SUBBARRIER FUSION OF HEAVY NUCLEI

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The total fusion cross section $\sigma_{fus}(E)$:

$$\sigma_{fus}(E) = \sum_{\ell=0}^{\infty} \sigma_{fus}(E, \ell).$$

The partial-wave cross sections $\sigma_{fus}(E, \ell)$

$$\sigma_{fus}(E, \ell) = \frac{\pi^2 \hbar^2}{2 M_N A_{12} E} (2\ell + 1) T_\ell(E).$$

The transmission coefficient at the collision energy $E \leq U_{bar}$

$$T_\ell(E) = [1 + exp(2\mathcal{A}_\ell(E)/\hbar)]^{-1}.$$

Here U_{bar} is the height of effective potential between ions.

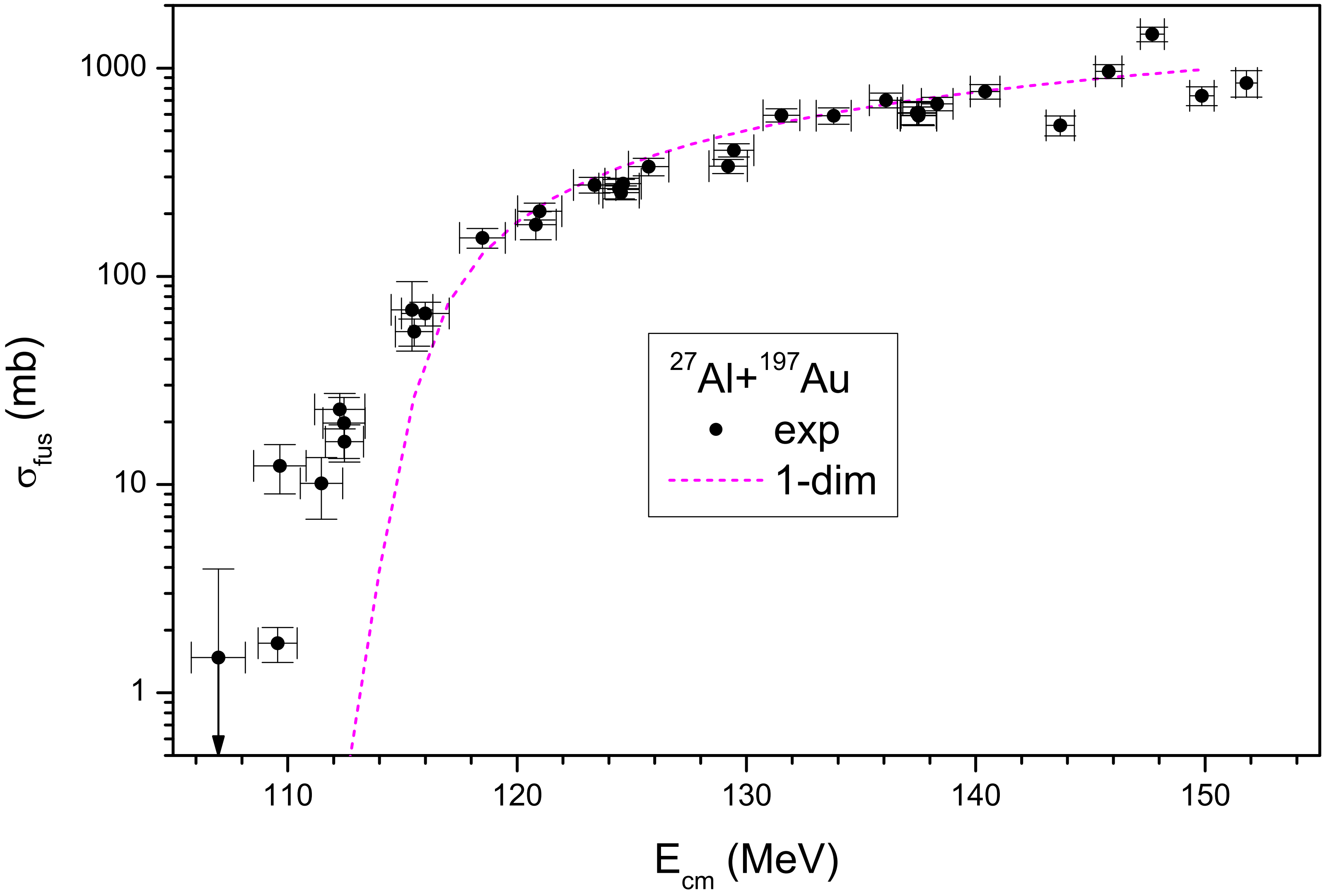
The action $\mathcal{A}_\ell(E)$ in WKB approximation:

$$\mathcal{A}_\ell(E) = \int_T dD [2(U_\ell(D) - E) M_{DD}]^{1/2},$$

where T is the fusion trajectory, $U_\ell(D)$ is the effective potential, $M_{DD} = M_N A_{12} = M_N \frac{A_1 A_2}{A_1 + A_2}$ is the mass parameter, D is the distance between the centers of masses of colliding nuclei.

The effective potential:

$$U_\ell(D) = \frac{Z_1 Z_2 e^2}{D} + V_{n-n}(D) + \frac{\hbar^2 \ell(\ell + 1)}{M_N A_{12} D^2}.$$



CONCLUSIONS: The one-dimensional model is strongly underestimated the cross section of subbarrier heavy-ion fusion.

Barrier penetration

Enhancement of barrier penetration due to coupling with the low-energy 2^+ and 3^- vibrational states

The system of coupled channel equations

$$\left[-\frac{\hbar^2}{2\mu_i}\frac{d^2}{dr^2}+\frac{\hbar^2\ell_i(\ell_i+1)}{2\mu_i r^2}+V(r)-Q_i-E\right]\varphi_i(r)=-\sum_j V_{ij}(r)\varphi_j(r),$$

where $\psi_i(r)=\varphi_i(r)/r$ is the wave function.

The coupling potential between the ground state and the channels connected with the low-energy surface vibrational state of multipolarity λ

$$V_{0i}=\frac{\beta_i R_i}{\sqrt{4\pi}}\left[\frac{dV_{\mathrm{i-i}}(r)}{dr}+\frac{3}{2\lambda+1}\frac{z_1 z_2 e^2 R_i^{\lambda-1}}{r^{\lambda+1}}\right].$$

Approximate solution: diagonalization at barrier. We diagonalize the system of coupled channel equations with the help of the substitution $\varphi_i(r)=\sum_k U_{ik}\xi_k(r)$

The coupling matrix $\mathcal{M}_{ij}$ takes the form

$$\sum_{ij}U_{ki}\mathcal{M}_{ij}U_{jl}=\sum_{ij}U_{ki}[-Q_i\delta_{ij}+V_{ij}(\overline{R})]U_{jl}=\epsilon_k\delta_{kl}.$$

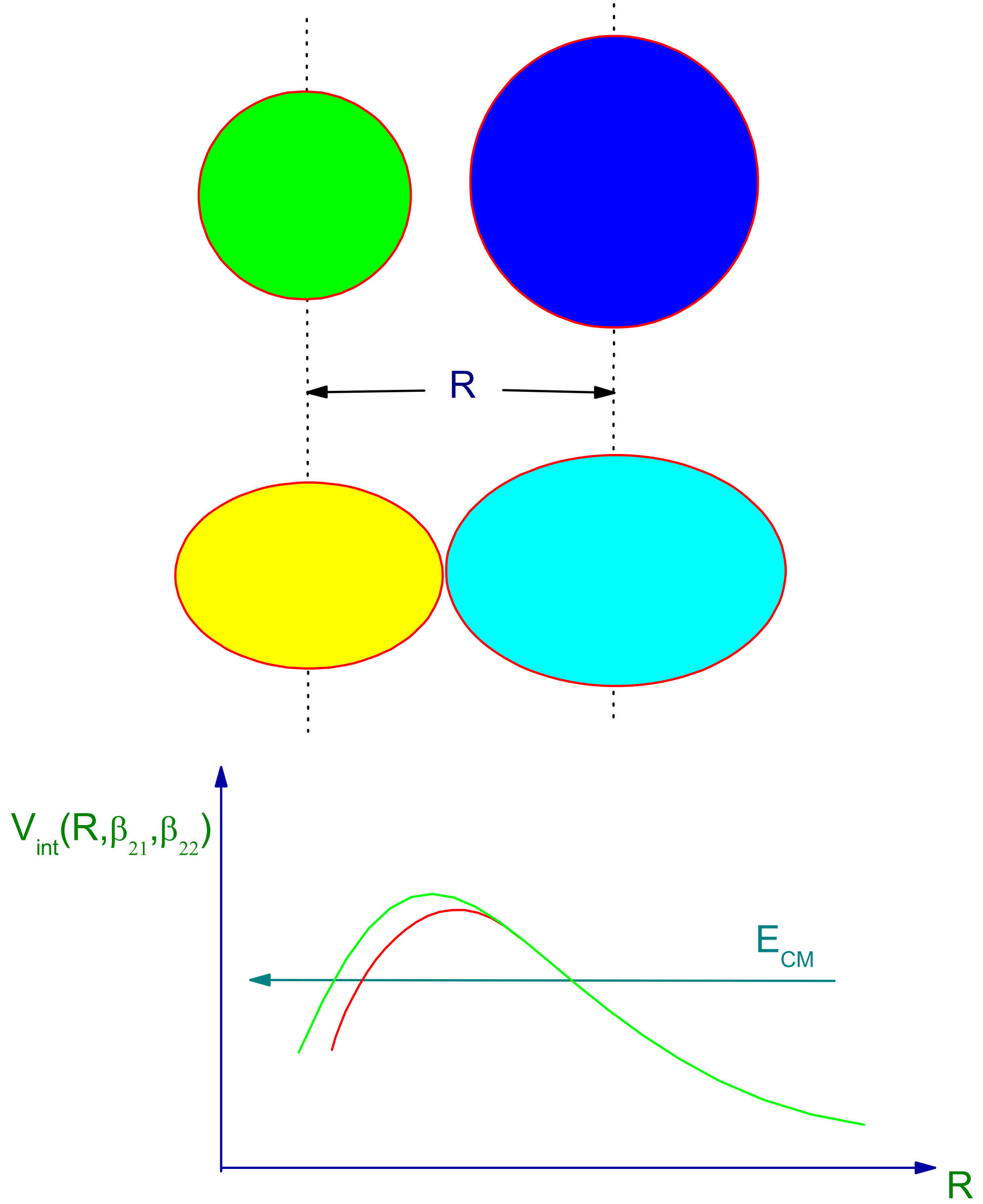
We find the eigenvalue ϵ_k by a diagonalization.

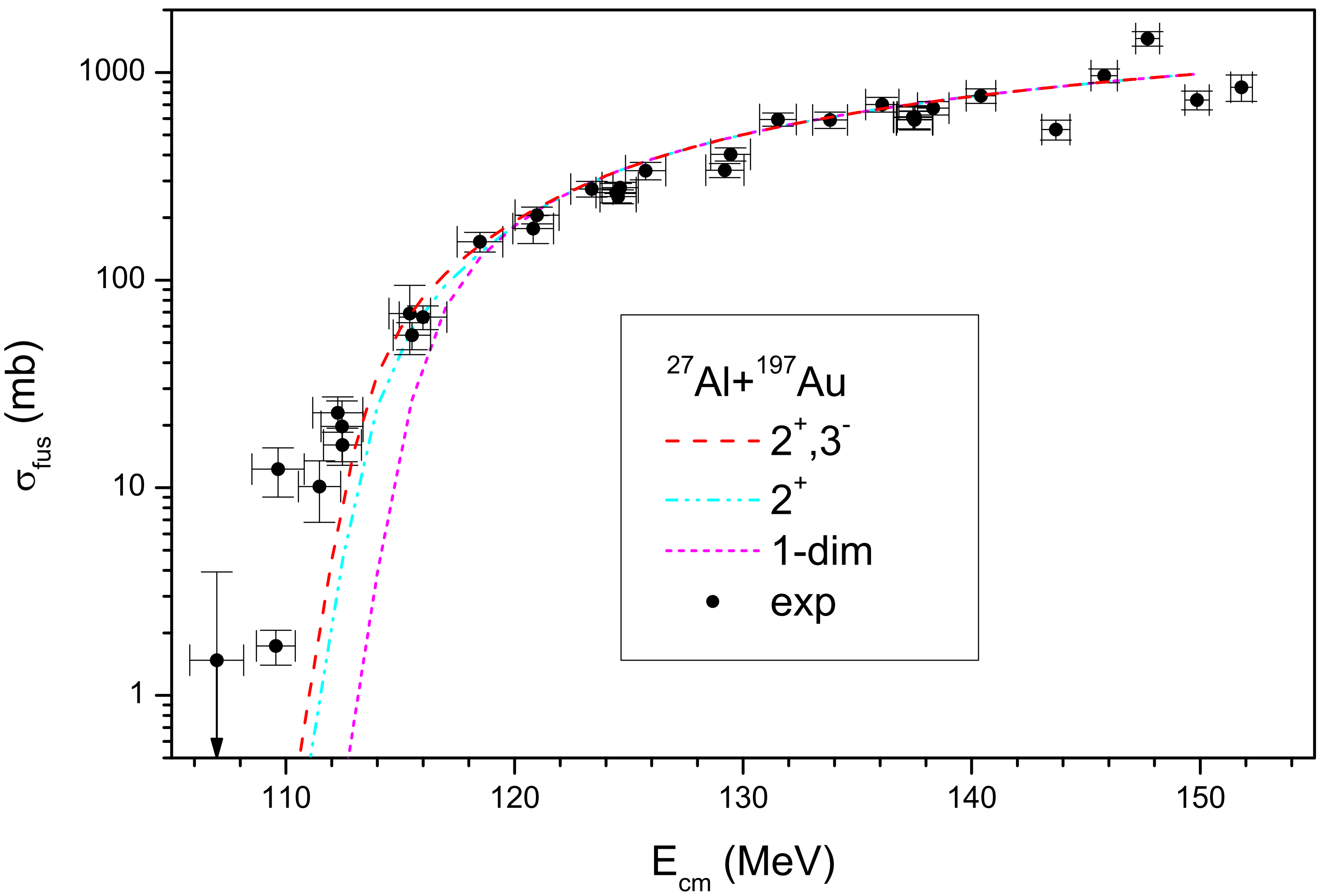
Transmission coefficient $\mathcal{T}(E,\ell,A,Z)$ is equal to

$$\mathcal{T}(E,\ell,A,Z)=\sum_k|U_{k0}|^2T(E,\mathcal{V}_{\ell k}),$$

$$\mathcal{V}_{\ell k}(r)=V_\ell(r)+\epsilon_k=V(r)+\hbar^2\ell(\ell+1)/(2\mu r^2)+\epsilon_k.$$

$$V_{int}(R,\beta_{21},\beta_{22})=V_{Nucl}(R,\beta_{21},\beta_{22})+V_{Coul}(R,\beta_{21},\beta_{22})$$





$$1\text{-dim} \Rightarrow R = R_0$$

$$2^+ \Rightarrow R = R_0(1 + \beta_2 Y_{20}(\theta)), \quad Y_{20}(\theta) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1);$$

$$2^+\&3^- \Rightarrow R = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_3 Y_{30}(\theta)), \quad Y_{30}(\theta) = \sqrt{\frac{7}{16\pi}}(5\cos^2\theta - 3)\cos\theta;$$

Enhancement of barrier penetration due to transfer of neutrons

DWBA approach.

