

Transfer reactions

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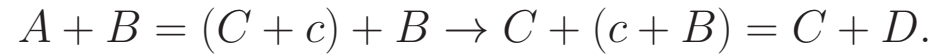
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PLAN

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1. Introduction

Transfer reactions is



As a rule transferred particle c is nucleon or few-nucleon cluster.

There are quasi-elastic transfer reactions, when the kinetic energies of nuclei in the incident and scattered channels at large distances between nuclei are similar.

At high-collision energies there are multinucleon transfer reactions induced by deep-inelastic heavy-ion collisions. The kinetic energies of nuclei the incident and scattered channels on large distances are very different.

2. Direct transfer reactions

Lets $H = H(R, \{r\})$ is the total hamiltonian of system of complex colliding nuclei and $\Psi(R, \{r\})$ is the total wave-function, which can be expanded as the sum on the eigen states of each nuclei (open channels, i.e. related to P projection)

$$\Psi(R, r) = \sum_n \psi_n(R) \varphi_n(\{r\}).$$

where $\psi_n(R)$ is the wave-function described the relative motion of two nuclei at channel n with distance between mass centers R , $\varphi_n(\{r\}) = \phi_{1n}(\{r_1\})\phi_{2n}(\{r_2\})$ is the wave function of intrinsic states of corresponding nuclei $\phi_{1n}(\{r_1\})$ and $\phi_{2n}(\{r_2\})$ at channel n , and $\{r_1\}$ and $\{r_2\}$ are intrinsic coordinate.

Note that at $R \rightarrow \infty$

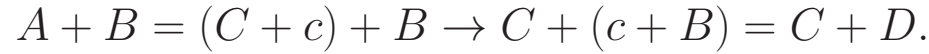
$$H\varphi_n(\{r\}) = \varepsilon_n\varphi_n(\{r\}),$$

where ε_n is eigenenergy of nuclei at channel n .

The Schrödinger equation is

$$(H - E)\Psi(R, r) = 0.$$

Transfer reaction:



The wave-function of nucleus A : $\phi_A(\{r_1\}) = \phi_C(\{r_1\}) \otimes \phi_c(R, \{r_1\})$,

the wave-function of nucleus B : $\phi_B(\{r_2\})$,

the wave-function of nucleus D : $\phi_D(\{r_2\}) = \phi_B(\{r_2\}) \otimes \phi_c(R, \{r_2\})$,

and which wave-function of transferred particle c : $\phi_c(R, \{r_1\})$ or $\phi_c(R, \{r_2\})$.

The total quantum numbers A, B, C, D, c are included and specified correspondingly the core eigen states with quantum eigen numbers $n_{C,B}$ and/or eigen states of transferred particles with quantum eigen numbers n_c .

Multiplying the Schrödinger equation

$$(H - E)\Psi(R, r) = (H - E) \sum_n \psi_n(R) \varphi_n(\{r\}) = 0.$$

on $\varphi_n^*(\{r\})$ and taking the integral on intrinsic coordinates we get

$$\begin{aligned} \int d\{r\} \varphi_n^*(\{r\})(H(R, \{r\}) - E)\Psi(R, r) &= \int d\{r\} \varphi_n^*(\{r\})(H(R, \{r\}) - E) \sum_m \psi_m(R) \varphi_m(\{r\}) \\ &= (h_{nn}(R) - E)\psi_n(R) + \sum_{m, m \neq n} h_{nm}(R)\psi_m(R) = 0, \end{aligned}$$

where

$$h_{nm}(R) = \int d\{r\} \varphi_n^*(\{r\})H(R, \{r\})\varphi_m(\{r\}) = \delta_{nm} \frac{-\hbar^2 \Delta_R}{2\mu_n} + V_{nm}^{Nucl}(R) + V_{nm}^{Coul}(R) + iW_{nn}(R)\delta_{nm} + \delta_{nm}\varepsilon_n,$$

$V_{nm}^{Nucl}(R)$ and $V_{nm}^{Coul}(R)$ are nuclear and Coulomb parts of nucleus-nucleus matrix elements, μ_n is the reduced mass in channel n , and $W_{nn}(R)$ is imaginary part of potential related to the coupling to the closed channels Q .

We consider spinless colliding nuclei, therefore we present $\psi_n(R) = \sum_{LM} \frac{\xi_{nL}(R)}{R} Y_{LM}(\Omega)$. Multiplying this system of equations on $Y_{LM}(\Omega)$ and taking the integral on angle Ω we have system of coupled-channel equations

$$\left[\frac{-\hbar^2}{2\mu_n} \frac{\partial^2}{\partial R^2} + \frac{\hbar^2 L(L+1)}{2\mu_n R^2} + V_{nn}^{Nucl}(R) + V_{nn}^{Coul}(R) + iW_{nn}(R) + \varepsilon_n - E \right] \xi_{nL}(R) = - \sum_{m, m \neq n} [V_{nm}^{Nucl}(R) + V_{nm}^{Coul}(R)] \xi_{mL}(R).$$

This is complex coupled-channels equations.

