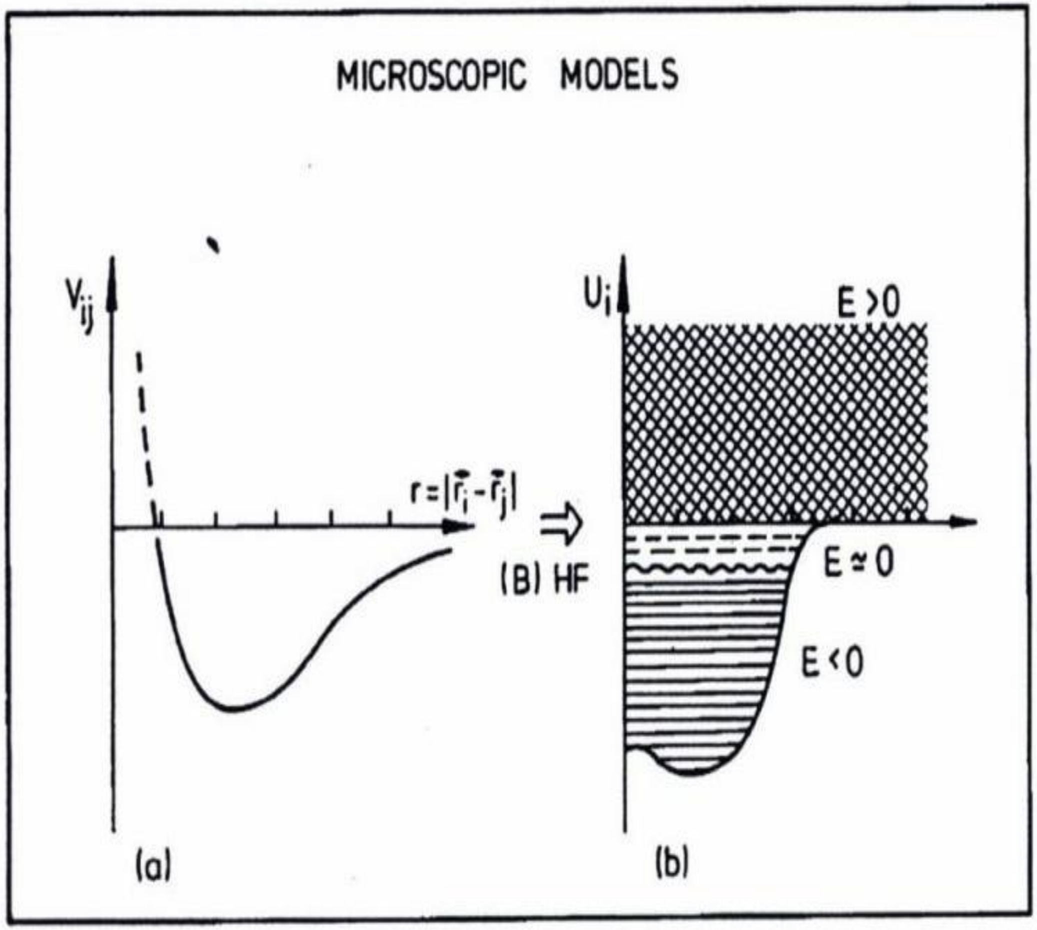




Microscopic Description

In the microscopic treatment of collective motion the nuclear wave function is described as *a linear combination of particle-hole excitations*.

The many-body Schrödinger equation $H\psi = E\,\psi$ is difficult to solve. In the mean-field approximation each particle moves independently from other nucleons in a single particle potential, representing its interactions with all other nucleons.



$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} + \sum_{i<j} V_{ij}$$

Mean-field approximation

$$H = \sum_i \left[\frac{\vec{p}_i^{\,2}}{2m_i} + U(\vec{r}_i) \right] + H_{res}$$

$$H_0 = \sum_i h_i, \qquad h_i = \frac{\vec{p}_i^{\,2}}{2m_i} + U(\vec{r}_i)$$

$$h_i \phi_i = E_i \phi_i$$

$$\Phi = \mathrm{A}\phi_i(1)\ldots\phi_A(A)$$

Skyrme Hartree Fock (SHF) model

- Skyrme interaction (effective zero-range force with density and momentum dependent terms)

$$V_{sky}(\vec{r}_1,\vec{r}_2)=t_0(1+x_0P_\sigma)\delta(\vec{r}_1-\vec{r}_2)+\frac{1}{2}t_1(1+x_1P_\sigma)\left\{\vec{k}'^2\delta(\vec{r}_1-\vec{r}_2)+\delta(\vec{r}_1-\vec{r}_2)\vec{k}^2\right\}$$

$$+t_2(1+x_2P_\sigma)\vec{k}'\cdot\delta(\vec{r}_1-\vec{r}_2)\vec{k}+\frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\vec{r})\delta(\vec{r}_1-\vec{r}_2)$$

$$+iW(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k}$$

$$\vec{k}=\frac{(\vec{\nabla}_1-\vec{\nabla}_2)}{2i},\quad \vec{k}'=-\frac{(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)}{2i}$$

- Hamiltonian density $H(\rho_n,\rho_p)=\langle \mathrm{HF}|T+V_{Sky}|\mathrm{HF}\rangle$

$$H(\rho_n,\rho_p)=\frac{\hbar^2}{2m}(\tau_n+\tau_p)+\frac{1}{4}t_0(1-x_0)(\rho_n{}^2+\rho_p{}^2)+t_0\left(1+\frac{1}{2}x_0\right)\rho_n\rho_p$$

$$+\frac{1}{12}t_3\left(1+\frac{1}{2}x_3\right)\rho^{\alpha+2}-\frac{1}{12}t_3\left(\frac{1}{2}+x_3\right)\rho^\alpha(\rho_n{}^2+\rho_p{}^2)$$

$$+\frac{1}{8}\left[t_1(1-x_1)+3t_2(1+x_2)\right](\rho_n\tau_n+\rho_p\tau_p)$$

$$+\frac{1}{4}\left[t_1\left(1+\frac{1}{2}x_1\right)+t_2\left(1+\frac{1}{2}x_2\right)\right](\rho_n\tau_p+\rho_p\tau_n)+H_{Coul}+(\text{derivative terms})$$

Physical properties of the infinite nuclear matter by the parameters of Skyrme interaction

- Isoscalar part

$$0=\frac{\hbar^2}{5m}c\rho_{nm}^{2/3}+\frac{3}{8}t_0\rho_{nm}+\frac{1}{16}\left\{3t_1+t_2\left(5+4x_2\right)\right\}c\rho_{nm}^{5/3}$$

$$-E_0=\frac{\hbar^2}{10m}c\rho_{nm}^{2/3}+\frac{3}{8}t_0\rho_{nm}+\frac{1}{16}t_3\rho_{nm}^{\alpha+1}+\frac{3}{80}\left\{3t_1+t_2\left(5+4x_2\right)\right\}c\rho_{nm}^{5/3}$$

$$K=-\frac{3\hbar^2}{5m}c\rho_{nm}^{2/3}+\frac{9}{16}t_3\alpha\left(\alpha+1\right)\rho_{nm}^{\alpha+1}+\frac{3}{8}\left\{3t_1+t_2\left(5+4x_2\right)\right\}c\rho_{nm}^{5/3}$$

- Isovector part

$$J=\frac{\hbar^2}{6m}c\rho_{nm}^{2/3}-\frac{1}{8}t_0\left(1+2x_0\right)\rho_{nm}-\frac{1}{48}t_3\left(1+2x_3\right)\rho_{nm}^{\alpha+1}-\frac{1}{24}\left\{3t_1x_1-t_2\left(4+5x_2\right)\right\}c\rho_{nm}^{5/3}$$

$$L=\frac{\hbar^2}{6m}c\rho_{nm}^{2/3}-\frac{3}{8}t_0\left(1+2x_0\right)\rho_{nm}-\frac{1}{16}t_3\left(1+2x_3\right)\left(\alpha+1\right)\rho_{nm}^{\alpha+1}$$
$$-\frac{5}{24}\left\{3t_1x_1-t_2\left(4+5x_2\right)\right\}c\rho_{nm}^{5/3}$$

$$K_{sym}=-\frac{\hbar^2}{3m}c\rho_{nm}^{2/3}-\frac{3}{16}t_3\left(1+2x_3\right)\left(\alpha+1\right)\alpha\rho_{nm}^{\alpha+1}-\frac{5}{12}\left\{3t_1x_1-t_2\left(4+5x_2\right)\right\}c\rho_{nm}^{5/3}$$

Деформовані ядра

$E_{sphere} \sim 1/R;$

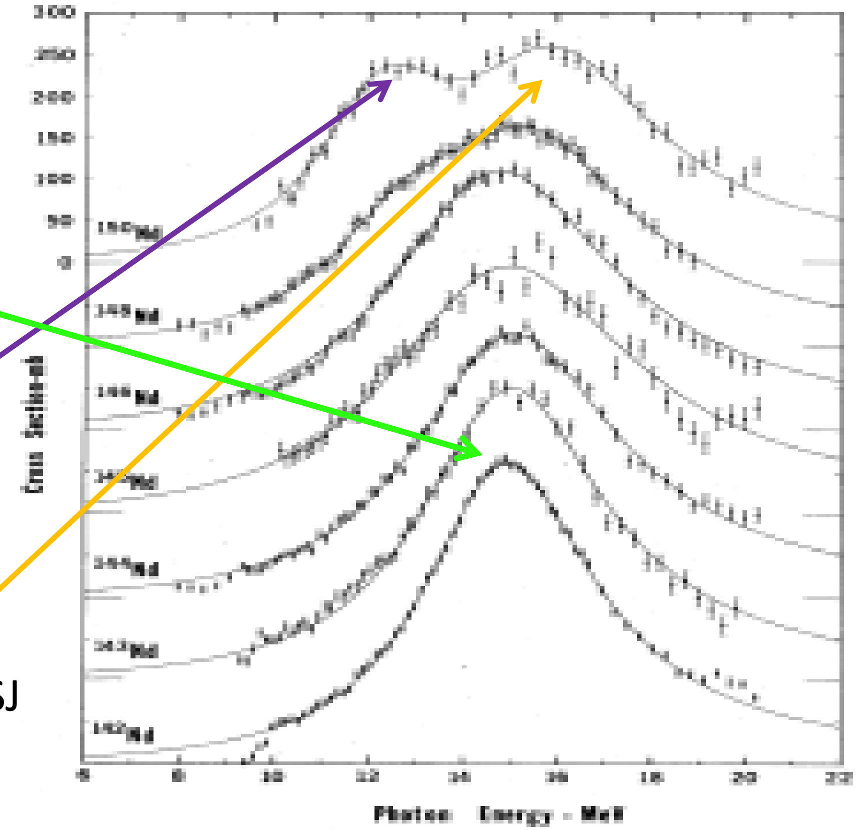
$\Gamma_{sphere} \sim 0.02\,(E_{sphere})^2$