If $E < V_{barrier}, V^{tr}_{barrier}$, transfer - at the distance $r_{\rm tr}$,

$$T(E, \mathcal{V}^i_{\ell k}, \mathcal{V}^f_{\ell k}) = 1/\{1 + \exp[\mathcal{A}(E, \mathcal{V}^i_{\ell k}, \mathcal{V}^f_{\ell k}, r_{\rm tr})]\},$$

$$\mathcal{A}(E, \mathcal{V}^i_{\ell k}, \mathcal{V}^f_{\ell k}, r_{\rm tr}) = \mathcal{A}^i(E, \mathcal{V}^i_{\ell k}, r_{\rm tr}) + \mathcal{A}^{\rm tr}(E, r_{\rm tr}) + \mathcal{A}^f(E, \mathcal{V}^f_{\ell k}, r_{\rm tr}).$$

The action related to the tunneling of nuclei in an effective potential before nucleons transfer

$$\mathcal{A}^i(E, \mathcal{V}^i_{\ell k}, r_{\rm tr}) = (2/\hbar) \int_{r_{\rm tr}}^{r^i_{\ell k}} \sqrt{2\mu_i(r)(\mathcal{V}^i_{\ell k}(r) - E)}dr,$$

the action related to the tunneling of nuclei in an effective potential after nucleons transfer

$$\mathcal{A}^f(E, \mathcal{V}^f_{\ell k}, r_{\rm tr}) = (2/\hbar) \int_{r^f_{\ell k}}^{r_{\rm tr}} \sqrt{2\mu_f(r)(\mathcal{V}^f_{\ell k}(r) - E)}dr,$$

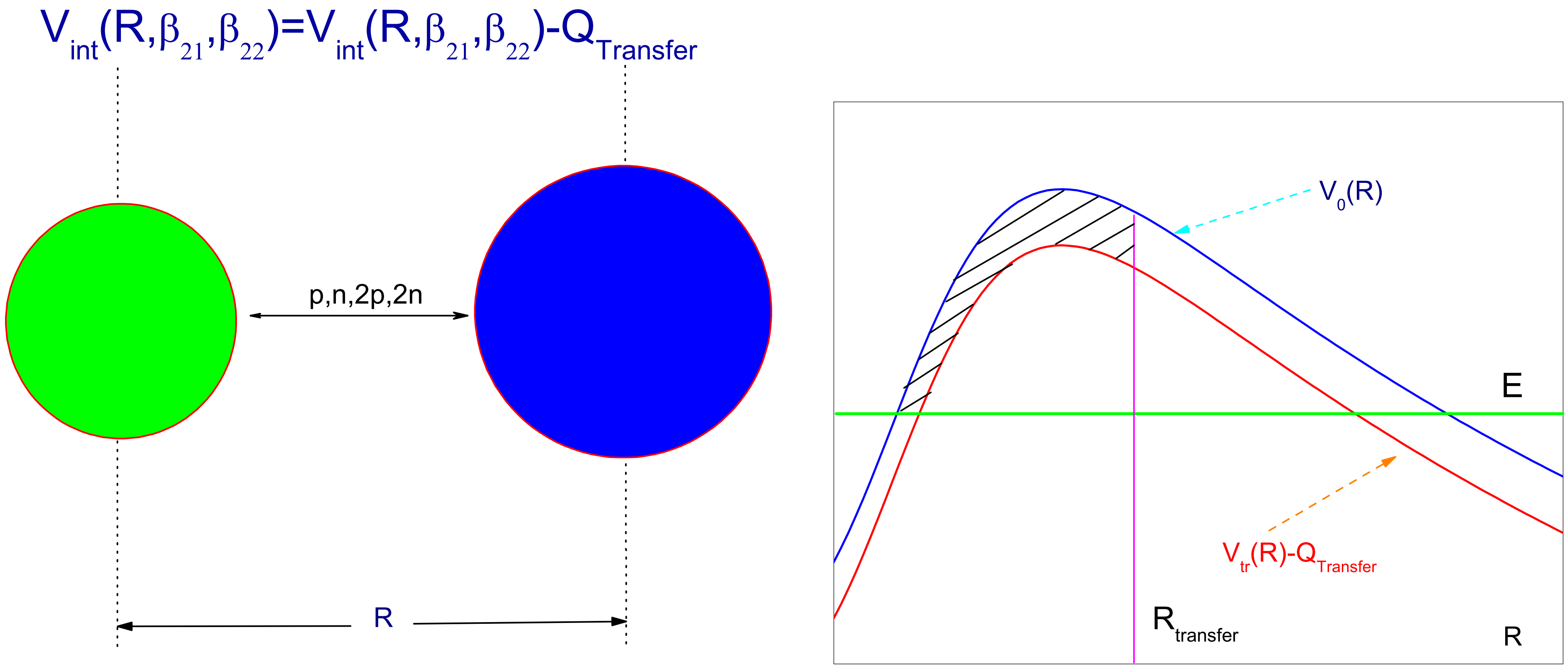
an effective potential

$$\mathcal{V}^f_{\ell k}(r) = V^f_\ell(r) + \epsilon_k - Q^f_{\rm transfer}.$$

The action related to transfer of m-neutrons

$$\mathcal{A}^{\rm tr}(E, r_{\rm tr}) = (2/\hbar) \sum_{i=1}^{m} \sqrt{2M\mathcal{E}_i}(r_{\rm tr} - R_{12} - \delta).$$

The tunneling of m neutrons between spherical square potential wells of the colliding ions. δ - simulate finite diffuseness of the realistic nucleon-nucleus potential


V_int(R,β_21,β_22)=V_int(R,β_21,β_22)-Q_Transfer
p,n,2p,2n
R
V_0(R)
E
V_tr(R)-Q_Transfer
R_transfer
R

The expression for the transmission coefficient is valid for collision energies E smaller than the effective barriers $\overline{\mathcal{V}}^i_{\ell k}$, before and $\overline{\mathcal{V}}^f_{\ell k}$, after the few-nucleon transfer.

In the case $\overline{\mathcal{V}}^f_{\ell k} < E < \overline{\mathcal{V}}^i_{\ell k}$ and $r_{\rm tr} > \overline{R}^f_{\ell k}$ the transmission coefficient has the form

$$T(E, \mathcal{V}^i_{\ell k}, \mathcal{V}^f_{\ell k}) = 1/\{1 + \exp[\mathcal{A}^i(E, \mathcal{V}^i_{\ell k}, r_{\rm tr}) + \mathcal{A}^{\rm tr}(E, r_{\rm tr})]\} \;\; T_{\rm HW}(E, \mathcal{V}^f_{\ell k}).$$

Here $\overline{R}^f_{\ell k}$ is the barrier distance of the effective potential $\mathcal{V}^f_{\ell k}$, $T_{\rm HW}(E, \mathcal{V}^f_{\ell k})$ is the transmission coefficient of the effective barrier after transfer obtained in the Hill-Wheeler approximation and taking into account the reflection during barrier penetration. (The Hill-Wheeler approximation is approximation for subbarrier tunneling through the "inverse oscillator" barrier or reflection from the "inverse oscillator" barrier at high energies.)

The subbarrier tunneling of ions before the nucleon transfer and the subbarrier nucleon transfer are described by the first factor.

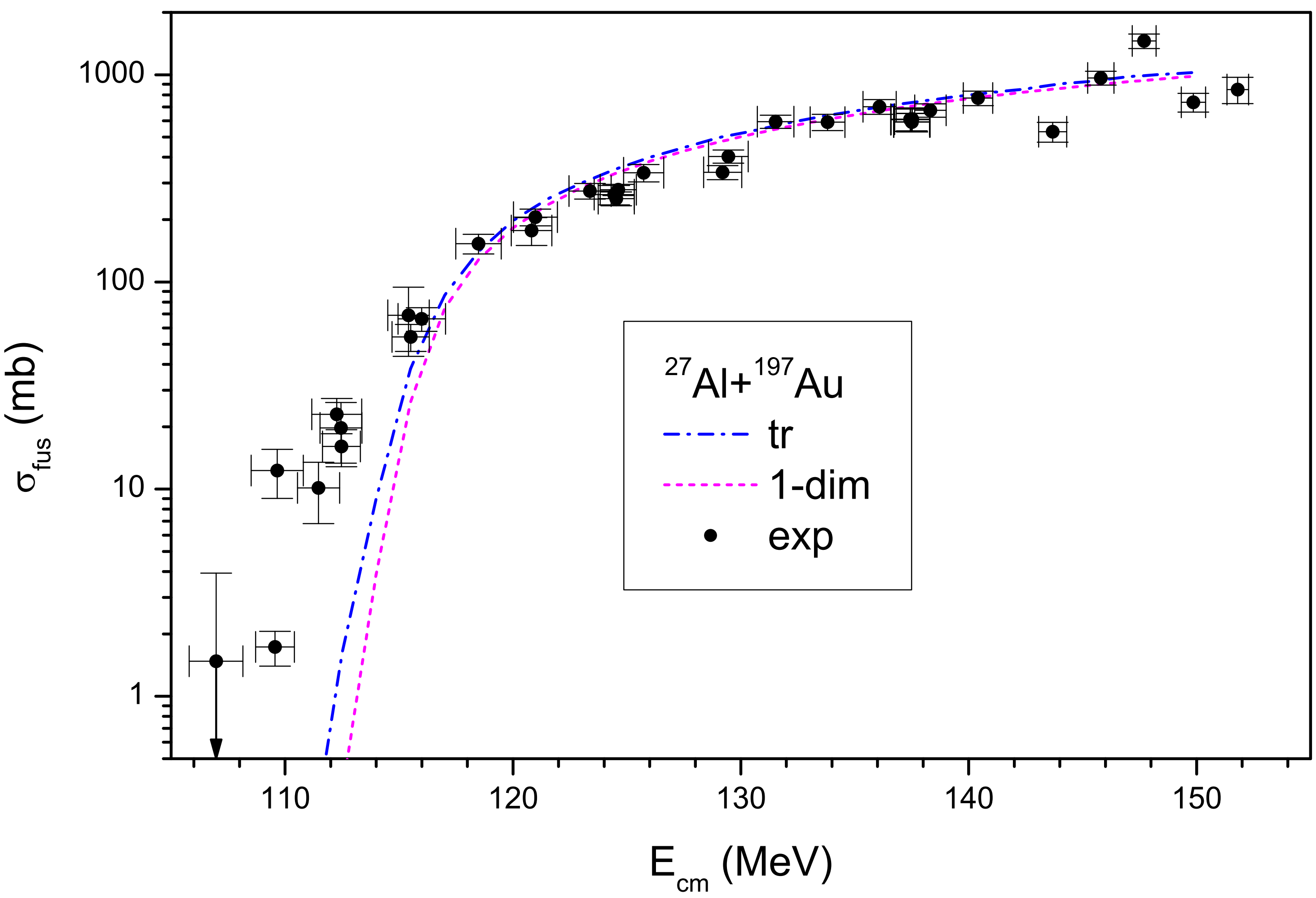
The second factor is related to the transmission above the barrier between the ions after nucleon transfer.

If $\overline{\mathcal{V}}^f_{\ell k} < E < \overline{\mathcal{V}}^i_{\ell k}$ and $r_{\rm tr} < \overline{R}^f_{\ell k}$, then we should take into account the decay of the system after the few-nucleon transfer. In this case the transmission coefficient may be written as

$$T(E, \mathcal{V}^i_{\ell k}, \mathcal{V}^f_{\ell k}) = 1/\{1 + \exp[\mathcal{A}^i(E, \mathcal{V}^i_{\ell k}, r_{\rm tr}) + \mathcal{A}^{\rm tr}(E, r_{\rm tr})]\}(1 - T_{\rm HW}(E, \mathcal{V}^f_{\ell k})).$$

We use the transmission coefficient in the Hill-Wheeler approximation at the high collision energy $E > \overline{\mathcal{V}}^f_{\ell k}$ and $E > \overline{\mathcal{V}}^i_{\ell k}$ and do not employ the enhancement of fusion due to nucleon transfer. Our expressions are written for the case $Q_{\rm transfer} > 0$ and may easily be transformed to the case $Q_{\rm transfer} < 0$.