The Distorted-Wave Born Approximation (DWBA) is simplest way to consider transfer reactions.

3. DWBA, single-particle and cluster states, spectroscopic factors

Lets consider two channels and propose that the influence of final channel (f) on the incident channel (i) is small.

Than we have system of two equations

$$\begin{aligned} & \left[\frac{-\hbar^2}{2\mu_i} \frac{\partial^2}{\partial R^2} + \frac{\hbar^2 L(L+1)}{2\mu_i R^2} + V_{ii}^{Nucl}(R) + V_{ii}^{Coul}(R) + iW_{ii}(R) - E \right] \xi_{iL}(R) \approx 0 \\ & \left[\frac{-\hbar^2}{2\mu_f} \frac{\partial^2}{\partial R^2} + \frac{\hbar^2 L(L+1)}{2\mu_f R^2} + V_{ff}^{Nucl}(R) + V_{ff}^{Coul}(R) + iW_{nn}(R) + \varepsilon_f - E \right] \xi_{fL}(R) \\ & \qquad \qquad \qquad = - \left[V_{fi}^{Nucl}(R) + V_{fi}^{Coul}(R) \right] \xi_{iL}(R). \end{aligned}$$

The application of Green function techniques leads to the formal solutions of these equations

$$\xi_{fL}(R) = \xi_{iL}^+(R) \delta_{if} + \int dR' G_{fL}^+(R, R') [V_{fi}^{Nucl}(R') + V_{fi}^{Coul}(R')] \xi_{iL}^+(R'),$$

where $G_{fL}^+(R, R')$ is the outgoing-wave Green function for the equation

$$\left[\frac{-\hbar^2}{2\mu_f} \frac{\partial^2}{\partial R^2} + \frac{\hbar^2 L(L+1)}{2\mu_f R^2} + V_{ff}^{Nucl}(R) + V_{ff}^{Coul}(R) + iW_{nn}(R) + \varepsilon_f - E^+ \right] G_{fL}^+(R, R') = \delta(R - R').$$

The amplitude of $i \rightarrow f$ transition is

$$T_{fiL} = T_{iiL} \delta_{if} + \int dR' \xi_{fL}^{-*}(R') [V_{fi}^{Nucl}(R') + V_{fi}^{Coul}(R')] \xi_{iL}^+(R'),$$

where $\xi_{fL}^-(R)$ is the solution of equation

$$\left[\frac{-\hbar^2}{2\mu_f} \frac{\partial^2}{\partial R^2} + \frac{\hbar^2 L(L+1)}{2\mu_f R^2} + V_{ff}^{Nucl}(R) + V_{ff}^{Coul}(R) + iW_{nn}(R) + \varepsilon_f - E^+ \right] \xi_{fL}^-(R) = 0$$

and T_{iiL} is the elastic transition amplitude.

For different initial i and final states f the DWBA transition amplitude of $i \rightarrow f$ transition is

$$\begin{aligned} T_{fiL} &= \int dR \xi_{fL}^{-*}(R) [V_{fi}^{NucL}(R) + V_{fi}^{Coul}(R)] \xi_{iL}^+(R) = \int dR \xi_{fL}^{-*}(R) F_{tr}(R) \xi_{iL}^+(R) \\ &= \int dR \xi_{fL}^{-*}(R) \left[\int d\{r\} \varphi_f^*(\{r\}) H(R, \{r\}) \varphi_i(\{r\}) \right] \xi_{iL}^+(R), \end{aligned}$$

where $F_{tr}(R')$ is the transfer form-factor. The differential cross section of transfer reaction can be written as

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{k_f}{k_i} \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \sum_L |T_{fiL}|^2 = \frac{k_f}{k_i} \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \sum_L \left| \int dR \xi_{fL}^{-*}(R) F_{tr}(R) \xi_{iL}^+(R) \right|^2,$$

where $k_f = \sqrt{2\mu_f E_f/\hbar^2}$, $k_i = \sqrt{2\mu_i E_i/\hbar^2}$, μ_i, μ_f, E_i, E_f are the reduced masses and energies in incident and final channels respectively.