$E_{long\ axis} \sim 1/[R(1+2c\beta)];$

$\Gamma_{long\ axis} \sim 0.02\,(E_{long\ axis})^2$

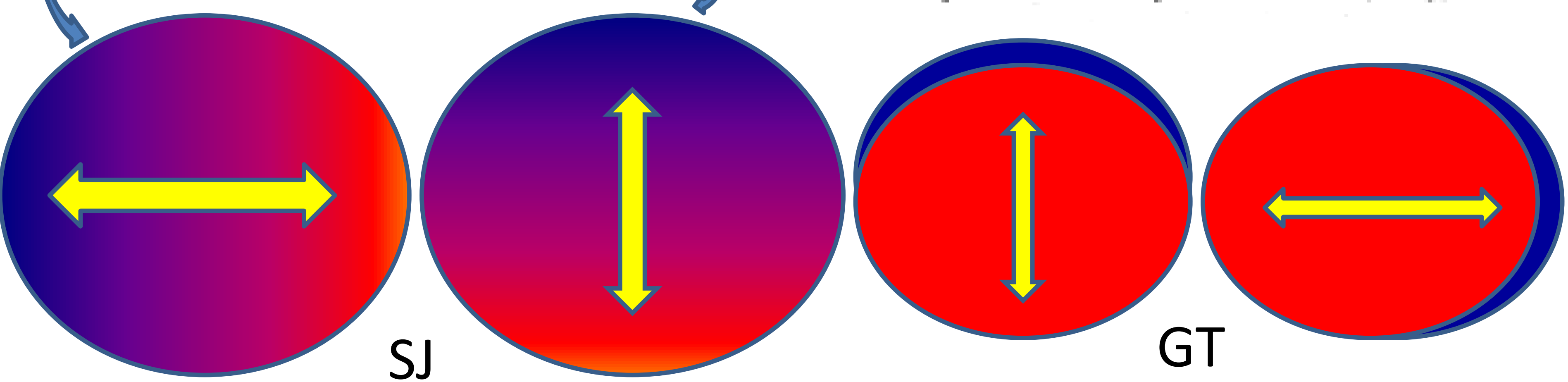
$E_{short\ axis} \sim 1/[R(1-c\beta)]$

$\Gamma_{short\ axis} \sim 0.02\,(E_{short\ axis})^2$



SJ

FIG. 27. Total photoneutron cross sections for the neodymium isotopes, showing the "evolution" of the giant resonance as one makes the transition from spherical to statically deformed nuclei (Saclay).



SJ

GT

Властивости GMR

Isoscalar: L=0 E=80 $A^{-1/3}$ MeV,

L=1 E~120-140 $A^{-1/3}$ MeV,

L=2 E=62-65 $A^{-1/3}$ MeV, Γ_{sphere}~0.02 $(E_{sphere})^2$

L=3 E~105-115 $A^{-1/3}$ MeV

Isovector: L=0 E~180-195 $A^{-1/3}$ MeV,

L=1 E=68-82 $A^{-1/3}$ MeV, Γ_{sphere}~0.02 $(E_{sphere})^2$

L=2 E~105-125$A^{-1/3}$ MeV,

L=3 E~145-170 $A^{-1/3}$ MeV

Time-Depending-Hartree-Fock equation:

$$i\hbar\frac{d\rho}{dt} = [H, \rho],$$

where

$$H = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r})$$

is the Hamiltonian and $V(\mathbf{r})$ is the potential:

$$V(\mathbf{r}) = \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}')\rho(\mathbf{r}')$$

Wigner transformation for one-particle operator:

$$A_W = A(\mathbf{r}, \mathbf{p}) = \int d\mathbf{s}\; exp(-\frac{i}{\hbar}\mathbf{p}\mathbf{s})\; A(\mathbf{r} + \mathbf{s}/2, \mathbf{r} - \mathbf{s}/2).$$

Wigner transformation of commutator:

$$([A, B])_W = 2iA_W sin\left[\frac{\hbar}{2}(\bar{\nabla}_r\vec{\nabla}_p - \bar{\nabla}_p\vec{\nabla}_r)\right] B_W =$$
$$= i\hbar A_W(\bar{\nabla}_r\vec{\nabla}_p - \bar{\nabla}_p\vec{\nabla}_r)B_W + O(\hbar^3).$$

Wigner transformation of TDHF equation:

$$\frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial t} + \frac{\mathbf{p}}{M}\vec{\nabla}_r f(\mathbf{r}, \mathbf{p}, t) - \vec{\nabla}_p f(\mathbf{r}, \mathbf{p}, t)\vec{\nabla}_r V(\mathbf{r}) = O(\hbar^2),$$

where

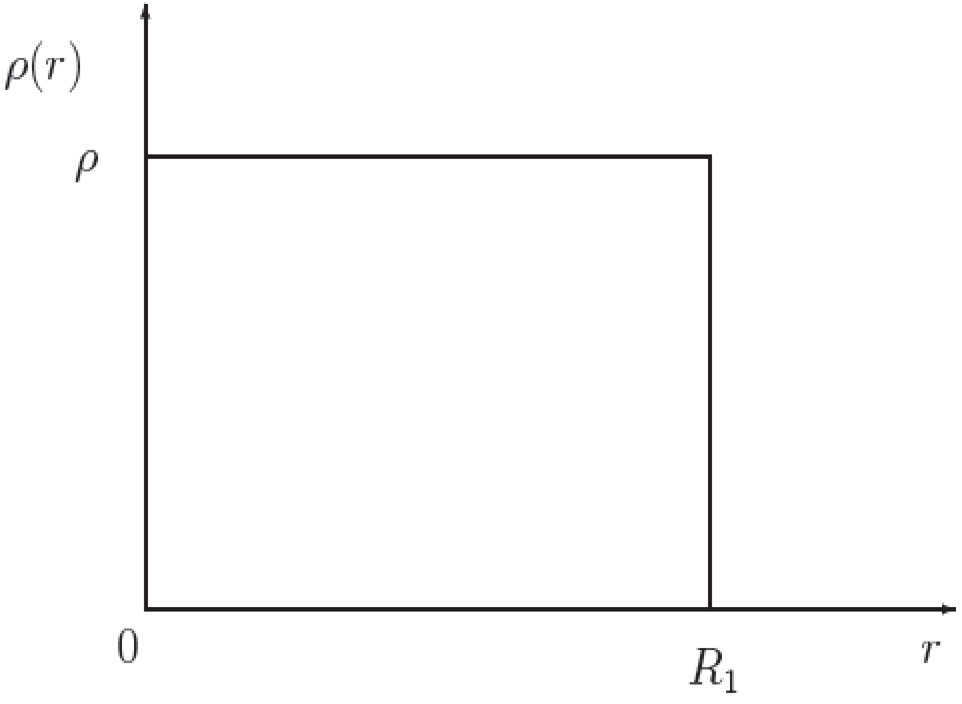
$$V(\mathbf{r}) = \int d\mathbf{r}' d\mathbf{p} U(\mathbf{r}, \mathbf{r}') f(\mathbf{r}', \mathbf{p}, t)$$

Approximations: 1. Sharp edge of density and potential in coordinate space

2. Sharp edge at the $p = p_f$ in momentum space

3. Neglecting $O(\hbar^2)$ terms

4. Linear approximation (small amplitude of vibrations $\Leftrightarrow f(\mathbf{r}, \mathbf{p}, t) \ll f(\mathbf{r}, \mathbf{p})$)



By using approximations we obtain the Vlasov equation into the nuclear volume for proton and neutron distribution functions:

$$\frac{\partial f_i(\mathbf{r},\mathbf{p},t)}{\partial t}+\frac{\mathbf{p}}{M}\vec{\nabla}_r\left[f_i(\mathbf{r},\mathbf{p},t)+\delta(\varepsilon-\varepsilon_f)\frac{\pi^2\hbar^3}{p_f M}\int d\tau\sum_{j=P,N}F_{ij}f_i(\mathbf{r},\mathbf{p},t)\right]=0,$$

where $i,j=P,N$, $d\tau=2d\mathbf{p}/(2\pi\hbar)^3$, F_{ij} are the constants of quasiparticle interaction.

THE DENSITY VIBRATIONS ARE DESCRIBED IN THE MODEL:

by the Vlasov equation ⇔ into the nuclear volume

by boundary conditions ⇔ into the nuclear edge

ISOSCALAR GIANT MULTIPOLE RESONANCES

Isoscalar distribution function:

$$f^+(\mathbf{r},\mathbf{p},t) = f_P(\mathbf{r},\mathbf{p},t) + f_N(\mathbf{r},\mathbf{p},t).$$

The Vlasov equation for $f^+(\mathbf{r},\mathbf{p},t)$:

$$\frac{\partial f^+(\mathbf{r},\mathbf{p},t)}{\partial t} + \frac{\mathbf{p}}{M}\vec{\nabla}_r \left[f^+(\mathbf{r},\mathbf{p},t) + \delta(\varepsilon - \varepsilon_f)\frac{\pi^2\hbar^3}{p_f M}\int d\tau F^+ f^+(\mathbf{r},\mathbf{p},t)\right] = 0,$$

where $F^+ = F_{PP} + F_{PN}$. The solution of the Vlasov equation, corresponded to the isoscalar density vibration of multipolarity ℓ is given by

$$f^+(\mathbf{r},\mathbf{p},t) = \alpha_v \delta(\varepsilon - \varepsilon_f)\int d\Omega_{\mathbf{k}} Y_{\ell 0}(\Omega_{\mathbf{k}}) exp(i(\mathbf{kr} - \omega t))\frac{cos(\vartheta_{pk})}{s - cos(\vartheta_{pk})},$$

where $s = \omega / k v_f$, $v_f = \sqrt{2\varepsilon_f / M}$.