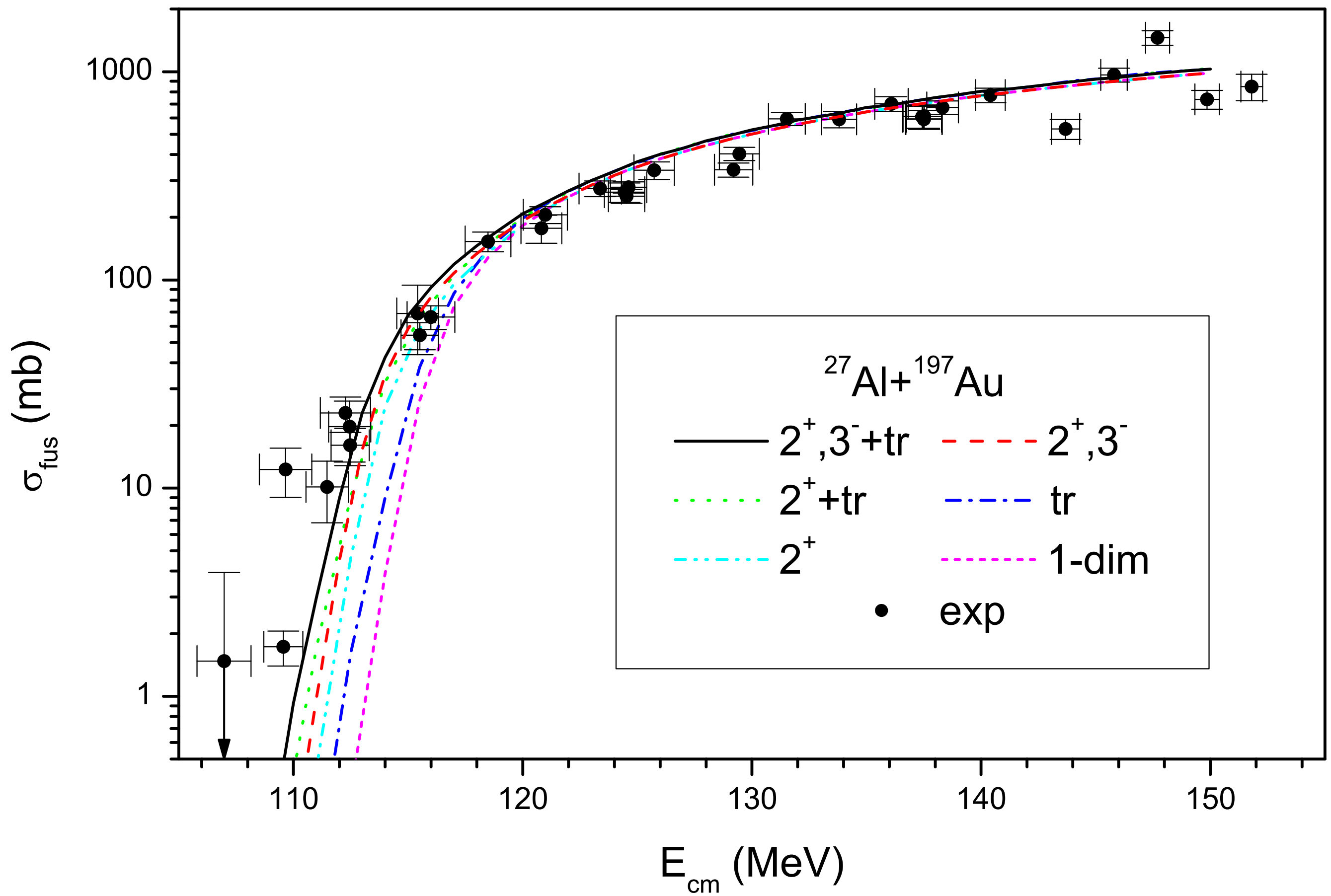
The compound nucleus is formed in any transfer channel. Therefore the total cross section is the sum of (5) and of all possible transfer channels f.



The compound nucleus is formed in any transfer channel and/or vibrational. Therefore the total cross section is the sum of and of all possible vibrational k and transfer f channels , i.e.

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E}\sum_\ell (2\ell+1)\sum_k |U_{k0}|^2[T(E,\mathcal{V}^i_{\ell k}) + \sum_f T(E,\mathcal{V}^i_{\ell k},\mathcal{V}^f_{\ell k})].$$

Note that the contributions of the channels with $Q_{\text{transfer}} \approx 0$ to the total cross section are small and negligible for $Q_{\text{transfer}} << 1$ MeV due to the exponential dependence of the transmission coefficient in the actions.



The interaction potential between two ions at distance r,

$$V(r) = z_1 z_2 e^2/r + V_{\rm i-i}(r).$$

We choose the Krappe-Nix-Sierk $V_{\rm KNS}(r)$ potential in our calculation for $r \geq R_{12} = R_1 + R_2$. The potential $V_{\rm KNS}(r)$ and the Coulomb energy depend on the shape of the ions at $r < R_{12}$. We would like to avoid a shape dependence of the potential $V(r)$. Hence we use a parameterization of the interaction potential $V(r)$ for $r < R_{12}$ in the form

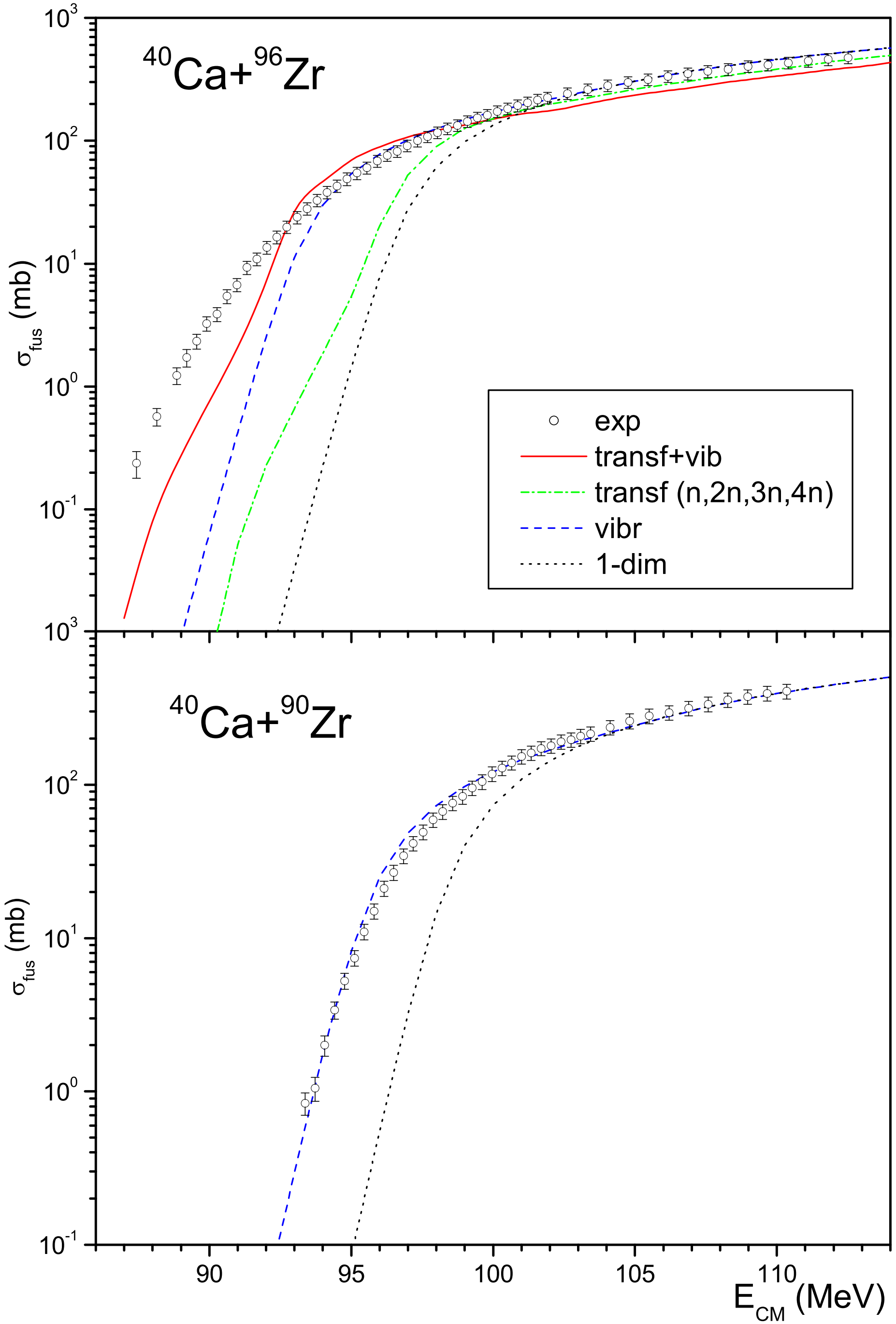
$$V_{\rm fus}(r) = -Q_{\rm fus} + x^2(c_1 + c_2 f(x)),$$

where $Q_{\rm fus}$ is the Q-value of the fusion reaction obtained by using the mass table or by using the mass formula, $x = (r - R_{\rm fus})/(R_{12} - R_{\rm fus})$, $R_{\rm fus}$ is the distance between the centers of mass of the left and right parts of the spherical compound nuclei. The coefficients c_1 and c_2 are obtained by matching at the touching point $R_{12} = R_1 + R_2$ for the potentials $V(r)$ and $V_{\rm fus}(r)$ and for its derivatives.

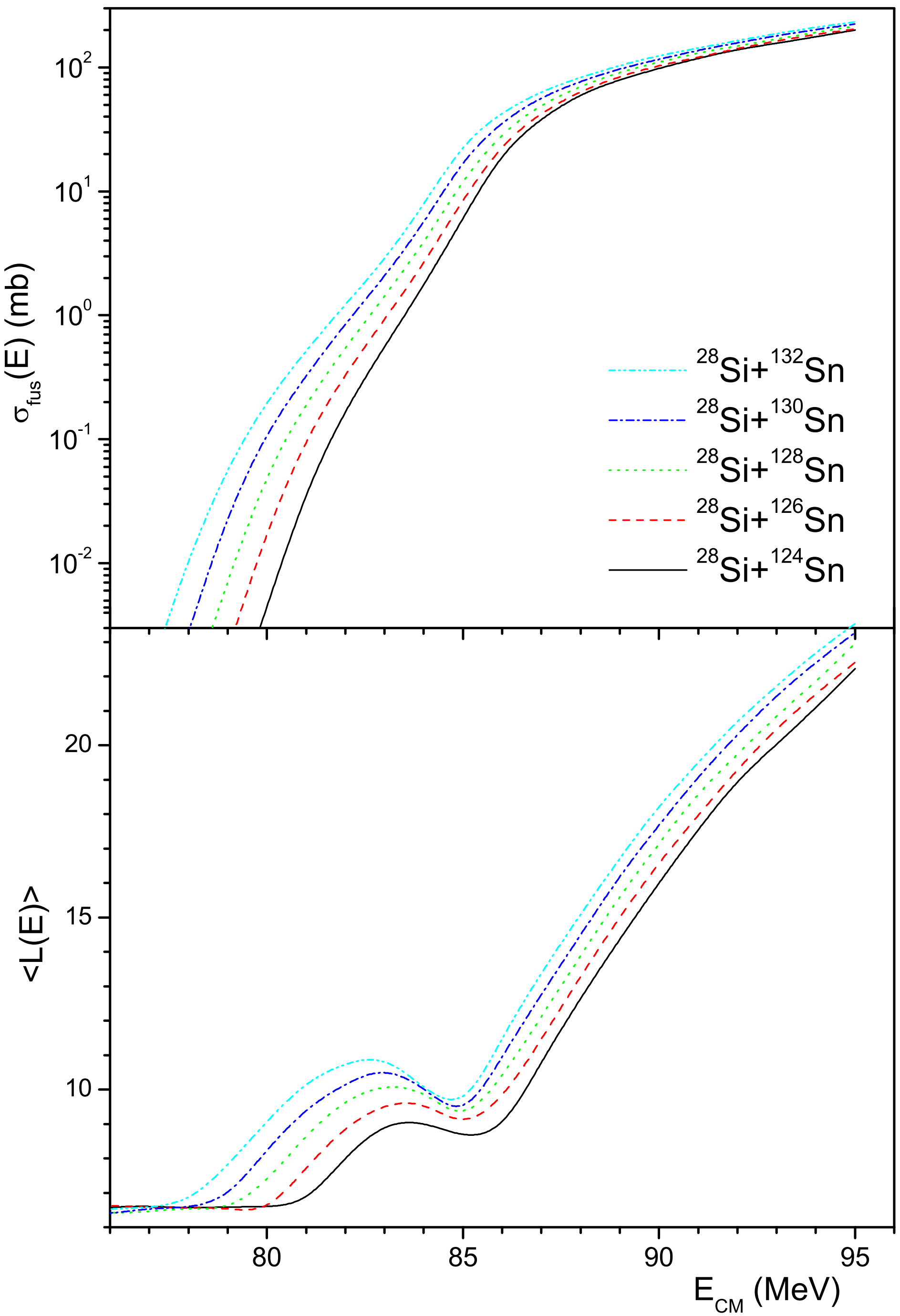
The reduced mass μ for $r > R_{12}$ is determined by using a standard expression. The reduced mass for $r < R_{12}$ is a function of r. We used the parameterization of $\mu(r)$ introduced in

$$\mu_{i(f)}(r) = \mu_{i(f)}\{(17/15)\, k[(R_{12} - r)/(R_{12} - R_{\rm fus})]^2 \exp[-(32/17)\,(r/R_{\rm fus} - 1)] + 1\},$$