In the realistic case the transfer amplitude of reaction $A + B = (C + c) + B \rightarrow C + (c + B) = C + D$ depends on the probability of representation of nucleus A as $(C + c)$ and nucleus D as $(c + B)$. These probabilities are the spectroscopic factors, which are defined as

$$S_{A,C+c} = \left| \int d\{r_1\} \phi_A^{exact}(\{r_1\}) \phi_C(\{r_1\}) \otimes \phi_c(\{r_1\}) \right|^2, \quad S_{D,B+c} = \left| \int d\{r_2\} \phi_D^{exact}(\{r_2\}) \phi_B(\{r_2\}) \otimes \phi_c(R, \{r_2\}) \right|^2.$$

Here $\phi_A^{exact}(\{r_1\})$ and $\phi_D^{exact}(\{r_2\})$ are exact wave-functions of nuclei A and D respectively. Note that $S_{A,C+c} \leq 1$, $S_{D,B+c} \leq 1$ as a rule. The realistic differential cross section of transfer reaction can be written as

$$\frac{d\sigma(\theta)}{d\Omega} = S_{A,C+c} S_{D,B+c} \frac{k_f}{k_i} \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \sum_L |T_{fiL}|^2.$$

If $S_{A,C+c} = S_{D,B+c} = 1$ we get previous expression for the differential cross section of transfer reaction.

4. Transfer reactions around barrier. Sub-Coulomb transfer.

The simplest description of the elastic transfer for scattering processes in the vicinity of the Coulomb barrier than would be in terms of a semiclassical model by Broglia and Winther PC19, 1 (1972) (it is easy derive from the DWBA taking into account the same ingoing and outgoing wave), where

$$T_{tr}(\theta) \simeq T_{\text{elastic}}(\theta) \sqrt{P_{tr}(R)},$$

with $P_{tr}(\theta)$ representing the transfer probability.