The quantities s is determined from the equation:

$$\frac{s}{2} ln\left(\frac{s+1}{s-1}\right) - 1 = \frac{1}{F^+}.$$

The boundary conditions for **free** surface:

$$v_r(\mathbf{r},t)|_R = \dot{R}(t)$$

Velocity of Particles = Velocity of Surface

$$\sigma_{rr}(\mathbf{r},t)|_R = P(t)$$

Normal to Surface Pressure of Particles = Pressure of Surface Tension

$$v_r(\mathbf{r},t) = \int d\tau \frac{p_r}{m\rho} f^+(\mathbf{r},\mathbf{p},t),$$
$$R(t) = R(1+\alpha_s(t)Y_{\ell 0}(\vartheta)),$$
$$\sigma_{rr}(\mathbf{r},t) = \frac{1}{M}\int d\tau p_r^2 f^+(\mathbf{r},\mathbf{p},t) + \rho\frac{\pi^2\hbar^3 F^+}{2p_f M}\int d\tau f^+(\mathbf{r},\mathbf{p},t),$$
$$P(t) = \frac{\alpha_s(t)\sigma(\ell-1)(\ell+2)}{R} Y_{\ell 0}(\vartheta).$$

In nucleus with A nucleons the energy of isoscalar giant multipole resonance is equal to:

$$E_{\ell n} = D_{\ell n}(A)A^{-1/3},$$

where

$$D_{\ell n}(A) = 2s\left(\frac{\hbar^2\varepsilon_f}{2Mr_0^2}\right)^{1/2} x_{\ell n}(A).$$

Here $x_{\ell n}(A)$ is the n-th root of equation:

$$j'_\ell(x) = \frac{3\varepsilon_f x A^{1/3}}{b^+(\ell-1)(\ell+2)}[(1-3s^2+F^+/3)j''_\ell(x) + (1-s^2+F^+)j_\ell(x)]$$

$j_\ell(x)$ is the spherical Bessel function, primes - derivatives, $b^+ = 4\pi r_0^2\sigma$, $R = r_0 A^{-1/3}$.

$\frac{E}{A} = a_0 + b^+ A^{-1/3} + JA\left(\frac{N-Z}{A}\right)^2(1 - \frac{9JA^{1/3}}{4Q}) + ...$ Note: $x_{\ell n}(A) \approx x_{\ell n} \Rightarrow$ weak A dependence.

Transition densities $\Rightarrow$ volume and surface contributions:

$$\delta\rho_{\ell n}(r) = \alpha_{\ell n}\rho_0[j_\ell(x_{\ell n}(r/R))w(r) - (j'_\ell(x_{\ell n})/x_{\ell n})Rw'(r)],$$

here $\rho = 3/(4\pi r_0^3)$ is density of nuclear matter, $w(r)$ is the radial dependence of static density in nucleus

$$w(r) = [1 + exp((r-R)/d)]^{-1}.$$

Parameters: $\varepsilon_f = 40$ MeV, $r_0 = 1.2$ fm, $b^+ = 19$ MeV, $F^+ = 1$.

Here $x_{\ell n}(A)$ is the n-th root of equation:

$$j'_\ell(x) = \frac{3\varepsilon_f x A^{1/3}}{b^+(\ell-1)(\ell+2)}[(1-3s^2+F^+/3)j''_\ell(x)+(1-s^2+F^+)j_\ell(x)]$$

$j_\ell(x)$ is the spherical Bessel function, primes - derivatives, $b^+ = 4\pi r_0^2\sigma$, $R = r_0 A^{-1/3}$.

$\frac{E}{A} = a_0 + b^+ A^{-1/3} + JA\left(\frac{N-Z}{A}\right)^2(1-\frac{9JA^{1/3}}{4Q}) + \ldots$ Note: $x_{\ell n}(A) \approx x_{\ell n} \Rightarrow$ weak A dependence.

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$$\delta\rho_{\ell n}(r) = \alpha_{\ell n}\rho_0[j_\ell(x_{\ell n}(r/R))w(r) - (j'_\ell(x_{\ell n})/x_{\ell n})Rw'(r)],$$

here $\rho = 3/(4\pi r_0^3)$ is density of nuclear matter, $w(r)$ is the radial dependence of static density in nucleus

$$w(r) = [1 + exp((r-R)/d)]^{-1}.$$

Parameters: $\varepsilon_f = 40$ MeV, $r_0 = 1.2$ fm, $b^+ = 19$ MeV, $F^+ = 1$.

ISOVECTOR GIANT MULTIPOLE RESONANCES

Isovector distribution function:

$$f^{-}(\mathbf{r},\mathbf{p},t) = f_N(\mathbf{r},\mathbf{p},t) - f_P(\mathbf{r},\mathbf{p},t).$$

The Vlasov equation for $f^{-}(\mathbf{r},\mathbf{p},t)$:

$$\frac{\partial f^{-}(\mathbf{r},\mathbf{p},t)}{\partial t} + \frac{\mathbf{p}}{M}\vec{\nabla}_r\left[f^{-}(\mathbf{r},\mathbf{p},t) + \delta(\varepsilon-\varepsilon_f)\frac{\pi^2\hbar^3}{p_f M}\int d\tau F^{-} f^{-}(\mathbf{r},\mathbf{p},t)\right] = 0,$$

where $F^{-} = F_{PP} - F_{PN}$. The solution of the Vlasov equation, corresponded to the isovector density vibration of multipolarity ℓ is given by

$$f^{-}(\mathbf{r},\mathbf{p},t) = \alpha_v\delta(\varepsilon-\varepsilon_f)\int d\Omega_{\mathbf{k}} Y_{\ell 0}(\Omega_{\mathbf{k}}) exp(i(\mathbf{kr}-\omega t))\frac{cos(\vartheta_{pk})}{s - cos(\vartheta_{pk})},$$

where $s = \omega/kv_f$, $v_f = \sqrt{2\varepsilon_f/M}$.

The quantities s is determined from the equation:

$$\frac{s}{2}ln\left(\frac{s+1}{s-1}\right) - 1 = \frac{1}{F^{-}}.$$

The boundary conditions for **free** proton and neutron surfaces:

$$v_r^{P,N}(\mathbf{r},t)|_R = \dot{R}^{P,N}(t)$$

Velocity of Protons (Neutrons) = Velocity of Proton (Neutron) Surface

$$\sigma_{rr}^{-}(\mathbf{r},t)|_R = P^{-}(t)$$

Normal to Surface Pressure of Particles = Pressure of Neutron Skin Deformation

$$v_r^{P,N}(\mathbf{r},t) = \int d\tau \frac{p_r}{m\rho} f^{P,N}(\mathbf{r},\mathbf{p},t),$$

$$R^{P,N}(t) = R^{P,N}(1 + \alpha_s^{P,N}(t) Y_{\ell 0}(\vartheta))$$

$$\sigma_{rr}^{-}(\mathbf{r},t) = \frac{1}{M} \int d\tau p_r^2 f^{-}(\mathbf{r},\mathbf{p},t) + \rho \frac{\pi^2 \hbar^3 F^{-}}{2 p_f M} \int d\tau f^{-}(\mathbf{r},\mathbf{p},t),$$

$$P^{-}(t) = \frac{Q}{2\pi r_0^4} (R^N \alpha_s^N(t) - R^P \alpha_s^P(t)) Y_{\ell 0}(\vartheta).$$

In nucleus with A nucleons the energy of isovector giant multipole resonance is equal to:

$$E_{\ell n} = D_{\ell n}(A) A^{-1/3},$$

where

$$D_{\ell n}(A) = 2s \left(\frac{\hbar^2 \varepsilon_f}{2 M r_0^2} \right)^{1/2} x_{\ell n}(A).$$

Here $x_{\ell n}(A)$ is the n-th root of equation:

$$j_\ell'(x) = \frac{3 \varepsilon_f A^{1/3} x}{4 Q A^{1/3}} [(1 - 3s^2 + F^{-}/3) j_\ell''(x) + (1 - s^2 + F^{-}) j_\ell(x)]$$

$\frac{E}{A} = a_0 + b^{+} A^{-1/3} + JA \left(\frac{N-Z}{A} \right)^2 (1 - \frac{9 J A^{1/3}}{4Q}) + \ldots.$

Note: The A dependence of $x_{\ell n}(A)$ is very strong and important.

Transition densities $\Rightarrow$ volume and surface contributions:

$$\delta\rho_{\ell n}(r) = \alpha^{-}_{\ell n}\rho_0[j_\ell(x_{\ell n}(r/R))w(r) - (j'_\ell(x_{\ell n})/x_{\ell n})Rw'(r)].$$

here $\rho = 3/(4\pi r_0^3)$ is density of nuclear matter, $w(r)$ is the radial dependence of static density in nucleus

$$w(r) = [1 + exp((r - R)/d)]^{-1}.$$

Parameters: $\varepsilon_f = 40$ MeV, $r_0 = 1.2$ fm, $F_0^- = 1.6$, $B^- = 43.5$MeV.

Hydrodynamic (1-sound) Isovector Giant Multipole Resonances

Model	Volume Osc.	Surface Osc.	$E(A)$
Steinwedel-Jensen	+	-	$A^{-1/3}$
Goldhaber-Teller	-	+	$A^{-1/6}$
Free Surface Model	+	+	between $A^{-1/6}$ and $A^{-1/3}$

Free Surface Model

The linearized hydrodynamical continuity equation for proton and neutron liquids are

$$\frac{\partial \delta\rho_P(\mathbf{r},t)}{\partial t} + \frac{Z}{A}\rho_0 \, \mathrm{div}(\mathbf{v}_P(\mathbf{r},t)) = 0, \qquad \frac{\partial \delta\rho_N(\mathbf{r},t)}{\partial t} + \frac{N}{A}\rho_0 \, \mathrm{div}(\mathbf{v}_N(\mathbf{r},t)) = 0,$$

where $\rho_0 = \frac{3}{4\pi r_0^3}$. Condition: $\delta\rho_P(\mathbf{r},t) + \delta\rho_N(\mathbf{r},t) = 0$.

The Euler hydrodynamical equations for two-component liquid are

$$M\frac{Z}{A}\rho_0 \frac{\partial \mathbf{v}_P(\mathbf{r},t)}{\partial t} + 2C_{PP} \, \mathrm{grad} \, \delta\rho_P(\mathbf{r},t) + 2C_{PN} \, \mathrm{grad} \, \delta\rho_N(\mathbf{r},t) = 0,$$
$$M\frac{N}{A}\rho_0 \frac{\partial \mathbf{v}_N(\mathbf{r},t)}{\partial t} + 2C_{PP} \, \mathrm{grad} \, \delta\rho_N(\mathbf{r},t) + 2C_{PN} \, \mathrm{grad} \, \delta\rho_P(\mathbf{r},t) = 0,$$

where M is the nucleon mass.

The linearized wave equation is

$$\frac{\partial^2 \delta\rho^-(\mathbf{r},t)}{\partial t} - \frac{2J}{M}\Delta\rho^-(\mathbf{r},t) = 0,$$

where $K/18 = C_{PP} + C_{PN}$ is incompressibility modulus and $J = C_{PP} - C_{PN}$ is isovector symmetry coefficient.