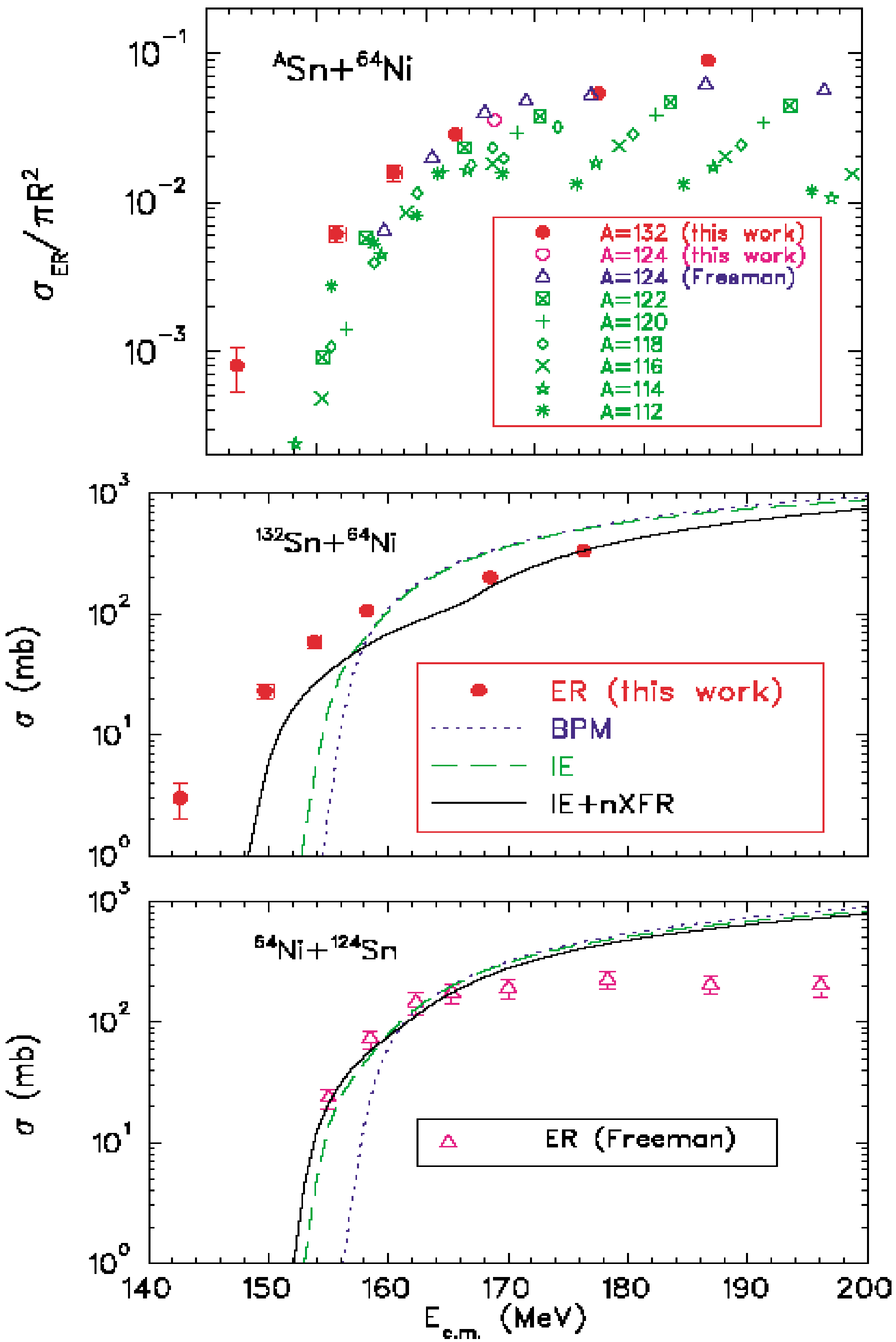
where $k = 16$.



Q_{2n}=5.525MeV (from ^{96}Zr to ^{40}Ca)

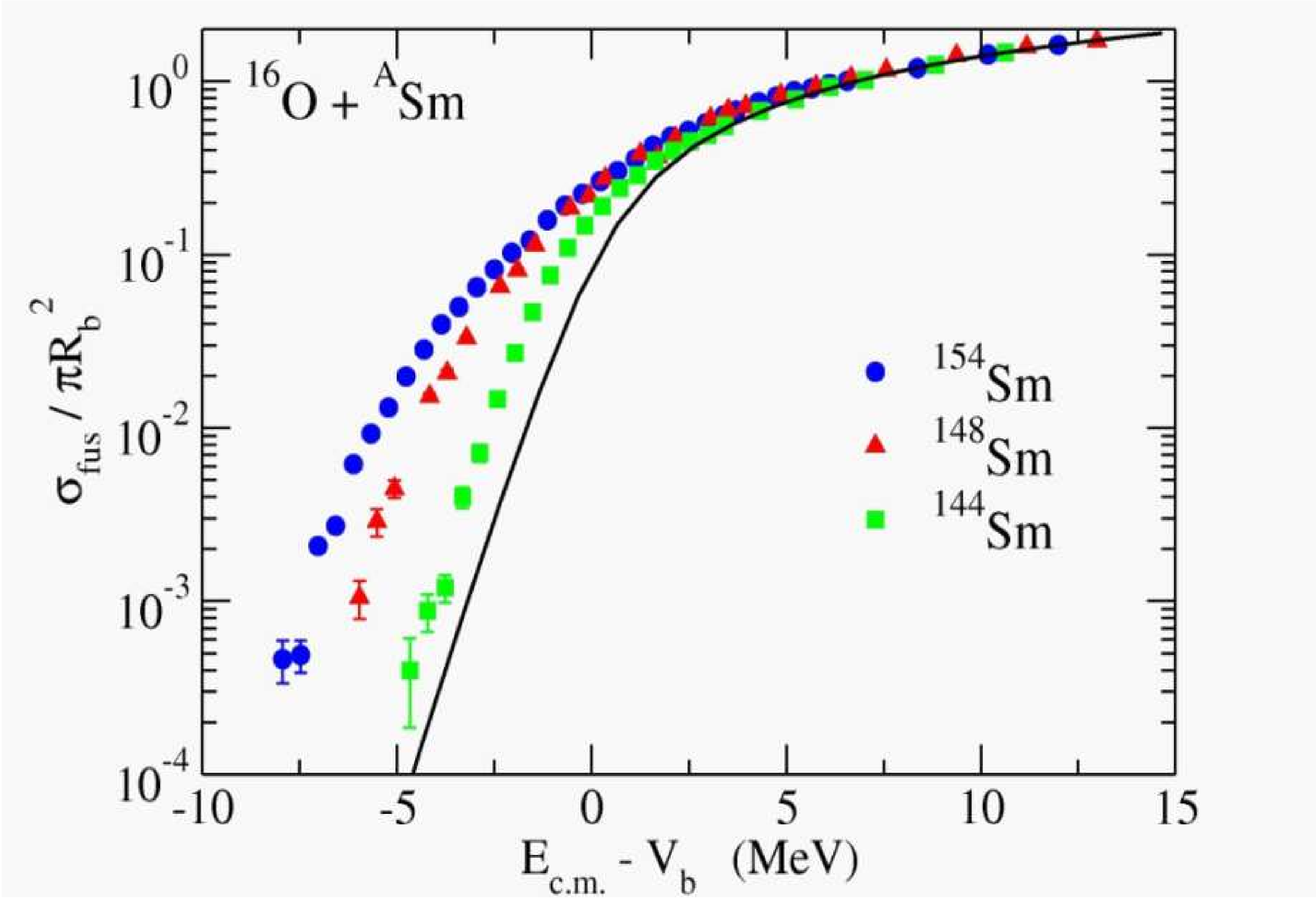


$\sigma_{fus}(E)$ (mb)
10^2
10^1
10^0
10^{-1}
10^{-2}
$^{28}Si+^{132}Sn$
$^{28}Si+^{130}Sn$
$^{28}Si+^{128}Sn$
$^{28}Si+^{126}Sn$
$^{28}Si+^{124}Sn$
<L(E)>
20
15
10
80
85
90
95
E_{CM} (MeV)



^{A}Sn+^{64}Ni
$\sigma_{ER}/\pi R^2$
A=132 (this work)
A=124 (this work)
A=124 (Freeman)
A=122
A=120
A=118
A=116
A=114
A=112
^{132}Sn+^{64}Ni
σ (mb)
ER (this work)
BPM
IE
IE+nXFR
^{64}Ni+^{124}Sn
σ (mb)
ER (Freeman)
140
150
160
170
180
190
200
$E_{c.m.}$ (MeV)

Fusion deformed nuclei in the ground state

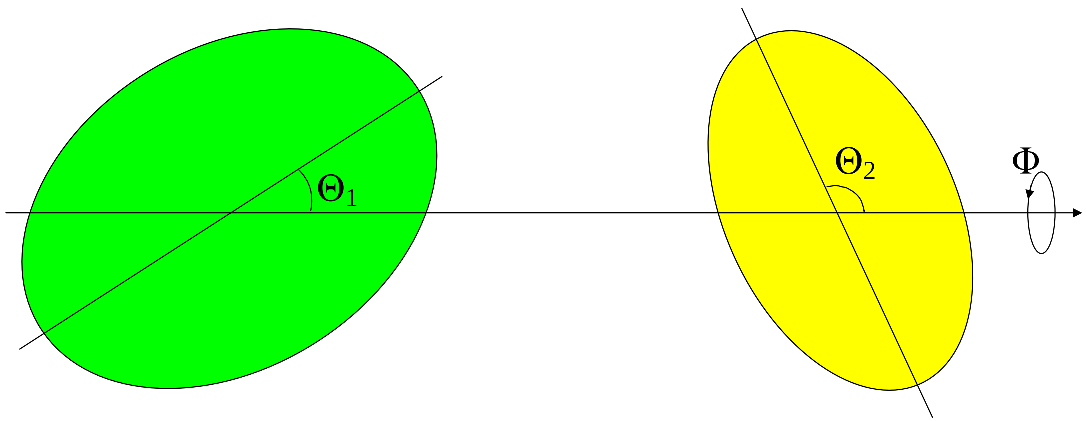


$\beta_2(^{144}Sm)=0$; $\beta_2(^{148}Sm)=0,14$; $\beta_2(^{154}Sm)=0,34$

Various orientations of deformed nuclei occur during collisions, therefore the fusion reaction cross section induced by two deformed nuclei should be averaged over all possible orientations of colliding nuclei

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E}\sum_{\ell}(2\ell+1)\frac{1}{8\pi}\int_0^{\pi}\sin(\Theta_1)d\Theta_1 \int_0^{\pi}\sin(\Theta_2)d\Theta_2\int_0^{2\pi}d\Phi\, T(E,\ell,\Theta_1,\Theta_2,\Phi).$$

Here μ is the reduced mass of colliding nuclei, E is the collision energy, $T(E,\ell,\Theta_1,\Theta_2,\Phi)$ is the transmission coefficient evaluated at orientation of colliding nuclei specified by angles Θ_1, Θ_2 and Φ:



We use the WKB approximation for evaluation of the transmission coefficient for sub-barrier energies

$$T(E,\ell,\Theta_1,\Theta_2,\Phi) = \left\{1+\exp\left[\frac{2}{\hbar}\int_{a(E,\ell,\Theta_1,\Theta_2,\Phi)}^{b(E,\ell,\Theta_1,\Theta_2,\Phi)} \sqrt{2\mu[V(R,\ell,\Theta_1,\Theta_2,\Phi)-E]}\,dR\right]\right\}^{-1}$$

and the Hill-Wheeler approach [?] for over-barrier collision energies. The inner $a(E,\ell,\Theta_1,\Theta_2,\Phi)$ and outer $b(E,\ell,\Theta_1,\Theta_2,\Phi)$ turning points in Eq. (2) are determined from corresponding equations

$$V(a(E,\ell,\Theta_1,\Theta_2,\Phi),\ell,\Theta_1,\Theta_2,\Phi) = E,$$

$$V(b(E,\ell,\Theta_1,\Theta_2,\Phi),\ell,\Theta_1,\Theta_2,\Phi) = E.$$

The interaction potential $V(R,\ell,\Theta_1,\Theta_2,\Phi)$ of two deformed nuclei at distance R between mass centers and mutual orientation described by angles Θ_1, Θ_2 and Φ consists of Coulomb $V_{\mathrm{C}}(R,\Theta_1,\Theta_2,\Phi)$, nuclear $V_{\mathrm{N}}(R,\Theta_1,\Theta_2,\Phi)$ and rotational $V_\ell(R) = \hbar^2\ell(\ell+1)/(2\mu R^2)$ parts

$$V(R,\ell,\Theta_1,\Theta_2,\Phi) = V_{\mathrm{C}}(R,\Theta_1,\Theta_2,\Phi) + V_{\mathrm{N}}(R,\Theta_1,\Theta_2,\Phi) + V_\ell(R).$$

The Coulomb interaction of two deformed nuclei is approximated as [?]