The transfer cross-section is

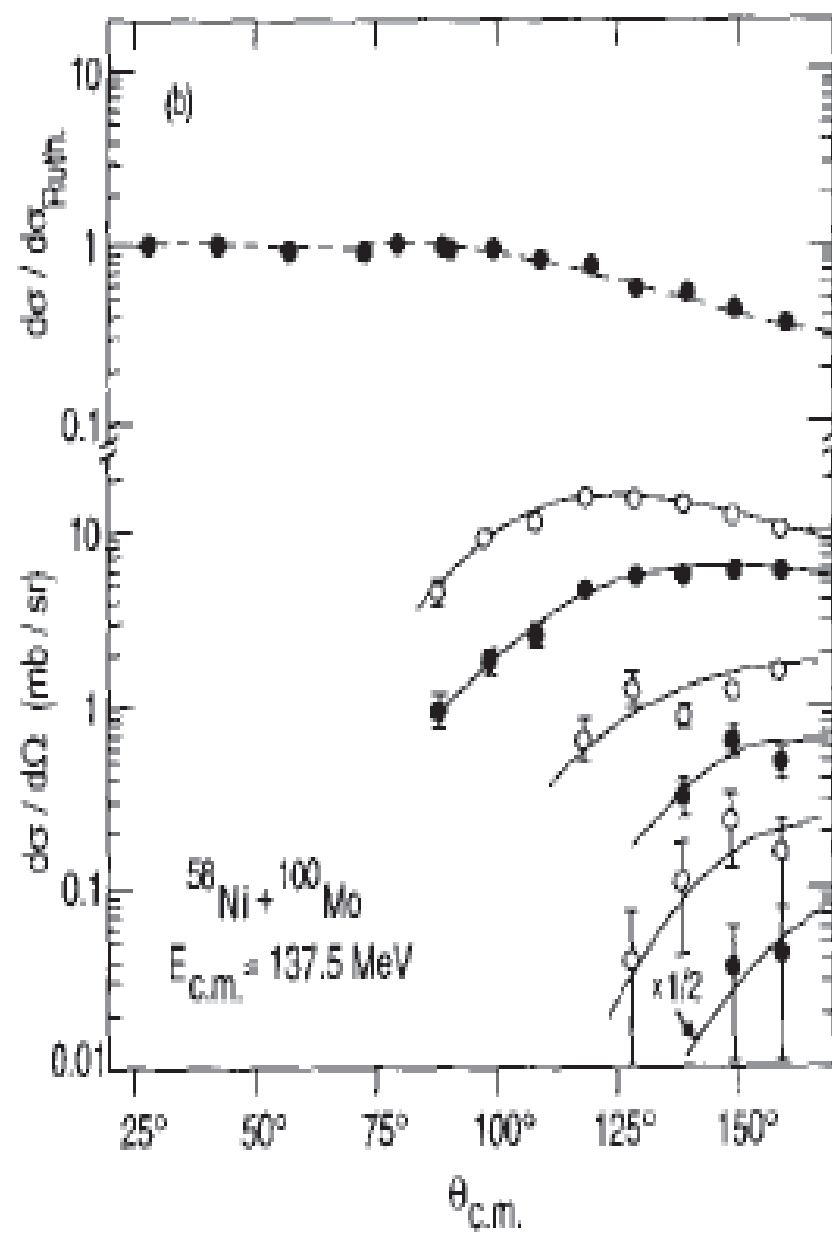
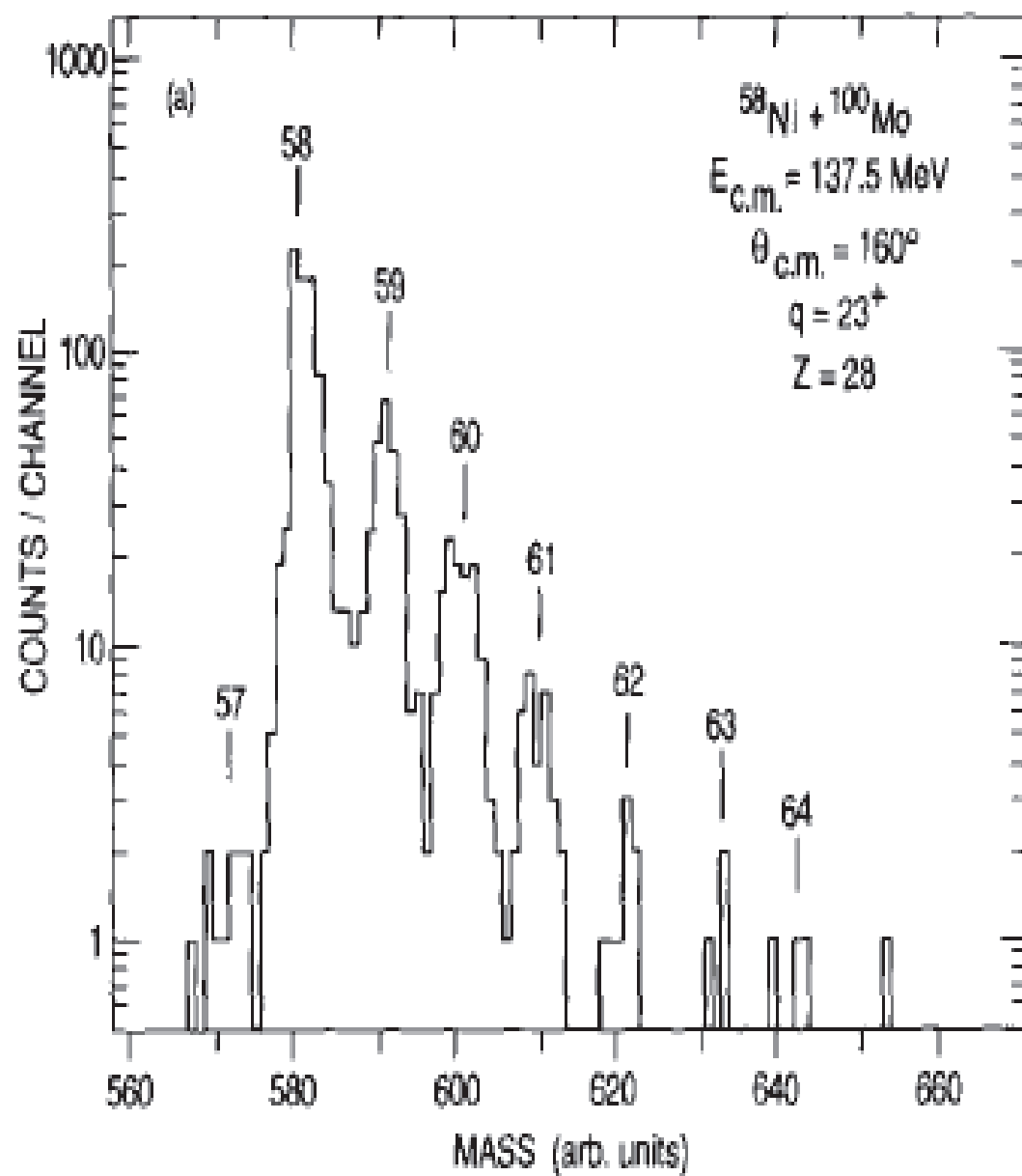
$$\frac{d\sigma(\theta)}{d\Omega} \simeq S_{A,C+c} S_{D,B+c} \left. \frac{d\sigma(\theta)}{d\Omega} \right|_{\text{elastic}} P_{tr}(\theta) = S_{A,C+c} S_{D,B+c} \left. \frac{d\sigma(\theta)}{d\Omega} \right|_{\text{Rutherford}} P_{tr}(\theta).$$