The linearized wave equation is

$$\frac{\partial^2 \delta\rho^-(\mathbf{r},t)}{\partial t} - \frac{2J}{M}\Delta\rho^-(\mathbf{r},t) = 0.$$

Solution:

$$\rho^-(\mathbf{r},t) = \delta\rho_N(\mathbf{r},t) - \delta\rho_P(\mathbf{r},t) = 2\delta\rho_N(\mathbf{r},t) = -2\delta\rho_P(\mathbf{r},t) = \alpha_{n\ell m}(t)\rho_0 j_\ell(k_{n\ell}r)Y_{\ell m}(\vartheta,\phi),$$

$$\mathbf{v}^-(\mathbf{r},t) = \mathbf{v}_N(\mathbf{r},t) - \mathbf{v}_P(\mathbf{r},t) = \frac{A}{Z}\mathbf{v}_N(\mathbf{r},t) = -\frac{A}{N}\mathbf{v}_P(\mathbf{r},t) = \frac{A^2}{2ZN}\frac{1}{k_{n\ell}^2}\dot{\alpha}_{n\ell m}(t)\,\mathrm{grad} j_\ell(k_{n\ell}r)Y_{\ell m}(\vartheta,\phi),$$

$$P^-_{vol}(\mathbf{r},t) = 2J\alpha_{n\ell m}(t)\rho_0 j_\ell(k_{n\ell}r)Y_{\ell m}(\vartheta,\phi).$$

The vibration of neutron skin surface:

$$T(t) = R_N(t) - R_P(t) = R_N - R_P + \alpha_s(t)r_0 Y_{\ell m}(\vartheta,\phi).$$

Boundary conditions:

$$v_r^-(\mathbf{r},t)|_R = \dot{T}(t)$$

$$P^-_{vol}(\mathbf{r},t)|_R = P^-_s(t) = \frac{Q}{2\pi r_0^4}\alpha_s(t)r_0 Y_{\ell m}(\vartheta,\phi)$$

Free Surface Model

The energy of resonance:

$$E_{\ell n} = \sqrt{\frac{2\hbar^2 J}{M r_0^2}} x_{\ell n}(A) A^{-1/3},$$

where $x_{\ell n}(A)$ is the root of equation

$$j_\ell'(x) = \frac{2NZ}{A^2}\frac{3Jx}{QA^{1/3}} j_\ell(x).$$

Transition densities $\Rightarrow$ volume and surface contributions:

$$\delta\rho_{\ell n}(r) = \alpha_{\ell n}^- \rho_0 [j_\ell(x_{\ell n}(r/R)) w(r) - (j_\ell'(x_{\ell n})/x_{\ell n}) R w'(r)].$$

The limiting cases:

Steinwedel-Jensen Model

$Q \to \infty$, $\Rightarrow j_\ell'(x) = 0$, $E_{\ell n} = \sqrt{\frac{2\hbar^2 J}{M r_0^2}} x_{\ell n} A^{-1/3}$ Transition densities: $\Rightarrow$ volume contribution:

$$\delta\rho_{\ell n}(r) = \alpha_{\ell n}^- \rho_0 j_\ell(x_{\ell n}(r/R)) w(r).$$

Goldhaber-Teller Model

$J \to \infty$, $\Rightarrow E_\ell(A) = \sqrt{\frac{\hbar^2 Q \ell}{3 M r_0^2}\frac{A^2}{NZ}} A^{-1/6}$

Transition densities: $\Rightarrow$ surface contribution:

$$\delta\rho_1(r) = \alpha_\ell^- \rho_0 R w'(r).$$

Isovector GMR

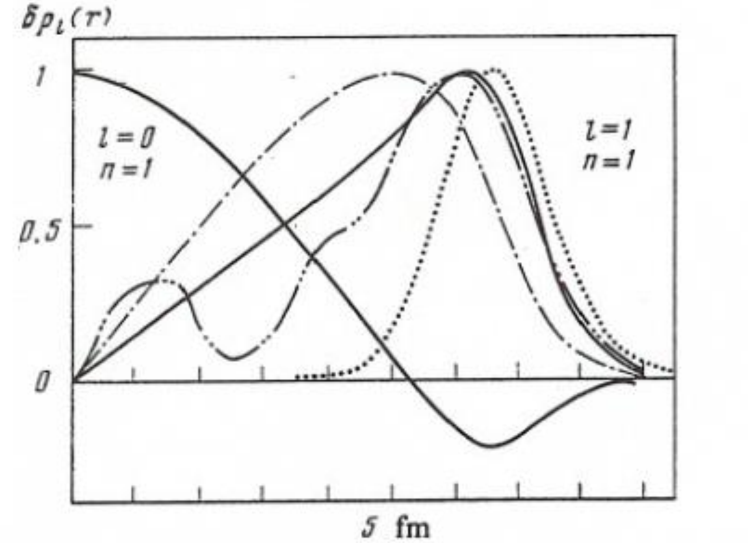
Transition densities:

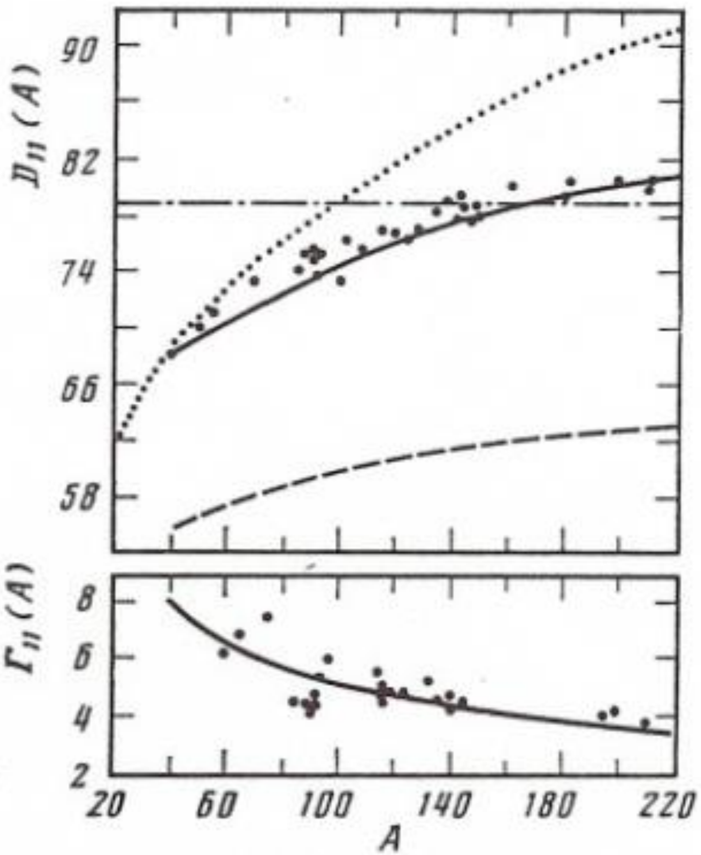
$$\Delta\rho(t) = \rho_p(t) - \rho_n(t)$$

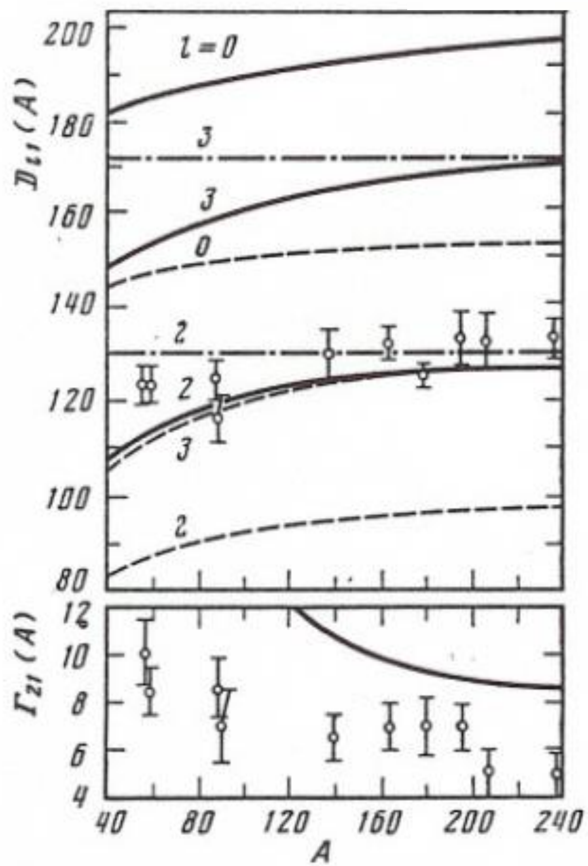
$$= \alpha_\ell(t)\, \rho_0\, f(r)\, Y_{\ell 0}(\theta) + \delta_\ell(t)\, \rho_0\, R\, f'(r)\, Y_{\ell 0}(\theta).$$

Energy

$$E_\ell = D_\ell\, A^{-1/3}$$







Deformed Nuclei

$E_{sphere} \sim 1/R$;
$\Gamma_{sphere} \sim 0.02\,(E_{sphere})^2$

$E_{long\ axis} \sim 1/[R(1+2c\beta)]$;
$\Gamma_{long\ axis} \sim 0.02\,(E_{long\ axis})^2$

$E_{short\ axis} \sim 1/[R(1-c\beta)]$
$\Gamma_{short\ axis} \sim 0.02\,(E_{short\ axis})^2$