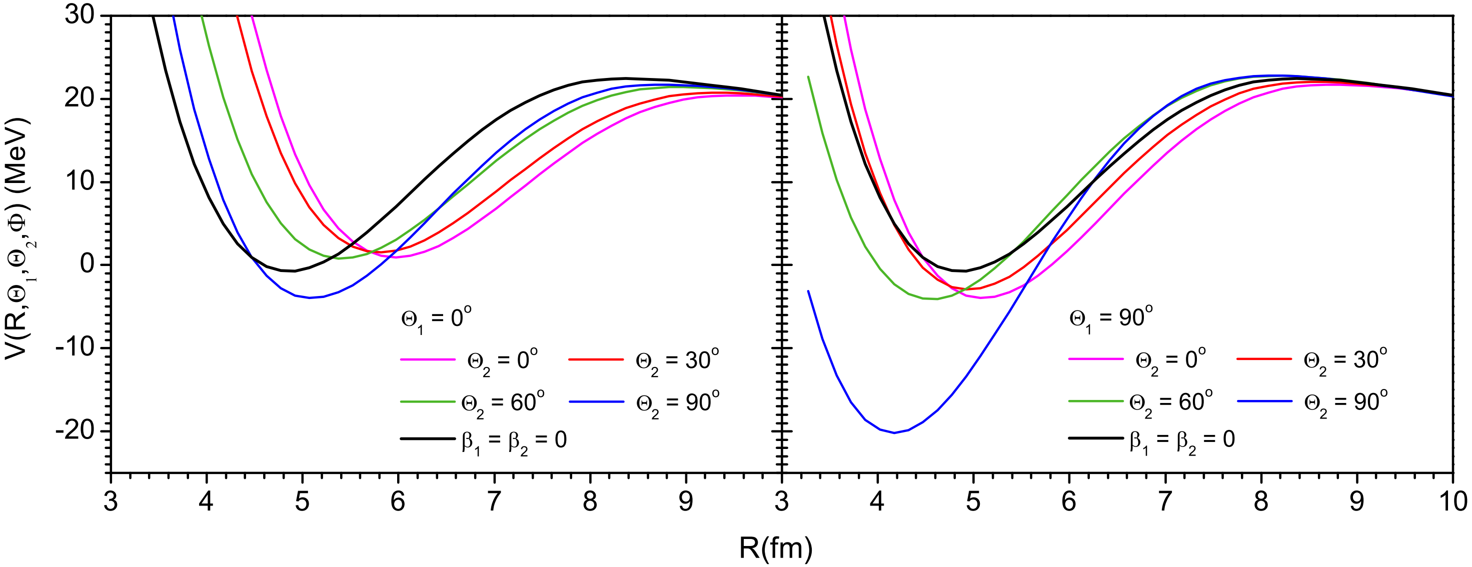
$$
\begin{aligned}
V_{\rm C}(R,\Theta_1,\Theta_2,\Phi) = \frac{Z_1 Z_2 e^2}{R}\,\{1 \\
+\sum_{\ell\geq 2}\left[f_{1\ell}(R,\Theta_1,R_{10})\beta_{1\ell}+f_{1\ell}(R,\Theta_2,R_{20})\beta_{2\ell}\right] \\
+f_2(R,\Theta_1,R_{10})\beta_{12}^2+f_2(R,\Theta_2,R_{20})\beta_{22}^2 \\
+f_3(R,\Theta_1,\Theta_2,R_{10},R_{20})\beta_{12}\beta_{22} \\
+f_4(R,\Theta_1,\Theta_2,\Phi,R_{10},R_{20})\beta_{12}\beta_{22}\}\,.
\end{aligned}
$$

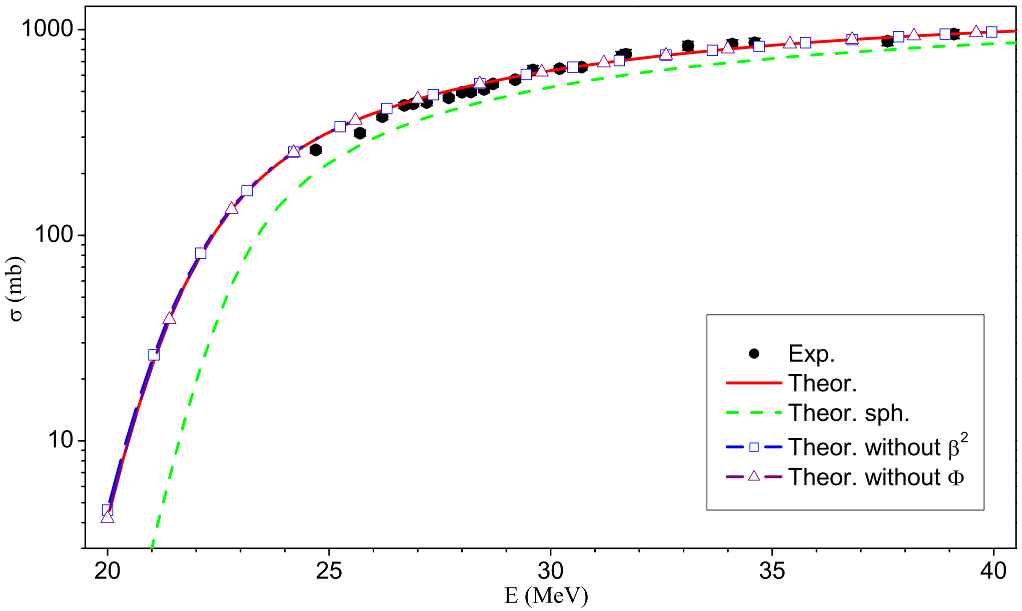
Applying the proximity theorem we can obtain a simple parametrization of the nuclear part of interaction potential between two deformed nuclei

$$
V_{\rm N}(R,\Theta_1,\Theta_2,\Phi) \approx \frac{1/R_{10}+1/R_{20}}{\left[(C_1^{\|}+C_2^{\|})(C_1^{\perp}+C_2^{\perp})\right]^{1/2}} \quad \times V_{\rm N}^{\rm sph}(d(R,\Theta_1,\Theta_2,\Phi,\beta_{i2},\beta_{i\ell})),
$$

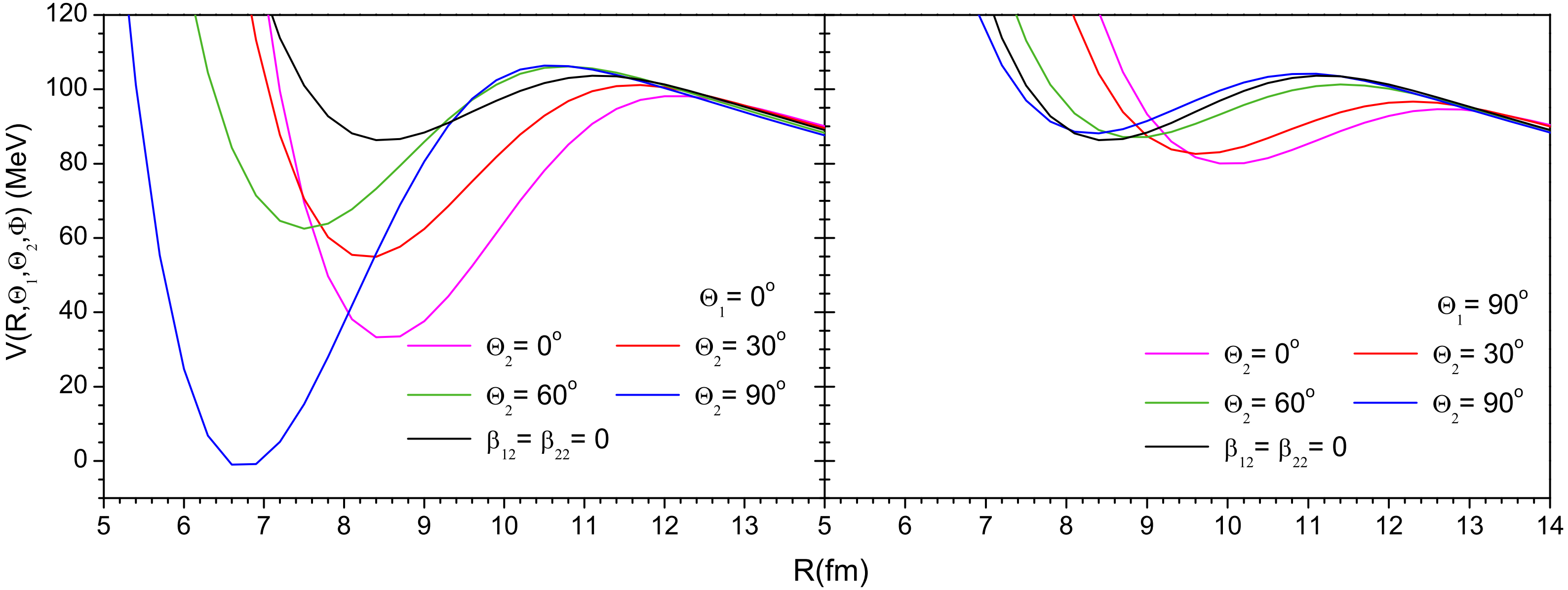
where $C_i^{\|}$ and $C_i^{\perp}$ are the main curvatures of deformed surface of nucleus i at the point closest to the surface of another nucleus, $d(R,\Theta_1,\Theta_2,\Phi,\beta_{i2},\beta_{i\ell})$ is the closest distance between surfaces of interacting nuclei, $V_{\rm N}^{\rm sph}(d)$ is the nuclear part of the interaction potential between spherical nuclei at $d=R-R_{10}-R_{20}$. The nuclear part of potential depends strongly on the value of the closest distance between surfaces of interacting nuclei, therefore we evaluate $d(R,\Theta_1,\Theta_2,\Phi,\beta_{i2},\beta_{i\ell})$ numerically.

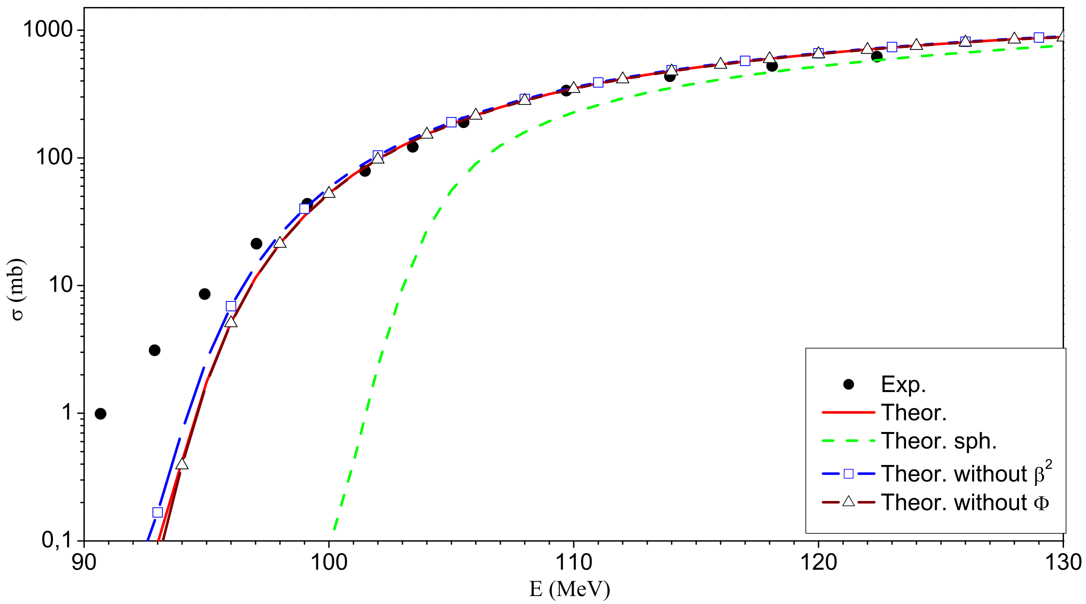
^{24}Mg+^{24}Mg $(\beta_2(^{24}Mg) = 0.438)$