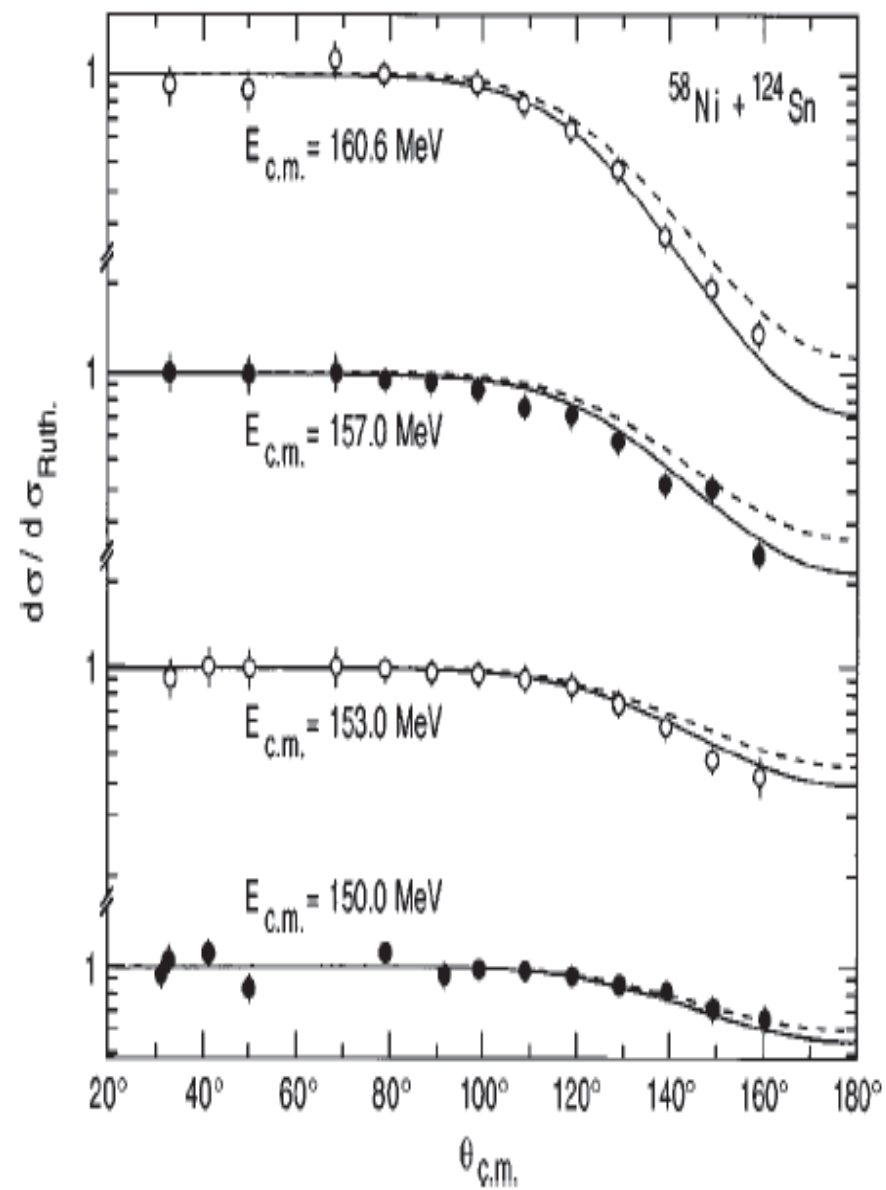
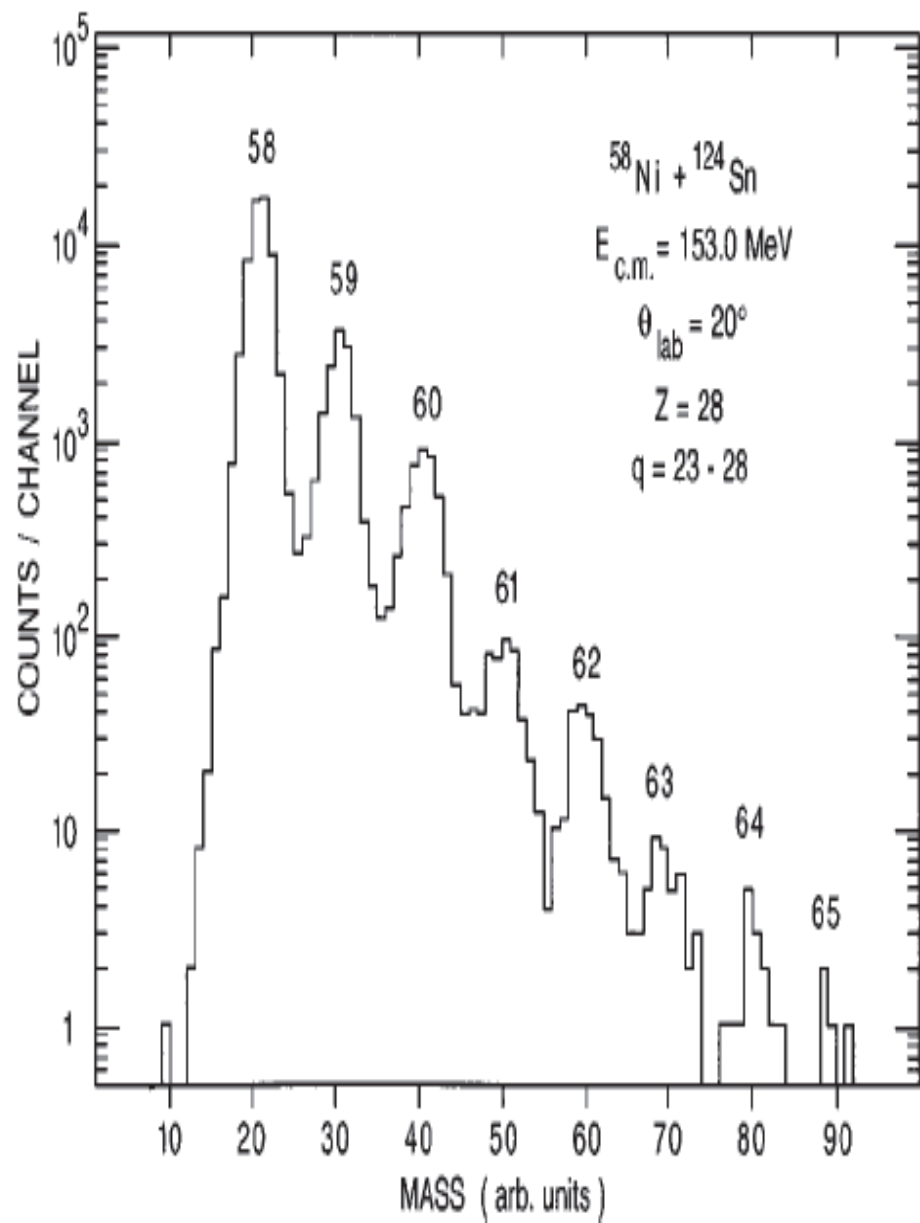
The simplest approximation for the transfer probability $P_{tr}(\theta)$ is semiclassical value for sub-barrier penetration of transferred particle under rectangle barrier. The barrier height for the neutron with $\ell = 0$ is zero, the neutron separation energy is ε and distance of tunneling (barrier width $d(\theta) = D(\theta) - (R_1 + R_2)$) equals to the distance between surfaces of colliding nuclei at the closest point. So