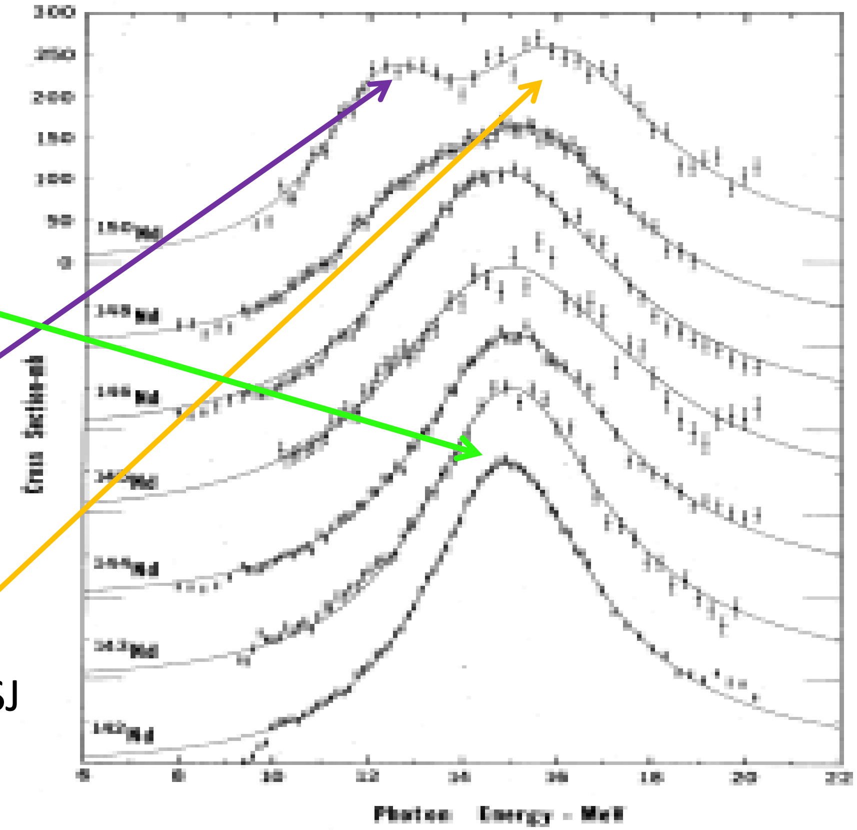
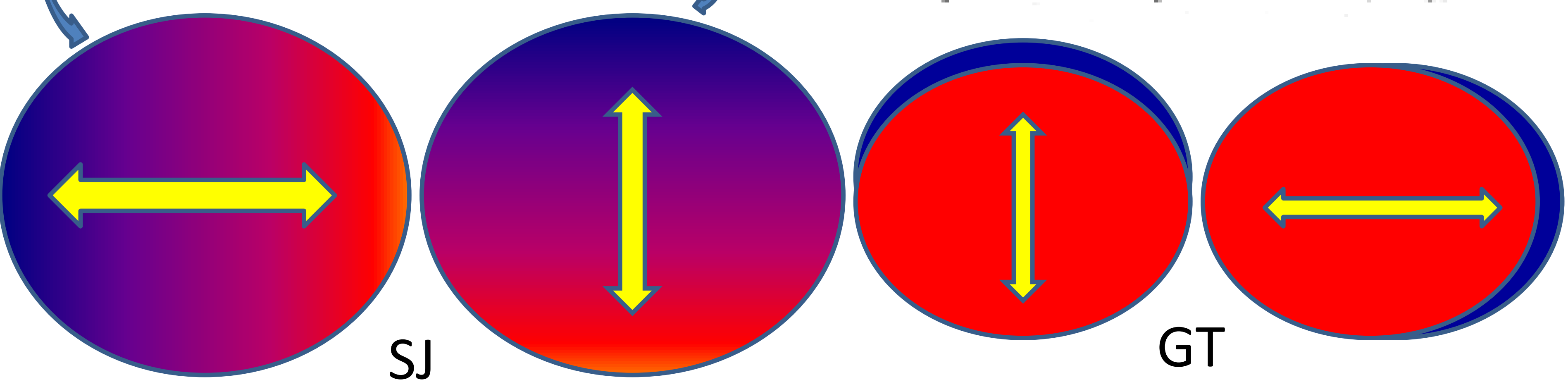
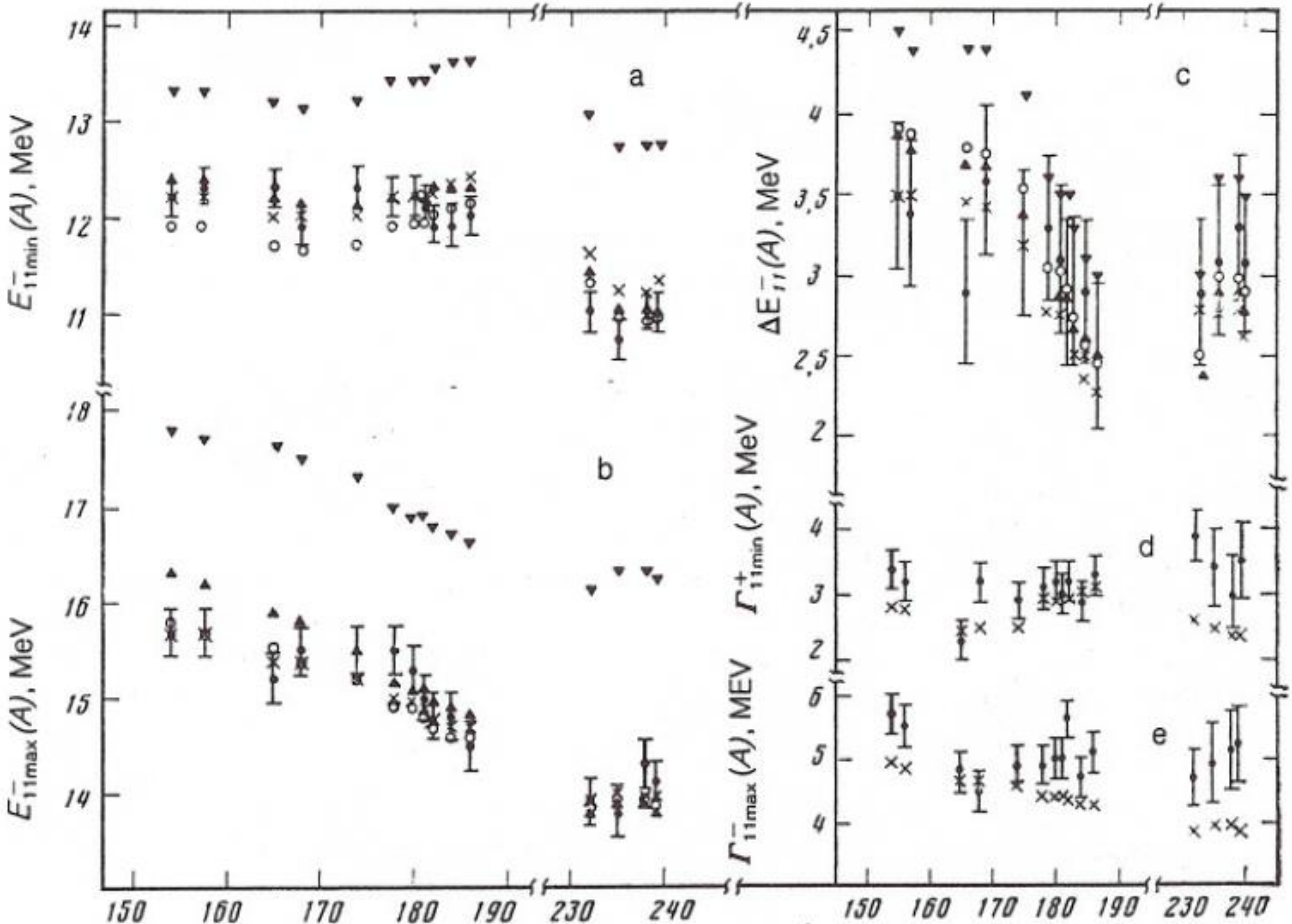


FIG. 27. Total photoneutron cross sections for the neodymium isotopes, showing the "evolution" of the giant resonance as one makes the transition from spherical to statically deformed nuclei (Saclay).



Isovector GDR in deformed nuclei



GIANT RESONANCES IN HOT NUCLEUS.

The Vlasov equation for $f^-(\mathbf{r},\mathbf{p},t)$ with the collision integral:

$$\frac{\partial f^{\pm}(\mathbf{r},\mathbf{p},t)}{\partial t}+$$
$$+\frac{\mathbf{p}}{M}\vec{\nabla}_r\left[f^{\pm}(\mathbf{r},\mathbf{p},t)+\delta(\varepsilon-\varepsilon_f)\frac{\pi^2\hbar^3}{p_f M}\int d\tau F^{\pm}f^{\pm}(\mathbf{r},\mathbf{p},t)\right]=$$
$$=\frac{1}{i\tau}\int d\Omega_k Y_{\ell m}(\vartheta,\phi)\left[f_k^{\pm}(\mathbf{r},\mathbf{p},t)-\right.$$
$$\left.-\frac{1}{p_f^3}\left(\int d\tau' f_k^{\pm}(\mathbf{r},\mathbf{p}',t)+3cos(\mathbf{pk})\int d\tau' cos(\mathbf{p}'\mathbf{k})f_k^{\pm}(\mathbf{r},\mathbf{p}',t)\right)\right],$$

where $F^{\pm}=F_{PP}\pm F_{PN}$.

The width of 0-sound is given by

$$\Gamma_0(A)=a_0\left(E_0^2(A)+(2\pi T)^2\right),$$

where T is the temperature of nucleus,

$$a_0=\frac{\hbar\rho\varepsilon_f}{10\pi^2\eta_0}=0,02\ MeV^{-1},$$

$$\eta_0=1,84\cdot 10^{-21}(MeV/fm)^3 sec$$

is the parameter of quasiparticle viscosity of nucleus $\eta=\eta_0/T^2$.

The mean free time in Fermi-liquid theory is equal

$$\tau = \frac{5\eta_0}{2\rho\varepsilon_F T^2}.$$

0-sound $\Rightarrow$ *rare collisions regime*: The frequency of vibrations $\omega = \frac{E(A)}{\hbar}$ is smaller then frequency of quasi-particle collisions $\omega_{col} = 1/\tau$.

0-sound $\Rightarrow$ $\omega\tau \ll 1$ rare collisions, small T (could and warm nuclei)

1-sound $\Rightarrow$ $\omega\tau \gg 1$ frequent collisions, high T (very hot nuclei)

The width of 1-sound vibration (hydrodynamic):

$$\Gamma_1(A) = a_1 \left(\frac{E_1(A)}{T}\right)^2,$$

where

$$a_1 = \frac{\eta_0}{(1+F_0^-)\varepsilon_f \rho\hbar} = 0,2MeV.$$

Transition region $\omega\tau \approx 1 \Leftrightarrow$ strong dissipation and attenuation of collective motion.

The critical temperature T_{cr} at $\omega\tau = 1$:

$$T_{cr} = \sqrt{\frac{5\eta_0 E(A)}{2\bar{\rho}\varepsilon_f \hbar}} \simeq 1,12\sqrt{E(A)}.$$

For $A \approx 100 - 150$ ($E(A) \sim 15 - 16$ MeV) $\Rightarrow T_{cr} \sim 4,5MeV$.

T_{cr}: frequency of collision coincides with frequency of collective vibrations and the collective motion strongly dissipated.