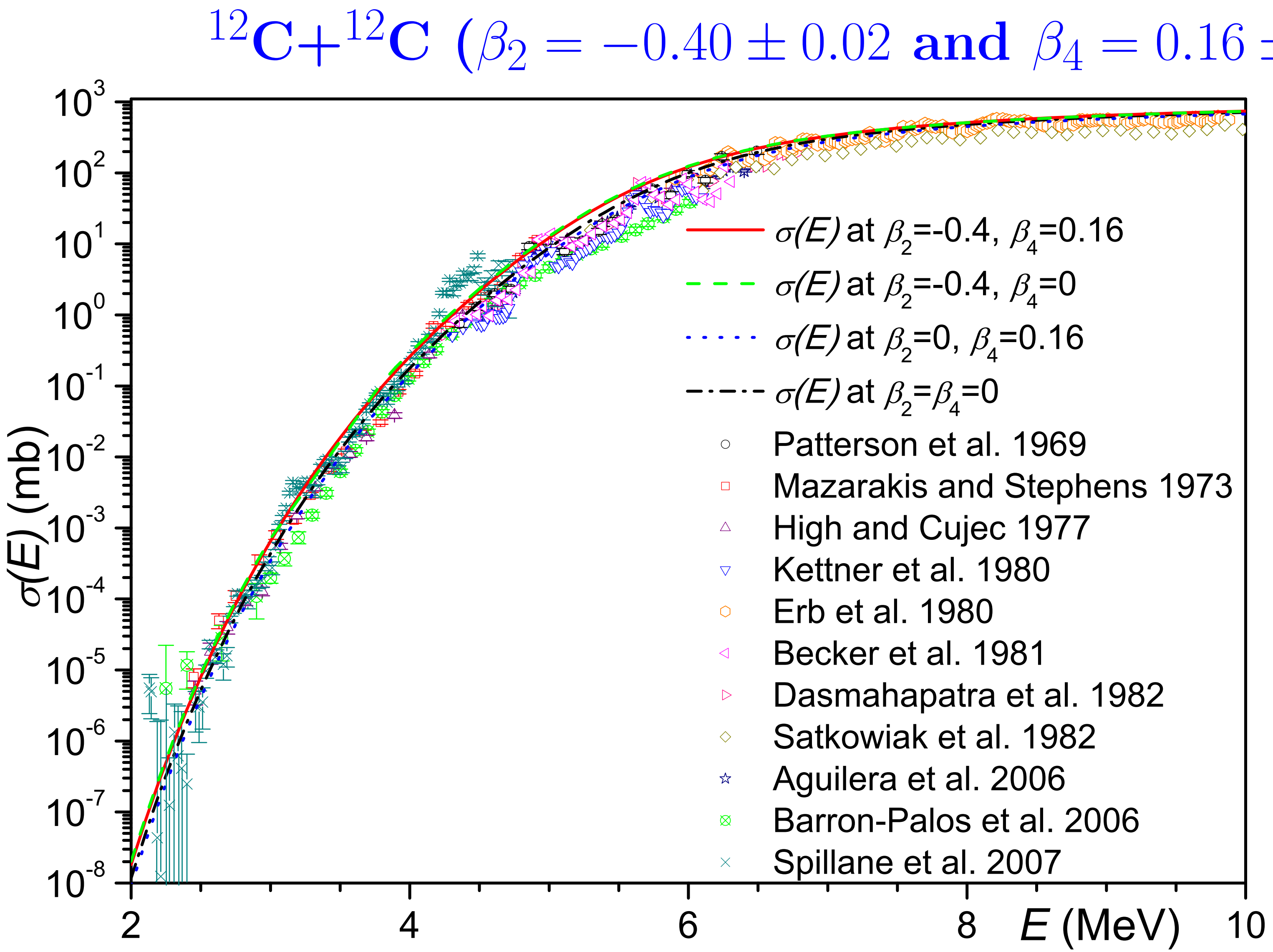




^{28}Si+^{154}Sm $(\beta_2(^{28}Si) = -0.407,\ \beta_2(^{154}Sm) = 0.34)$

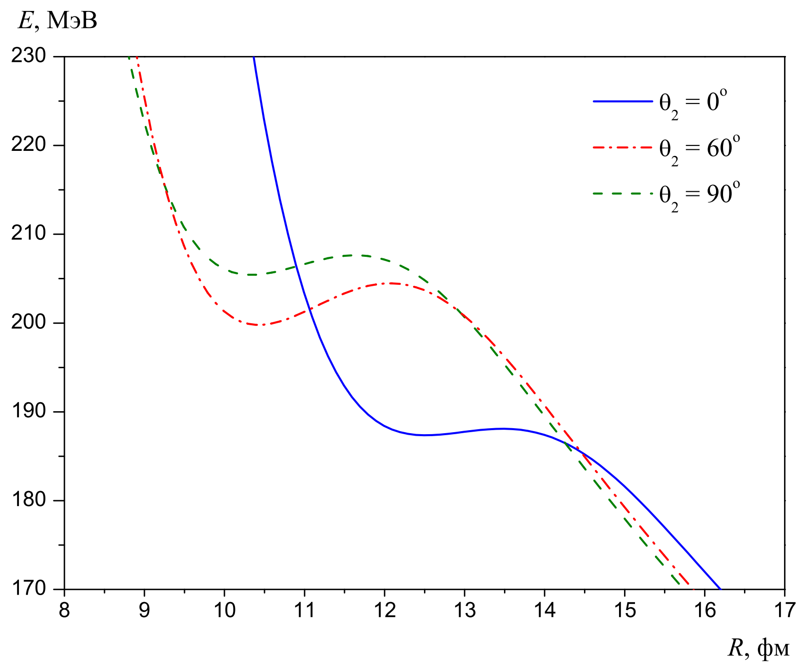


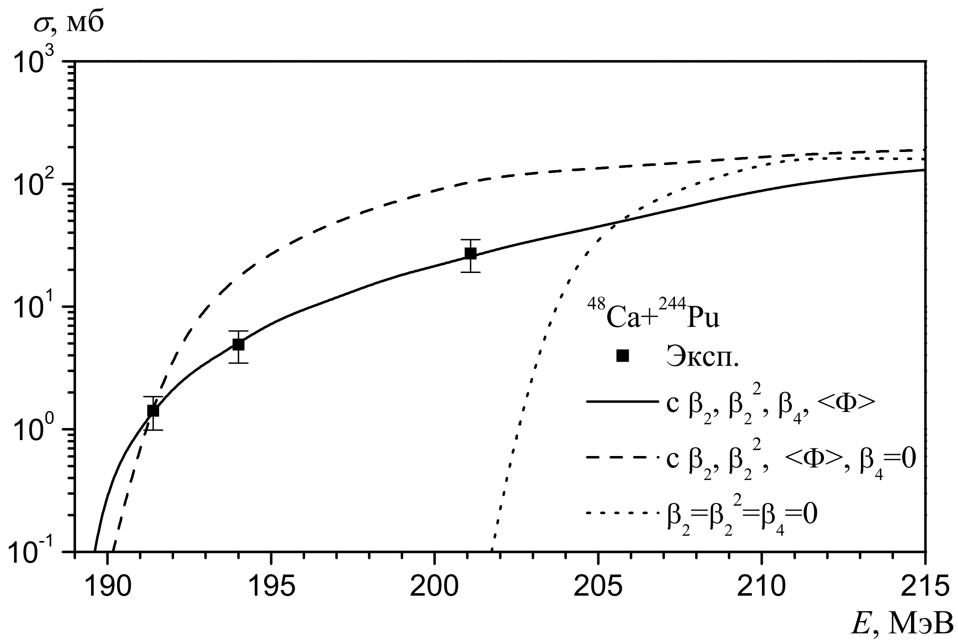




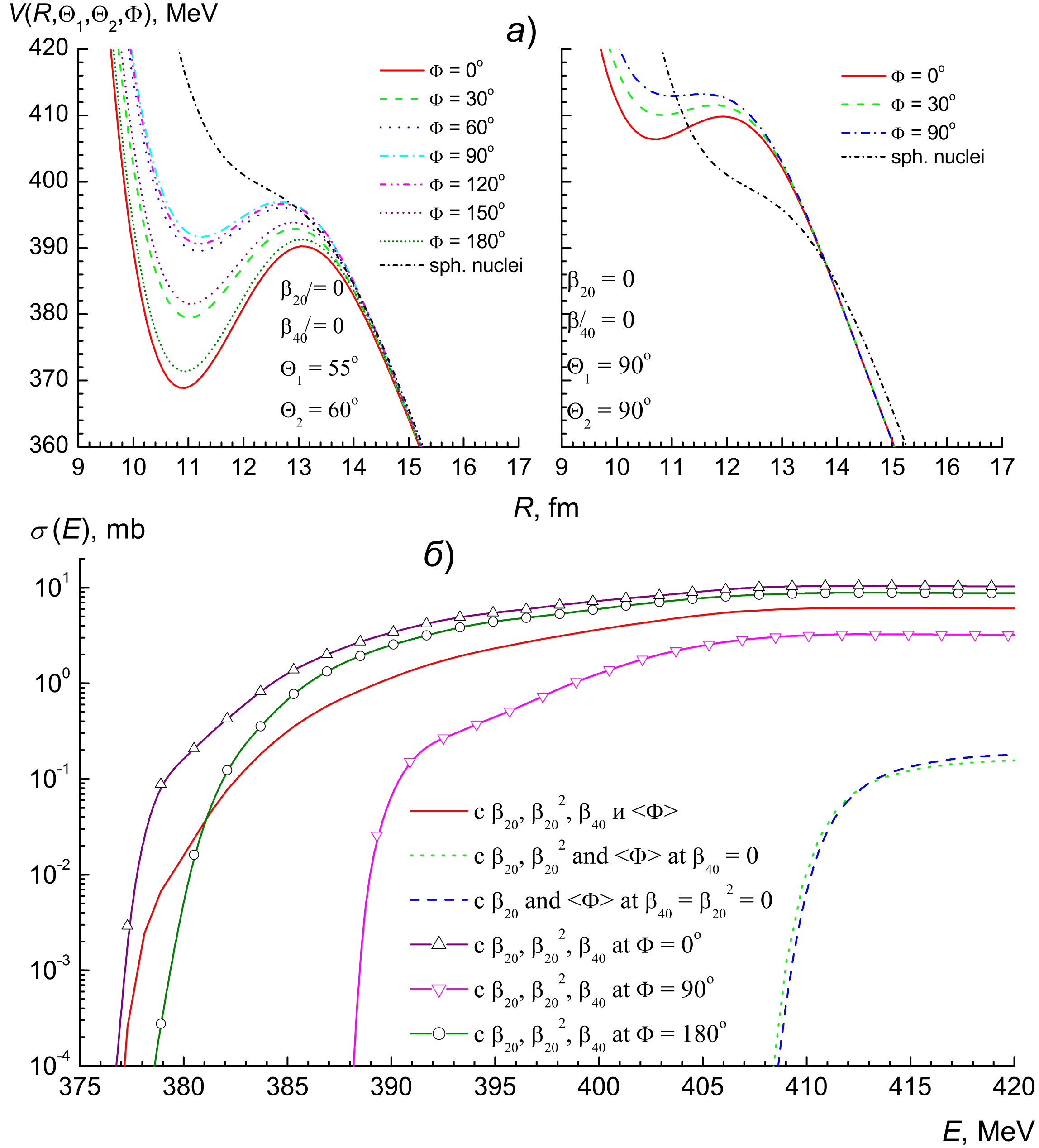
^{12}C+^{12}C ($\beta_2 = -0.40 \pm 0.02$ and $\beta_4 = 0.16 \pm 0.03$)
$\sigma(E)$ (mb)
E (MeV)
$\sigma(E)$ at β_2=-0.4, β_4=0.16
$\sigma(E)$ at β_2=-0.4, β_4=0
$\sigma(E)$ at β_2=0, β_4=0.16
$\sigma(E)$ at β_2=β_4=0
Patterson et al. 1969
Mazarakis and Stephens 1973
High and Cujec 1977
Kettner et al. 1980
Erb et al. 1980
Becker et al. 1981
Dasmahapatra et al. 1982
Satkowiak et al. 1982
Aguilera et al. 2006
Barron-Palos et al. 2006
Spillane et al. 2007

^{48}Ca+^{244}Pu: effect of shallow capture well





^{150}Nd ($\beta_2 = 0.285, \beta_4 = 0.107$)+^{158}Gd ($\beta_2 = 0.348, \beta_4 = 0.079$): effects of shallow capture well, orientation and hexadecapole deformation



NUCLEAR REACTIONS IN HOT STELLAR MATTER AND NUCLEAR SURFACE DEFORMATION

Various nuclear reactions (capture, fusion, decay, disintegration, excitation, elastic and inelastic collisions, γ-emission, γ-decay etc.) take place in the star matter during star burning, supernova explosions and other stages of star evolution.

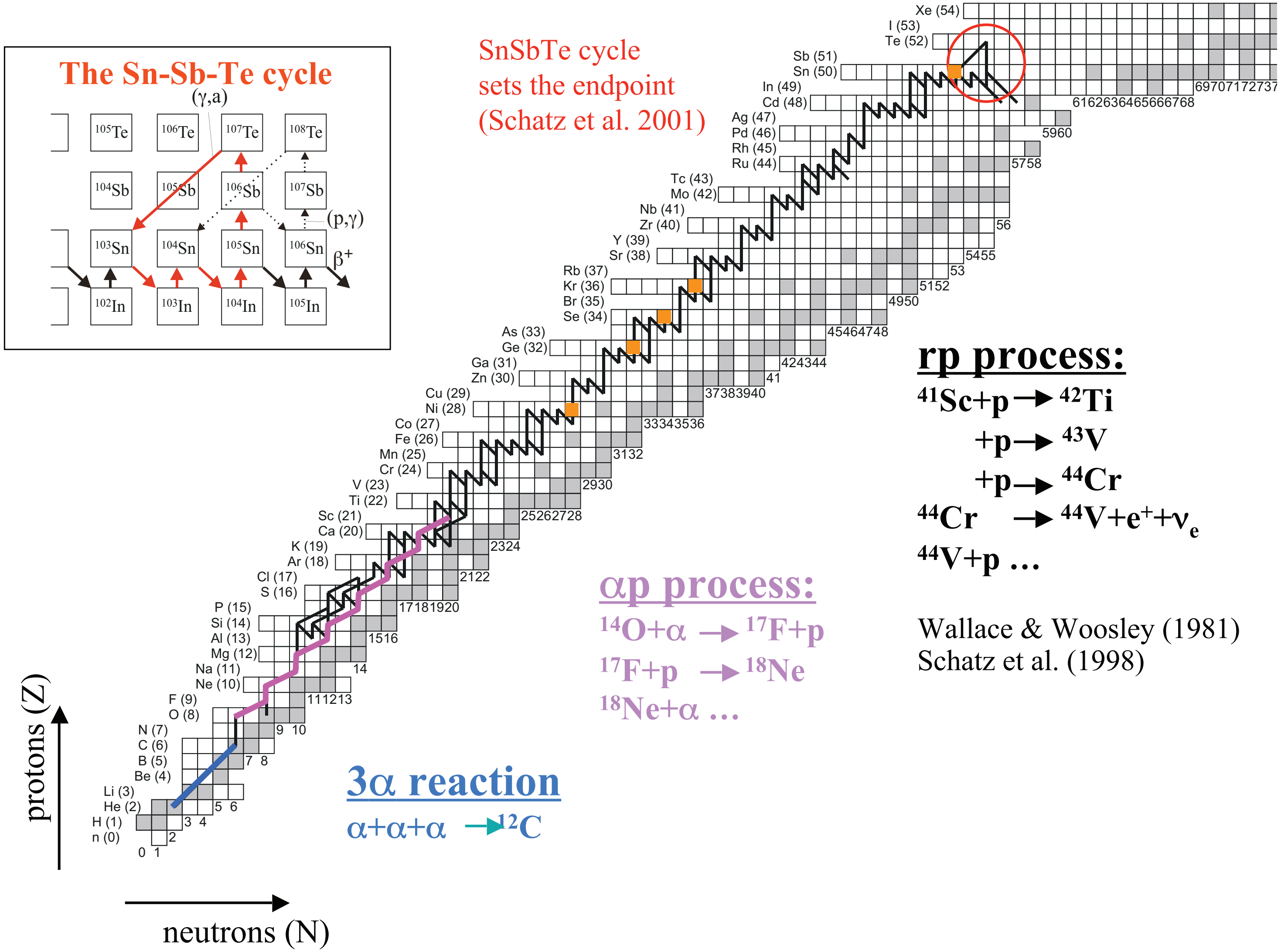
The cross-sections of nuclear reactions determine diverse properties of the stars, the evolution of star and the nucleosynthesis of elements in stellar matter.

The star matter mainly consists of nuclei, α-particles, nucleons, electrons and γ-quanta.