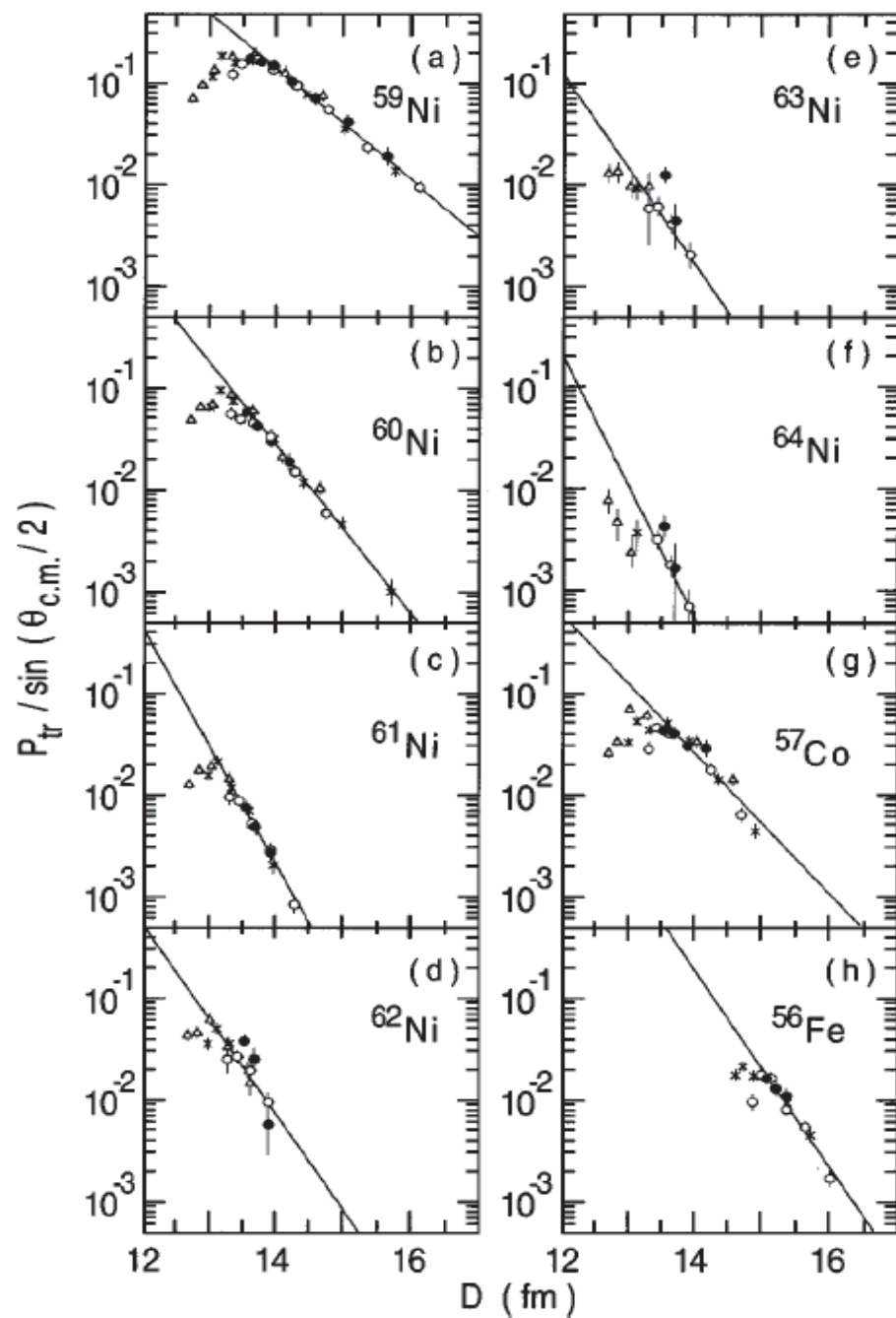
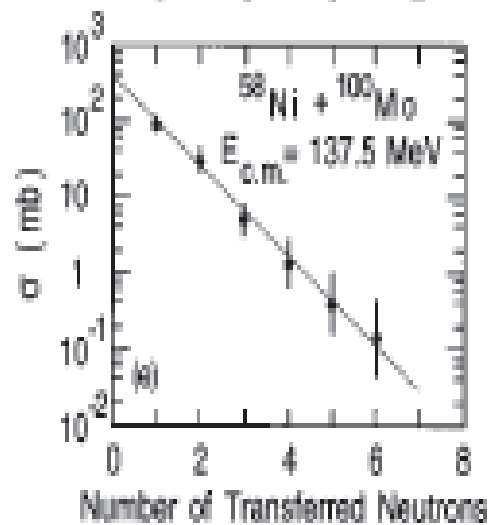
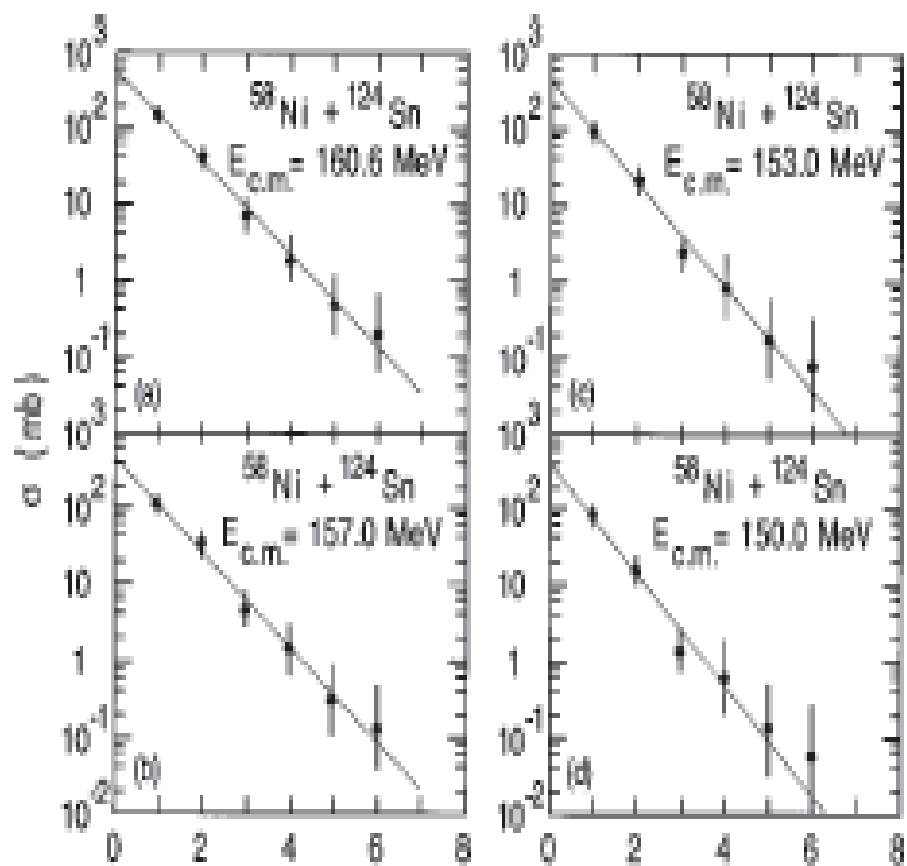
$$P_{tr}(\theta) \simeq \exp \left[-\sqrt{2M_n \varepsilon / \hbar^2} D(\theta) \right] \sin(\theta/2).$$