Before we proposed that $\Gamma_{total} = \Gamma_{collision}$, but

$$= \Gamma_{collision} + \Gamma_{escape} + \Gamma_{Landau} + \Gamma_{fragmentation} + \ldots$$

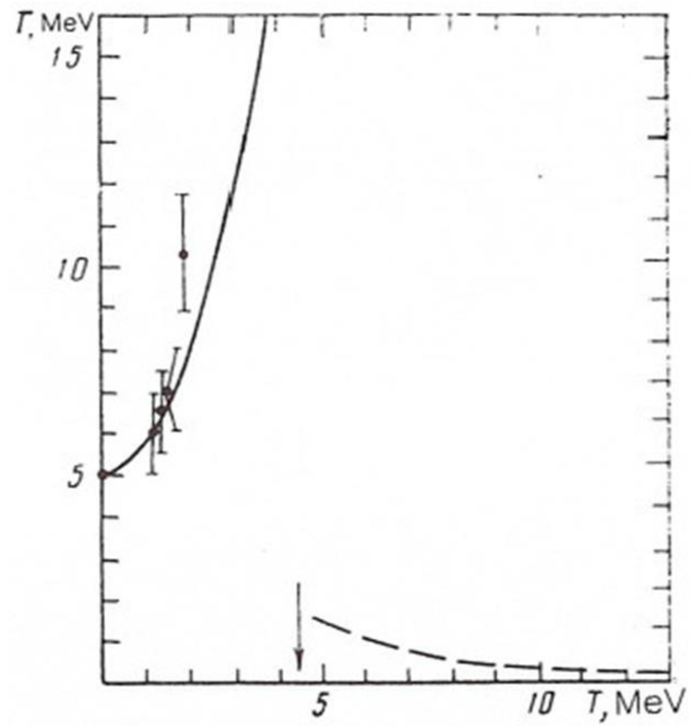
We propose that

$$\Gamma_{collision} = \frac{1}{\gamma}\Gamma_{total},$$

where $\gamma > 1$. In this case

$$T_{cr}(\gamma) = \sqrt{\gamma \frac{5\eta_0 E(A)}{2\bar{\rho}\varepsilon_f \hbar}} = \sqrt{\gamma} T_{cr}(\gamma = 1).$$

$T_{cr}(\gamma = 1) = 4.5$ MeV, $T_{cr}(\gamma = 2) = 6.3$ MeV, $T_{cr}(\gamma = 4) = 9$ MeV



Corsi, et al., APPB42,619, (2011): Analysis has shown that the hindrance of GDR decay in the selfconjugate nucleus 80Zr makes possible the evaluation of the degree of isospin mixing present in a highly excited compound nucleus

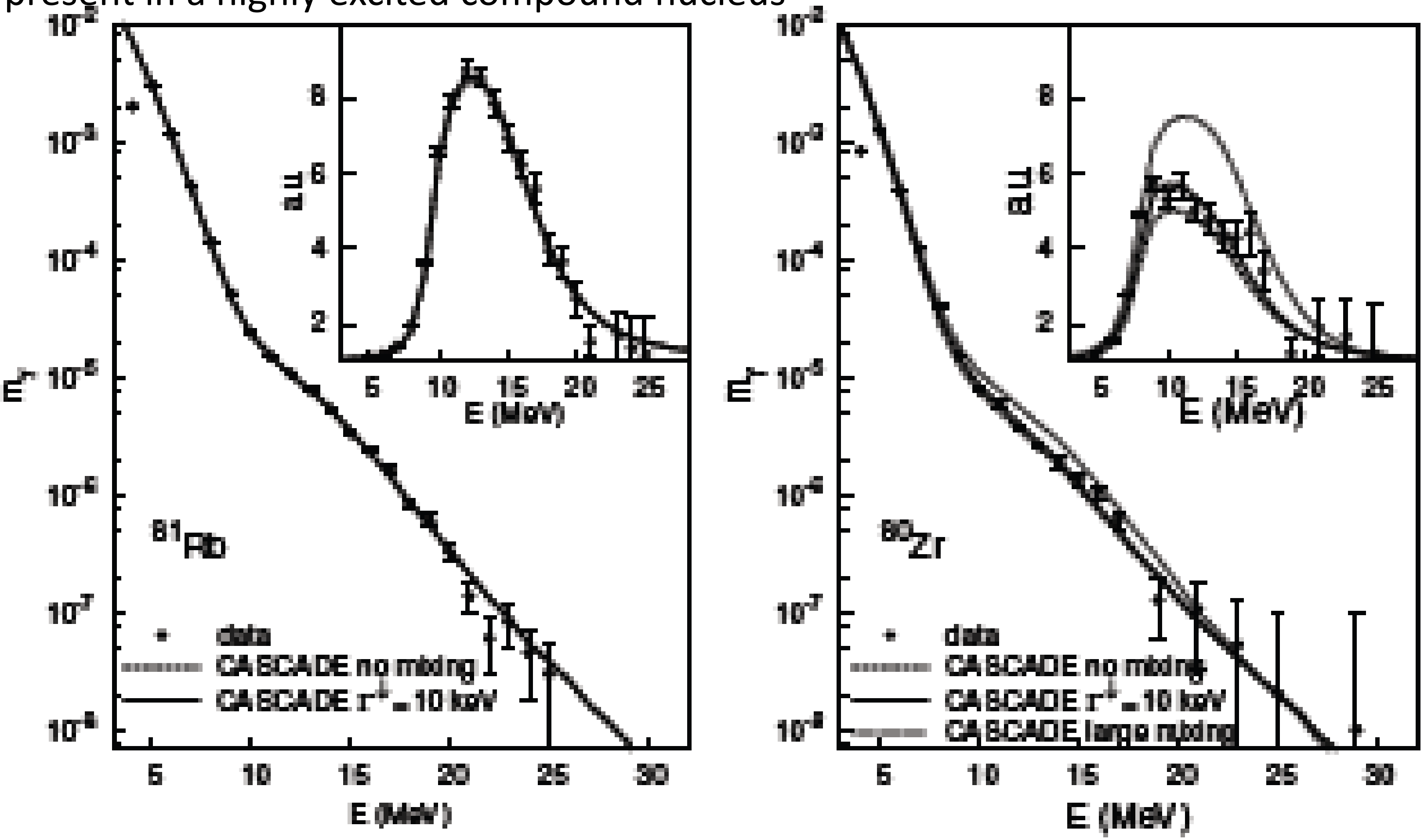


Fig. 2. The high energy γ-ray spectra measured in the reaction $^{37}Cl+^{44}Ca \to ^{81}Rb$ at $E^* = 83$ MeV (left panel) and in the $^{40}Ca+^{40}Ca \to ^{80}Zr$ ($E^* = 83$ MeV) reaction (right panel). In the insets, the linearised spectra are shown. The statistical model calculations (see text) are displayed with continuous line [22, 21].

In general light-particle emission is much more probable than γ-decay, but the latter, which has a probability of the order of 10^{-3}, is a more useful probe of the GDR properties since the γ-ray carries all the energy of the resonance. The decay rate R_γ is given by

$$R_\gamma \mathrm{d}E_\gamma = \frac{\rho(E_2)}{\hbar\rho(E_1)} f_{GDR}(E_\gamma)\mathrm{d}E_\gamma\,, \qquad (4)$$

where $\rho(E_1)$ and $\rho(E_2)$ are, respectively, the level densities for the initial and final states which differ by an energy $E_\gamma = E_1 - E_2$ and $f_{GDR}(E_\gamma) \propto \sigma_{abs} E_\gamma^2$. It can be wri as

$$f_{GDR}(E_\gamma) = \frac{4e^2}{3\pi\hbar m c^3} S_{GDR} \frac{NZ}{A} \times \frac{\Gamma_{GDR} E_\gamma^4}{(E_\gamma^2 - E_{GDR}^2)^2 + E_\gamma^2 \Gamma_{GDR}^2}\,,$$

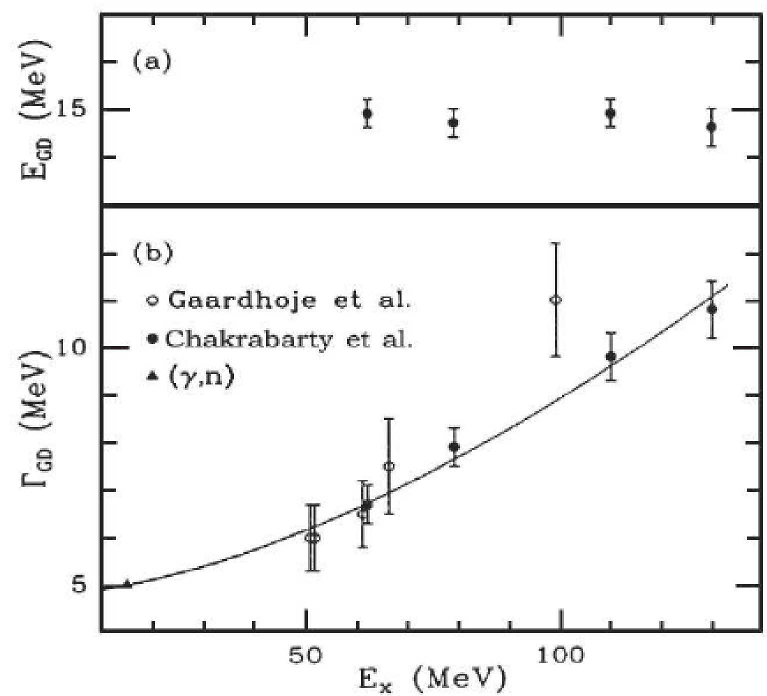


Fig. 2. The systematics for the energy (a) and width (b) of the GDR as a function of E^*. Open symbols are from refs. [23, 24] while full circles are from ref. [25]. The full line corresponds to the parametrization of the width given by eq. (6).