	Typical temperatures T	
	K	MeV
The Sun (center) (**H** burning)	$\approx 1.57 \cdot 10^7$	≈ 0.0015
He burning stage of massive stars	$\approx 1 \div 2 \cdot 10^8$	$\approx 0.009 \div 0.017$
C burning stage of massive stars	$\approx 0.8 \div 1 \cdot 10^9$	≈ 0.09
O burning stage of massive stars	$\approx 2 \cdot 10^9$	≈ 0.17
Si burning stage of massive stars	$\approx 3.5 \cdot 10^9$	≈ 0.3

There are many nuclei with energies of excited states around $\varepsilon_i \sim 0.3$ MeV: $\Rightarrow$ nuclei in star matter exist in both the ground and excited states due to various reactions at $T \gtrsim 0.2$ MeV.

rp-, rα-processes, fusion of heavy nuclei



SnSbTe cycle
sets the endpoint
(Schatz et al. 2001)

rp process:

^{41}Sc+p → ^{42}Ti

+p → ^{43}V

+p → ^{44}Cr

^{44}Cr → ^{44}V+e^{+}+ν_e

^{44}V+p …

Wallace & Woosley (1981)
Schatz et al. (1998)

αp process:

^{14}O+α → ^{17}F+p

^{17}F+p → ^{18}Ne

^{18}Ne+α …

3α reaction

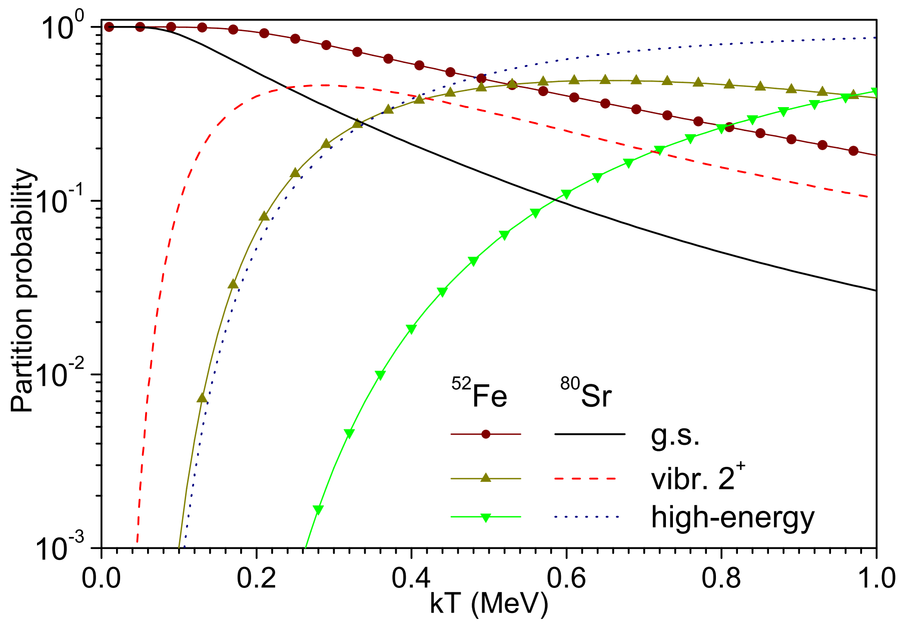
α+α+α → ^{12}C

Capture of charged particles on nuclei with low-energy quadrupole vibrations

The probability to find a nucleus in a state with excitation energy ε_i and spin j_i in stellar matter at temperature T can be estimated within the statistical approach as

$$P(\varepsilon_i, j_i, kT) = \frac{(2j_i + 1)\exp(-\varepsilon_i/kT)}{\sum_{i=0}^{\infty}(2j_i + 1)\exp(-\varepsilon_i/kT)}.$$

Here k is the Boltzmann constant; $i = 0 \Rightarrow$ the ground-state of the nucleus with $\varepsilon_i = 0$, $i = 1$ $\Rightarrow$ the lowest 2^+ surface oscillation state with $\varepsilon_1 = \varepsilon_{\rm vib}$ and $i \geq 2$ for other excited states.



Occupation probability for the ground state, the first 2^+ surface oscillation state and the net occupation probability of high-energy states $P(\varepsilon_i > \varepsilon_{\rm vib})$ in ^{52}Fe ($\varepsilon_{\rm vib} = 0.849$ MeV, $\beta_{\rm vib} = 0.308$) and ^{80}Sr ($\varepsilon_{\rm vib} = 0.386$ MeV, $\beta_{\rm vib} = 0.404$) at different temperatures of stellar matter kT.

Therefore evaluating the reaction cross-sections between charged particle and heavy soft nucleus in star matter we should take into account contributions from both the ground and well-deformed excited states of the nucleus.

α-capture reactions in stars

The α-particle may be considered the most rigid nucleus, because the energy of the first excited state is 20.21 MeV. The population of such high-energy state is negligible in stellar matter at temperatures $kT \lesssim 1$ MeV.

On the other hand, many states with energies $\varepsilon_i \lesssim 1$ MeV in soft nuclei can be noticeably populated at $kT \lesssim 1$ MeV. Therefore, the α-capture reaction cross-section in star matter at temperature T can be estimated as

$$\sigma(E, kT) = \sum_{i=0}^{\infty} P(\varepsilon_i, j_i, kT)\sigma_i(E),$$

where $\sigma_i(E)$ is the fusion cross-section between the α-particle and a nucleus in a state i with energy ε_i and spin j_i, E is the collision energy.

The capture (fusion) cross-section of two particles with corresponding values of spins 0 and j is

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E(2j+1)} \sum_{J\ell\ell'} (2J+1) t_{J\ell\ell'}(E),$$

where μ is the reduced mass, ℓ and ℓ' are the orbital moment of ingoing and outgoing channels, J is the total angular momentum and $t_{J\ell\ell'}(E)$ is the generalized transmission coefficient.

The shape of a nucleus in highly-excited states $i \geq 2$ can be spherical or deformed. The kind of surface deformation can be different for different high-energy states. The high-multipolarity $\lambda \geq 3$ axial or nonaxial multipole $\lambda \geq 2$ nuclear surface deformations usually lead to the smaller reduction of the barrier than those induced by the axial quadrupole surface deformation.

Therefore $\sigma_i(E)|_{i\geq 2} \approx \sigma_0(E)$.

As a result, the α-capture reaction cross-section in star matter can be rewritten as

$$\sigma(E,kT) \approx [P(0,0,kT) + \sum_{i=2}^{\infty} P(\varepsilon_i, j_i, kT)]\sigma_0(E) + P(\varepsilon_{\rm vib}, 2, kT)\sigma_1(E).$$

Using the identity

$$\sum_{i=0}^{\infty} P(\varepsilon_i, j_i, kT) \equiv [P(0,0,kT) + \sum_{i=2}^{\infty} P(\varepsilon_i, j_i, kT)] + P(\varepsilon_{\rm vib}, 2, kT) \equiv 1$$

we get the simple form

$$\sigma(E,kT) \approx \sigma_0(E) + P(\varepsilon_{\rm vib}, 2, kT)[\sigma_1(E) - \sigma_0(E)] = \sigma_0(E)\{1 + P(\varepsilon_{\rm vib}, 2, kT)[s(E) - 1]\}.$$

Here term containing $P(\varepsilon_{\rm vib}, 2, kT)$ is related to the cross-section enhancement induced by the population of the first 2^+ surface oscillation state in soft nuclei in the stellar matter, and

$$s(E) = \sigma_1(E)/\sigma_0(E).$$

Ratio $s(E)$ directly shows the effect of cross-section enhancement caused by deformation of the nuclear surface in 2^+ states, because if the surface deformation is neglected, then $\sigma_1(E) = \sigma_0(E)$ and $s(E) = 1$.

Two important effects on $t_{J\ell\ell'}(E)$:

1. During the α-nucleus fusion reaction the α-particle can arrive from any direction, therefore we should make averaging over space angles, if surface of the nucleus is deformed.

2. Nuclear surface in 2^+ state oscillates about the spherical equilibrium shape. Therefore we should make averaging over all possible values of the deformation parameter.

The reaction S-factor is proportional to the cross-section

$$S(E, kT) = E \exp(2\pi\eta(E))\sigma(E, kT),$$

where $\eta(E) = zZe^2/(\hbar v)$ is the Sommerfeld parameter, $v = (2E/\mu)^{1/2}$ is the collision velocity.

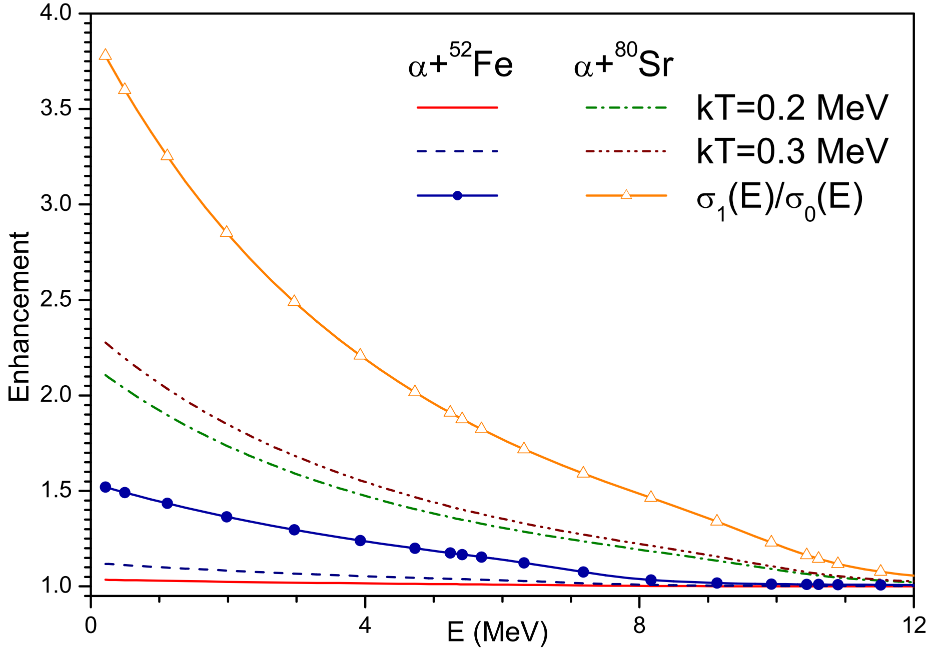
The enhancement of the S-factor or the reaction cross-section in stellar matter induced by 2^+ surface oscillation is described by the ratio

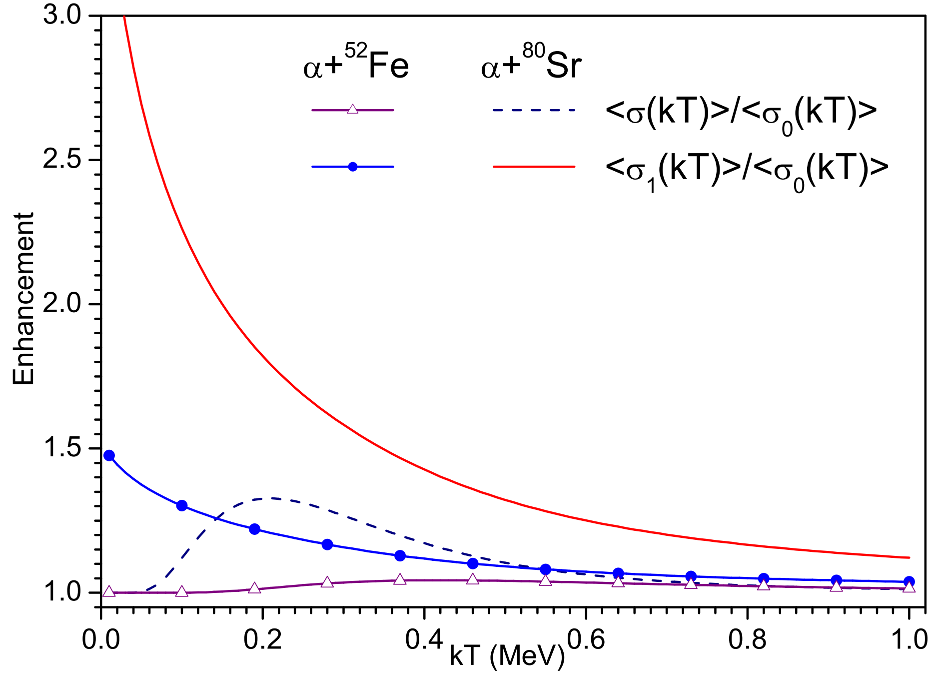
$$s(E, kT) = S(E, kT)/S_{\text{sph}}(E),$$

$S_{\text{sph}}(E) = E \exp(2\pi\eta(E))\sigma_{\text{sph}}(E, kT)$, $\sigma_{\text{sph}}(E, kT)$ is the cross-section on spherical nucleus.

The stellar reaction cross-sections are often averaged over the Maxwell–Boltzmann distribution of collision velocities v and can be presented in the form

$$\langle\sigma(kT)\rangle = 2\int_0^\infty \sigma(E, kT) E \exp(-E/kT) dE \Big/ \left[\sqrt{\pi}\int_0^\infty E \exp(-E/kT) dE\right].$$

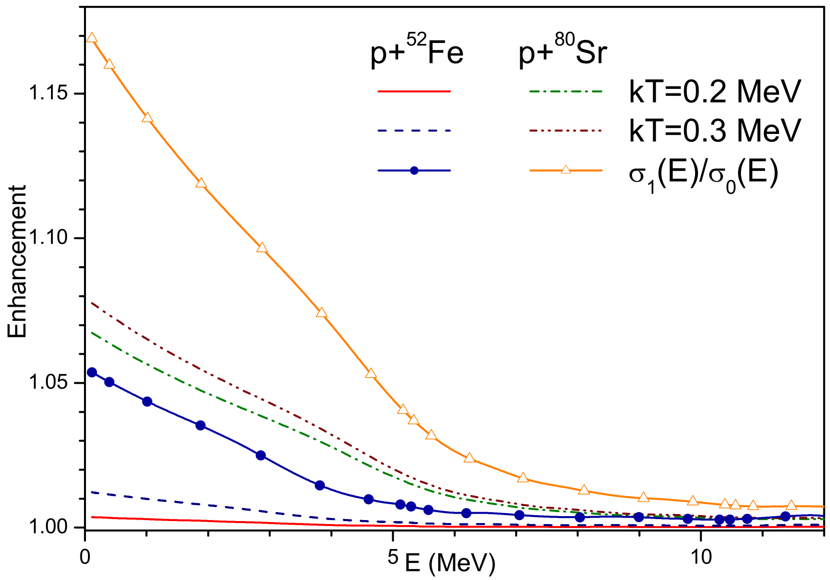




Spherical ground-state and vibrational deformation of 2^+ state in nuclei are taken into account.

$^{52}\mathrm{Fe} \Rightarrow \varepsilon_{\mathrm{vib}} = 0.849$ MeV, $\beta_{\mathrm{vib}} = 0.308$

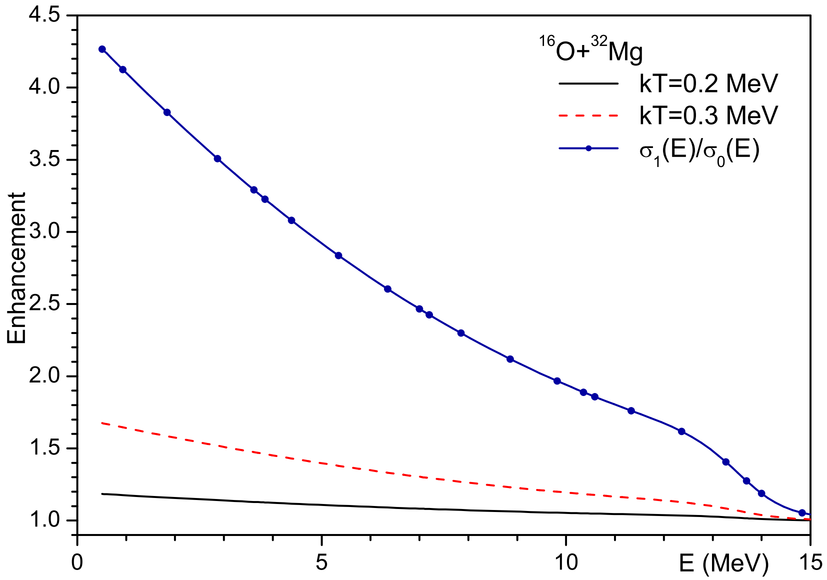
$^{80}\mathrm{Sr} \Rightarrow \varepsilon_{\mathrm{vib}} = 0.386$ MeV, $\beta_{\mathrm{vib}} = 0.404$

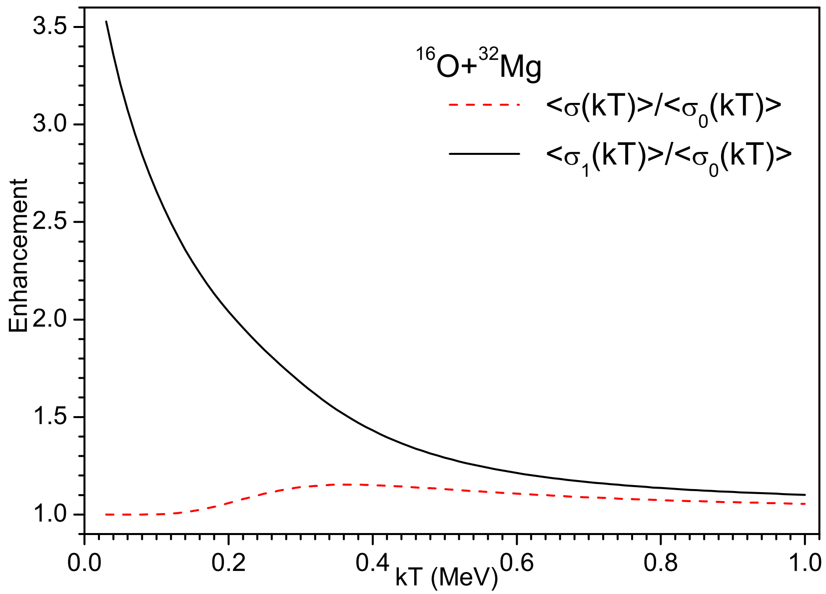


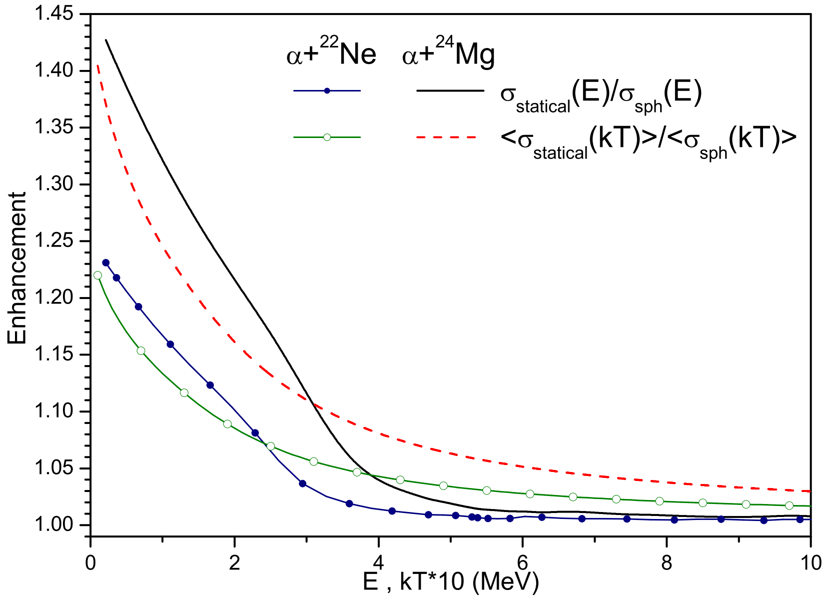
$$^{52}\mathrm{Fe} \Rightarrow \varepsilon_{\mathrm{vib}} = 0.849\ \mathrm{MeV},\ \beta_{\mathrm{vib}} = 0.308$$

$$^{80}\mathrm{Sr} \Rightarrow \varepsilon_{\mathrm{vib}} = 0.386\ \mathrm{MeV},\ \beta_{\mathrm{vib}} = 0.404$$

$$^{32}\mathrm{Mg} \Rightarrow \varepsilon_{\mathrm{vib}} = 0.8855\ \mathrm{MeV},\ \beta_{\mathrm{vib}} = 0.473$$







Statical deformation of nuclei is taken into account.

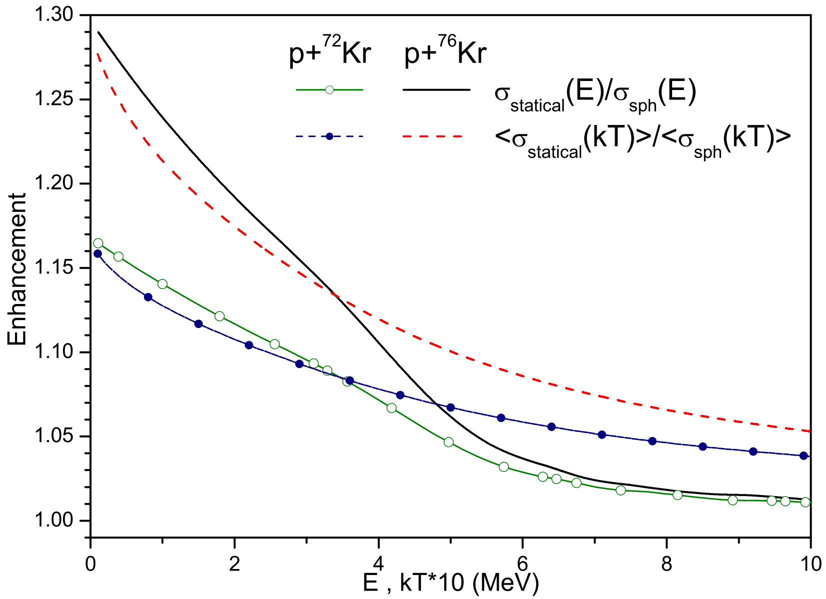
^{22}Ne: $\beta_2 = 0.326$

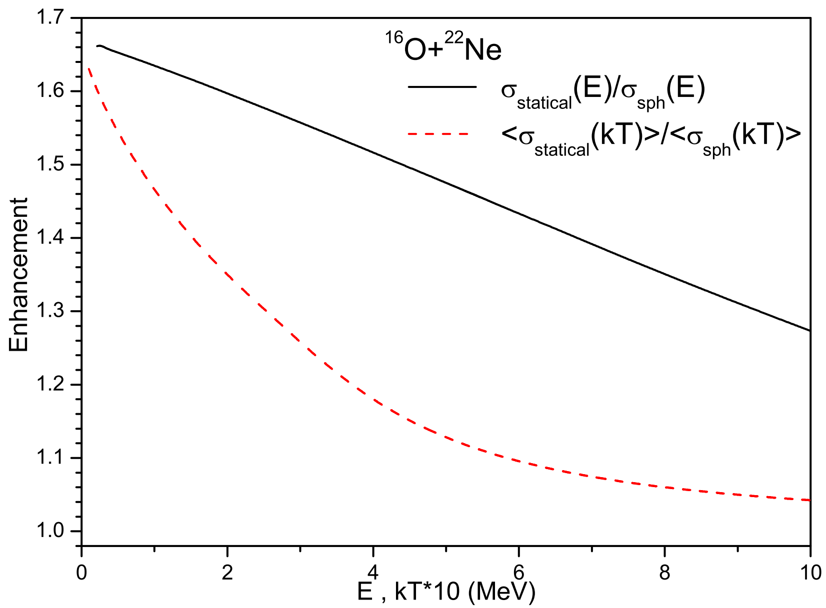
^{24}Mg: $\beta_2 = 0.374$

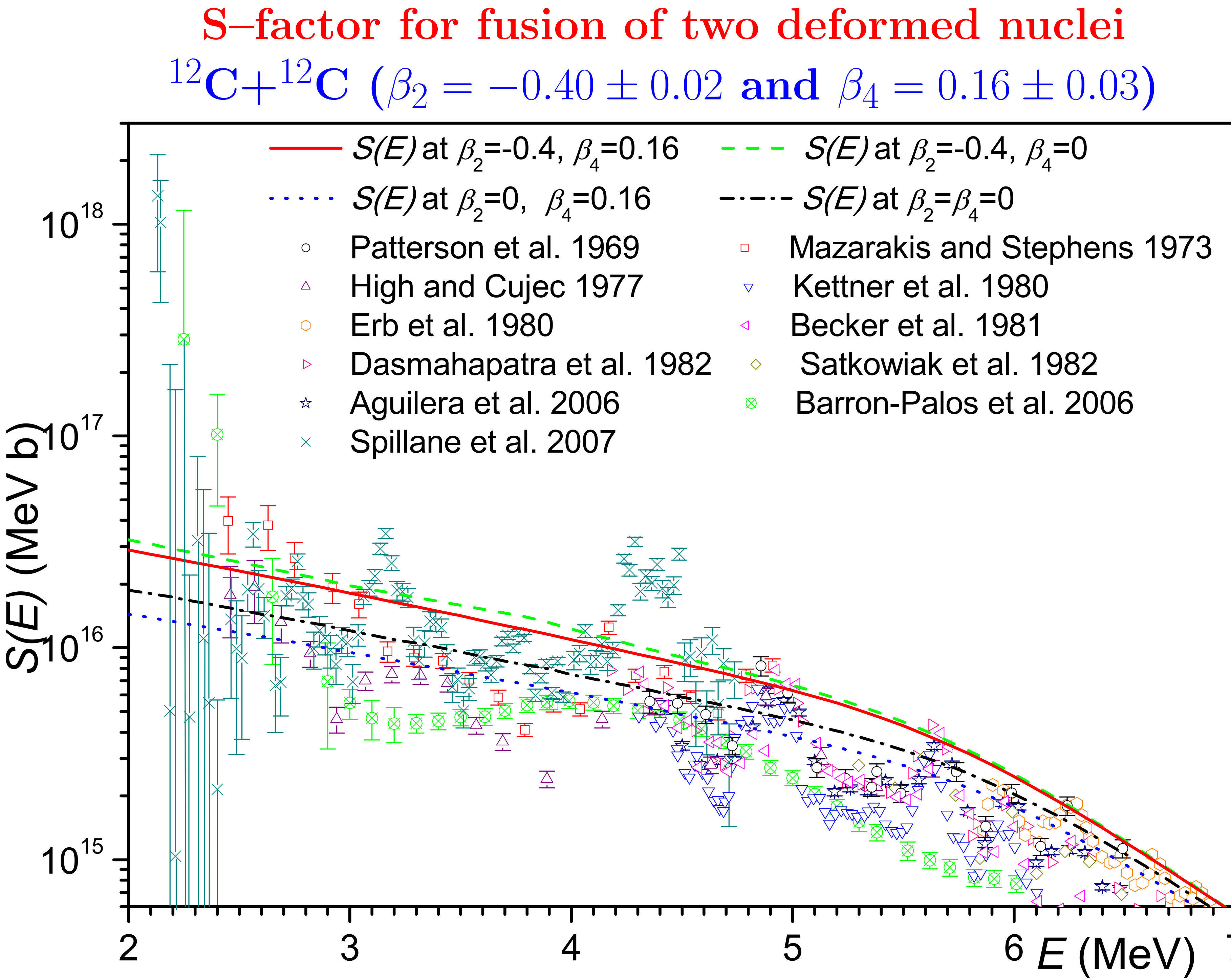
^{72}Kr: $\beta_2 = -0.349$

^{76}Kr: $\beta_2 = 0.4$

β_2 from Moller, ADNDT 59, 185 (1995)






S–factor for fusion of two deformed nuclei
^{12}C+^{12}C ($\beta_2 = -0.40 \pm 0.02$ and $\beta_4 = 0.16 \pm 0.03$)
S(E) at β_2=-0.4, β_4=0.16
S(E) at β_2=-0.4, β_4=0
S(E) at β_2=0, β_4=0.16
S(E) at β_2=β_4=0
Patterson et al. 1969
Mazarakis and Stephens 1973
High and Cujec 1977
Kettner et al. 1980
Erb et al. 1980
Becker et al. 1981
Dasmahapatra et al. 1982
Satkowiak et al. 1982
Aguilera et al. 2006
Barron-Palos et al. 2006
Spillane et al. 2007
S(E) (MeV b)
E (MeV)

Conclusions

Subbarrier heavy-ion fusion cross-sections are enhanced by

- statical deformation of nuclei

- dynamical (vibrational) deformations of nuclei, due to coupling effects

- nucleon transfer with positive transfer reaction Q-value.

Thanks for your attention!