D. Santonocito and Y. Blumenfeld, Eur. Phys. J. A 30, 183 (2006)

GDR in hot nuclei

Heavy-ion fusion → Compound nucleus → Gamma-Decay

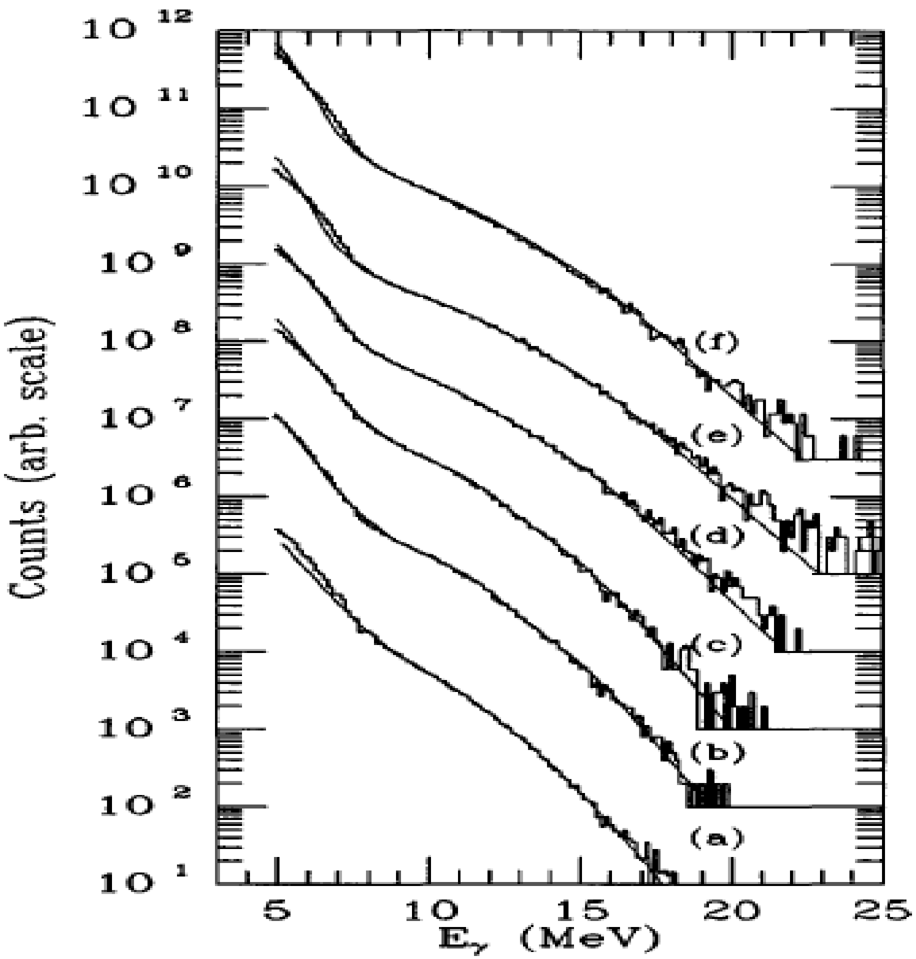


Figure 1
Experimental γ-spectra and the CASCADE fits in the reaction $^{19}F + {}^{181}Ta$ at 93(a), 105(c), 126(d), 141(f) MeV, and in $^{16}O + {}^{nat}W$ at 100(b), 140(e) MeV.

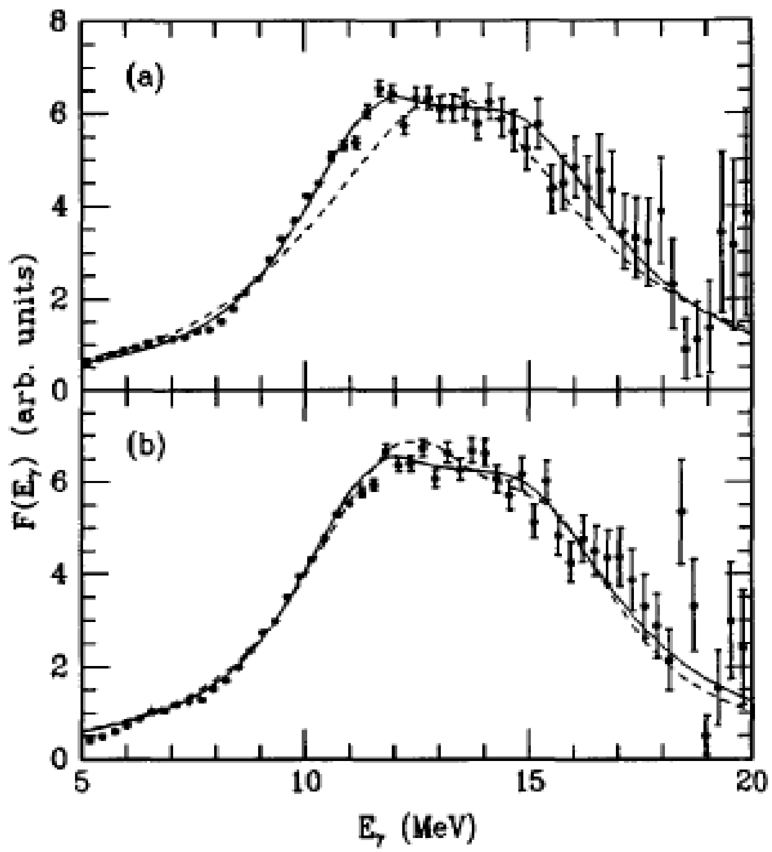


Figure 2
Divided plots of data and fits in (a)$^{16}O + {}^{nat}W$ at 100 MeV and (b) $^{19}F + {}^{181}Ta$ at 105 MeV. Solid curves are prolate fits. Dashed curve in (a) is a single component and in (b) is an oblate fit.

Multi-phonon giant resonances

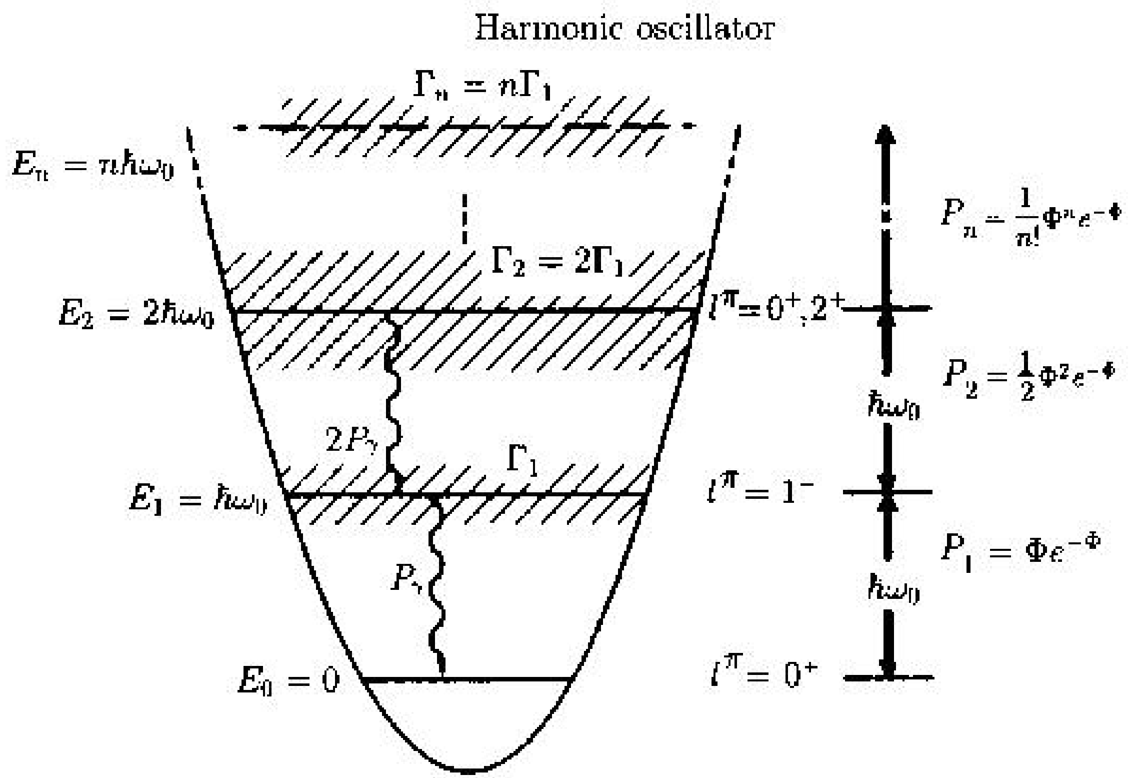


FIG. 9.2. Multiphonon states of a one-dimensional linear harmonic vibrator of $J^\pi = 1^-$ phonons (IVGDR). After (EML94).

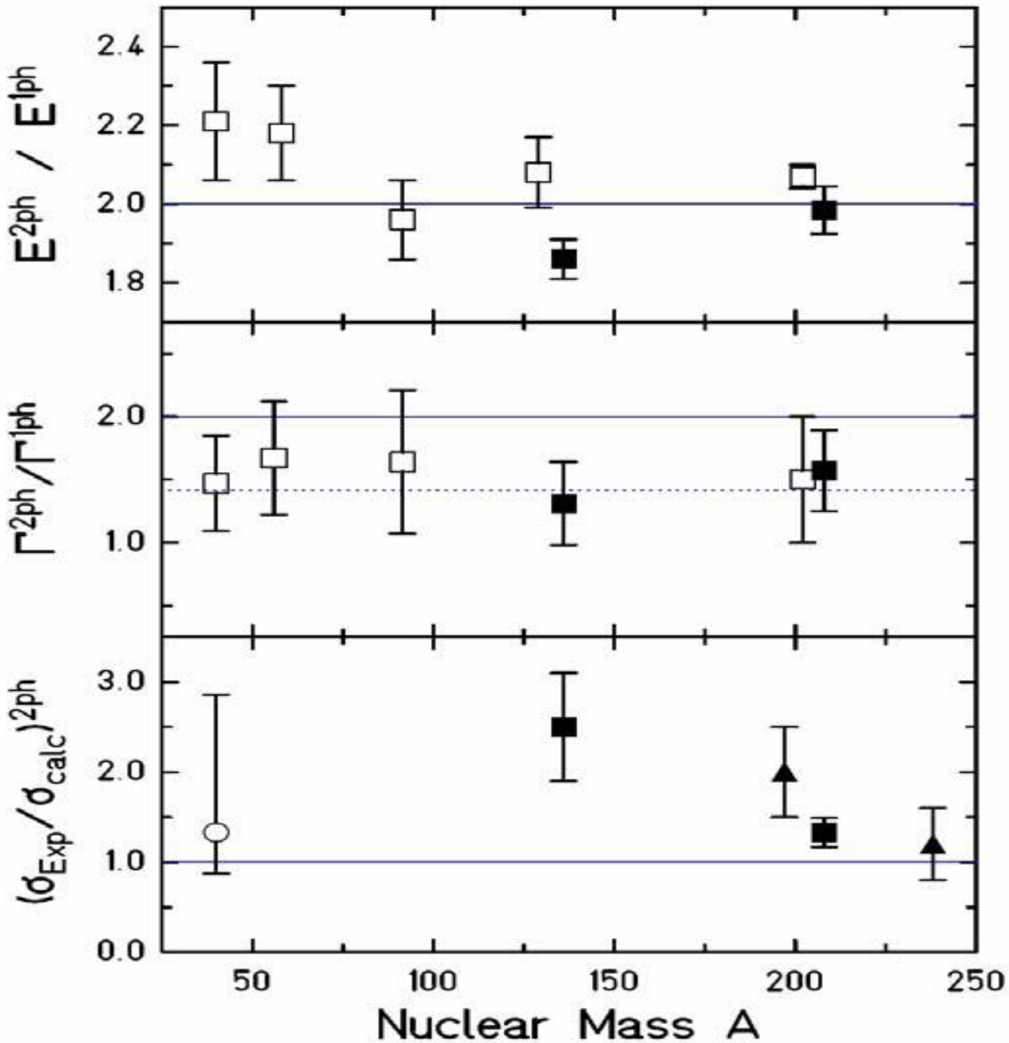
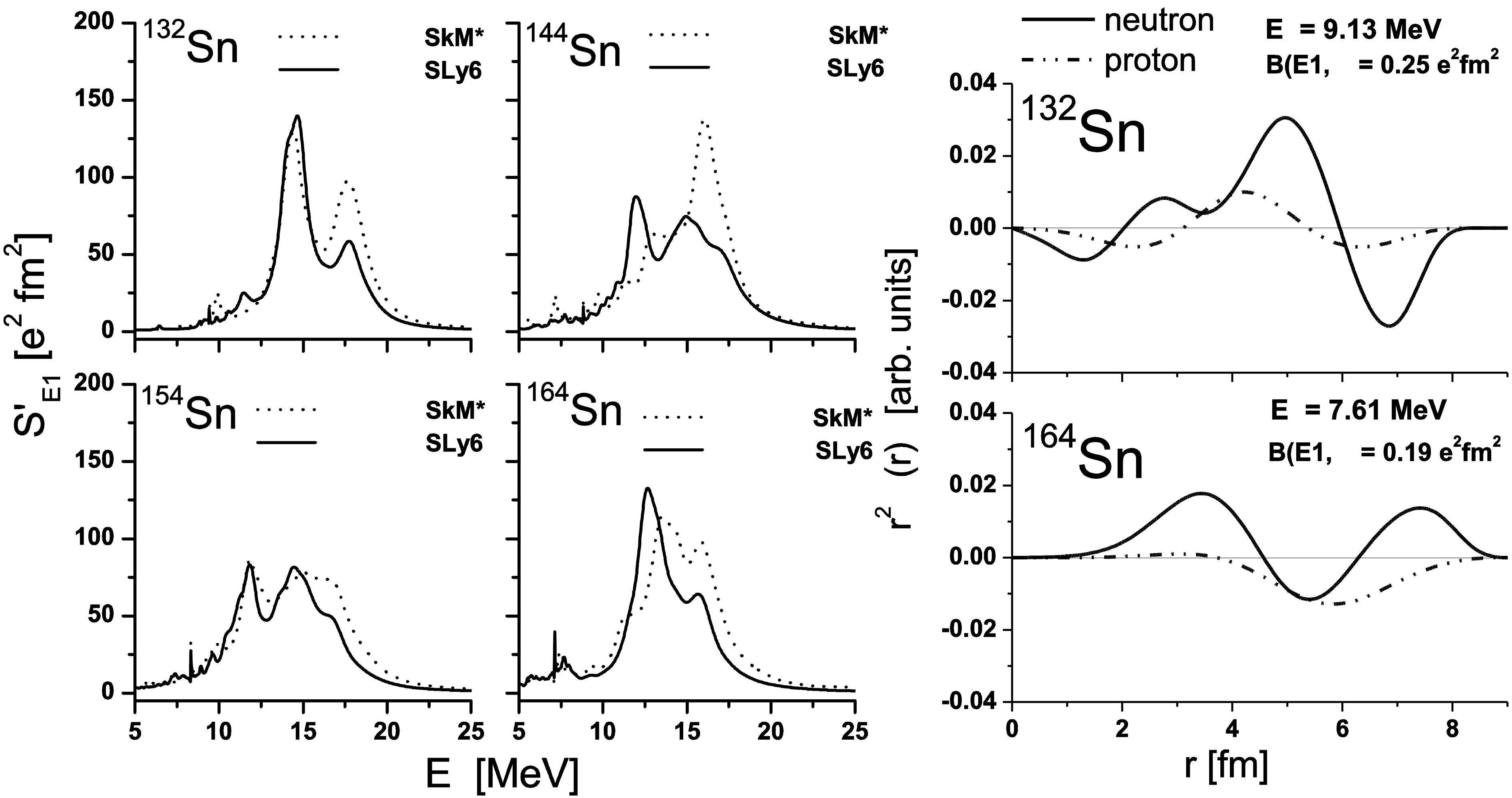
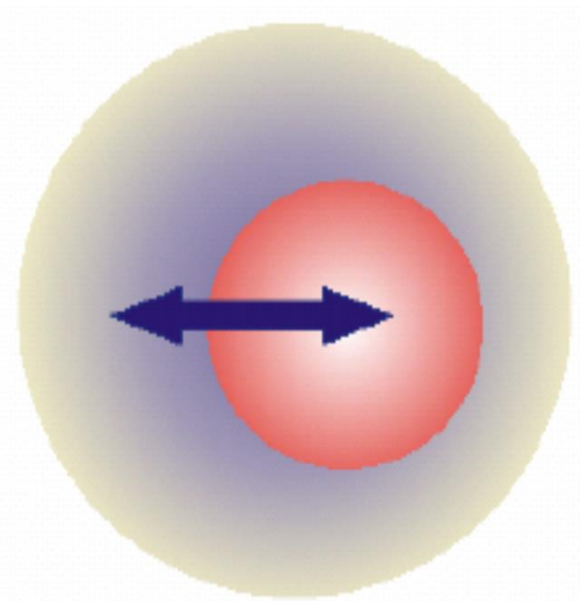


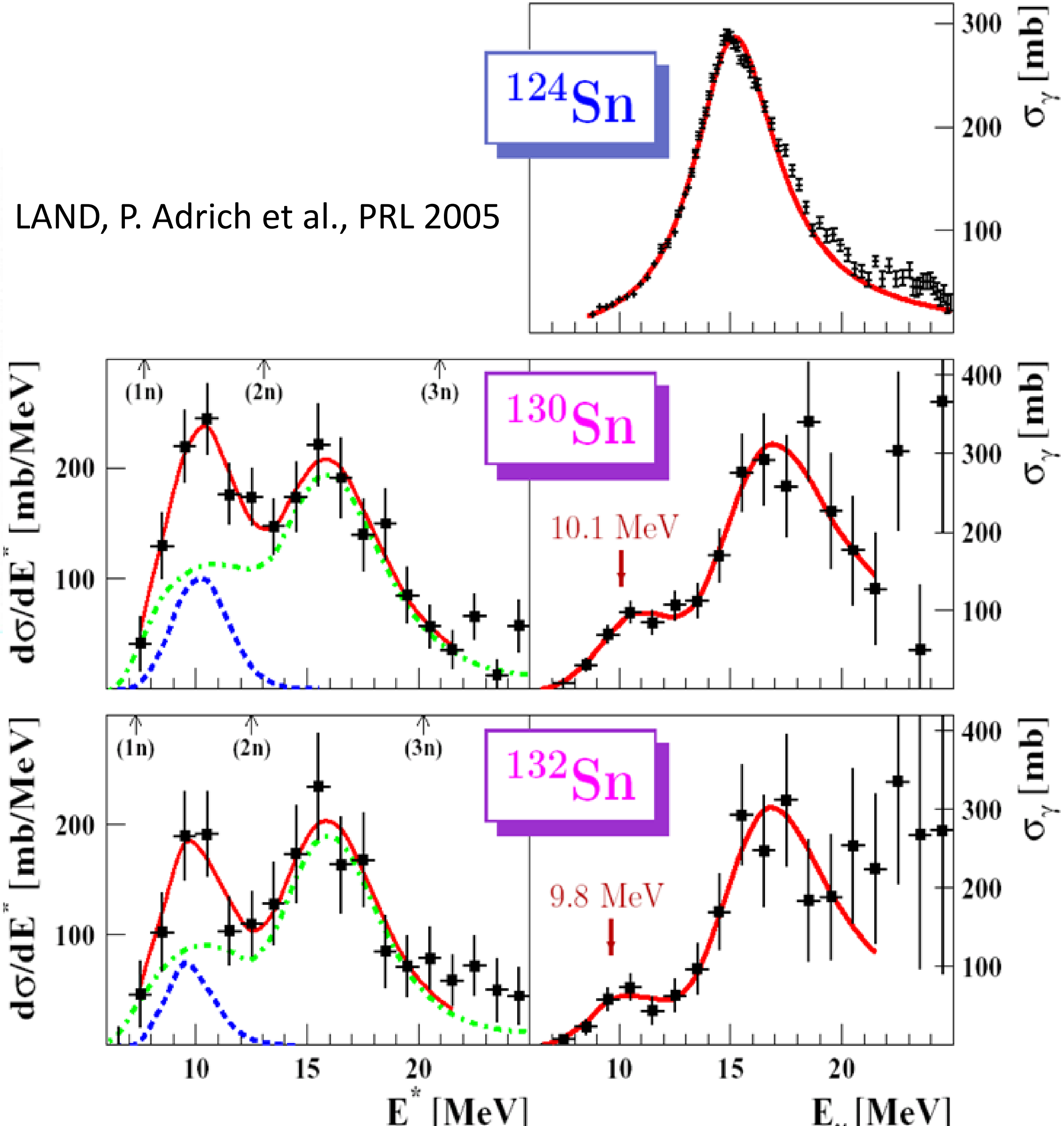
Figure 16 *Top*: Peak energies E of two-phonon states relative to those of one-phonon states. *Full* and *open symbols* denote data for dipole resonances from heavy-ion experiments and from pion charge exchange reactions, respectively. The *solid line* indicates the value expected in the harmonic limit. *Middle*: Same as above, but for the width Γ. The *dashed lines* correspond to a value of $\sqrt{2}$. *Bottom*: Ratio of experimental to calculated electromagnetic cross sections for the double giant dipole resonance (*triangles*: inclusive measurements; *squares*: exclusive measurements). The calculations were made in the harmonic approximation using the folding model. The *circle* denotes a corresponding value for the double giant quadrupole resonance.

Pigmi Resonances





LAND, P. Adrich et al., PRL 2005



Висновки

- Розрізняють повільні (поверхневі) та швидкі (об’ємні та поверхневі) ізоскалярні та ізовекторні коливання різної мультипольності
- Об'ємні та поверхневі ізоскалярні та ізовекторні коливання пов'язані граничними умовами.
- У ядрах є два різних режими: нульовий та перший звук.

Дякую за увагу!!!