The distance of closest point between surfaces is evaluated by using classical Coulomb trajectory

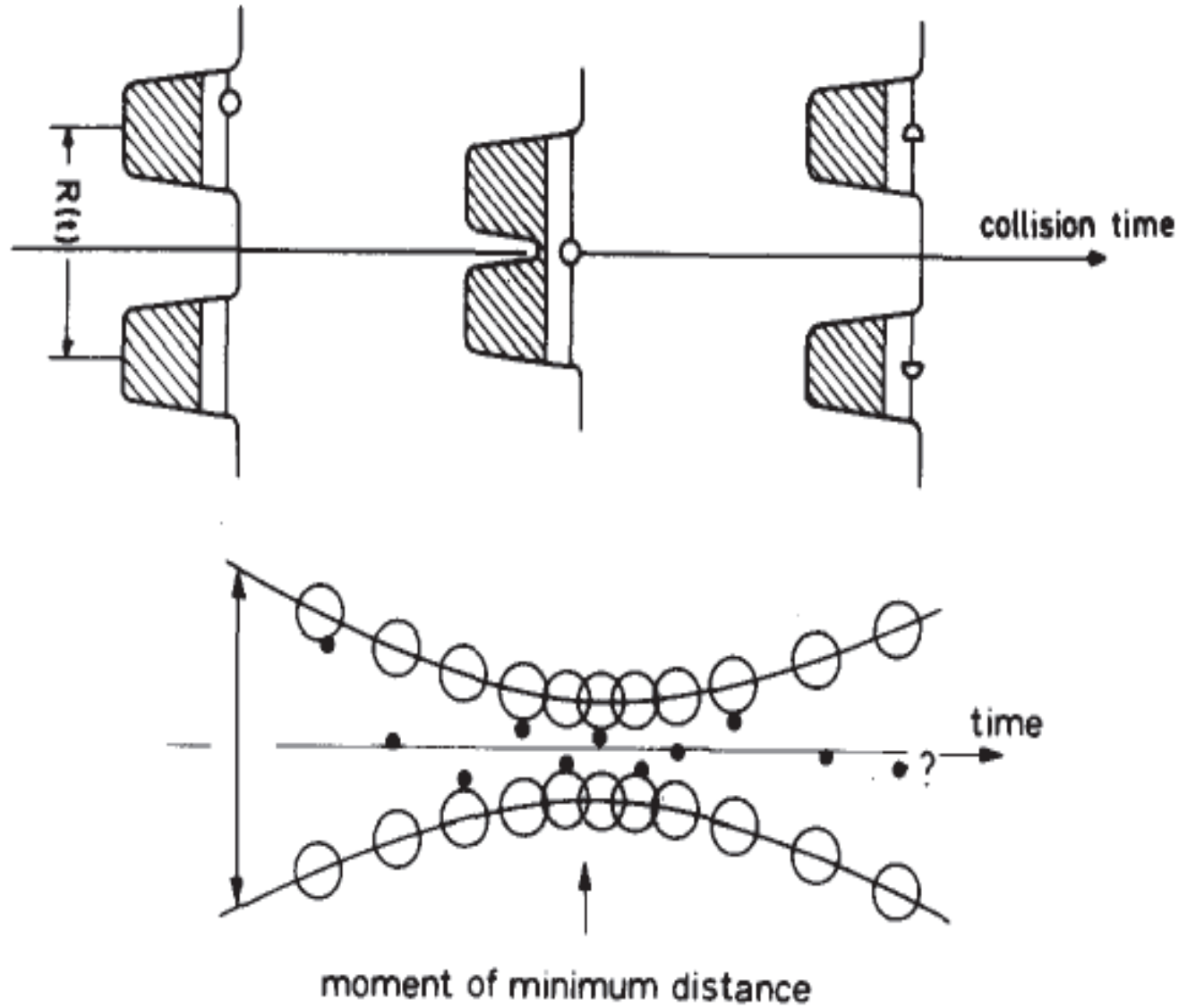
$$D(\theta) \simeq \frac{Z_1 Z_2 e^2}{2E_{cm}} \left(1 + \frac{1}{\sin(\theta/2)} \right).$$







4. Transfer reactions around barrier. Quasi-elastic transfer.



Schematic representation of the exchange of valence nucleons in a collision between two complex nuclei.

$$\Psi = \psi_{n,C_1}(R, r)\phi_{n+C_1}(r)/r + (-1)^\pi \psi_{n,C_2}(R, r)\phi_{n+C_2}(r')/r'$$

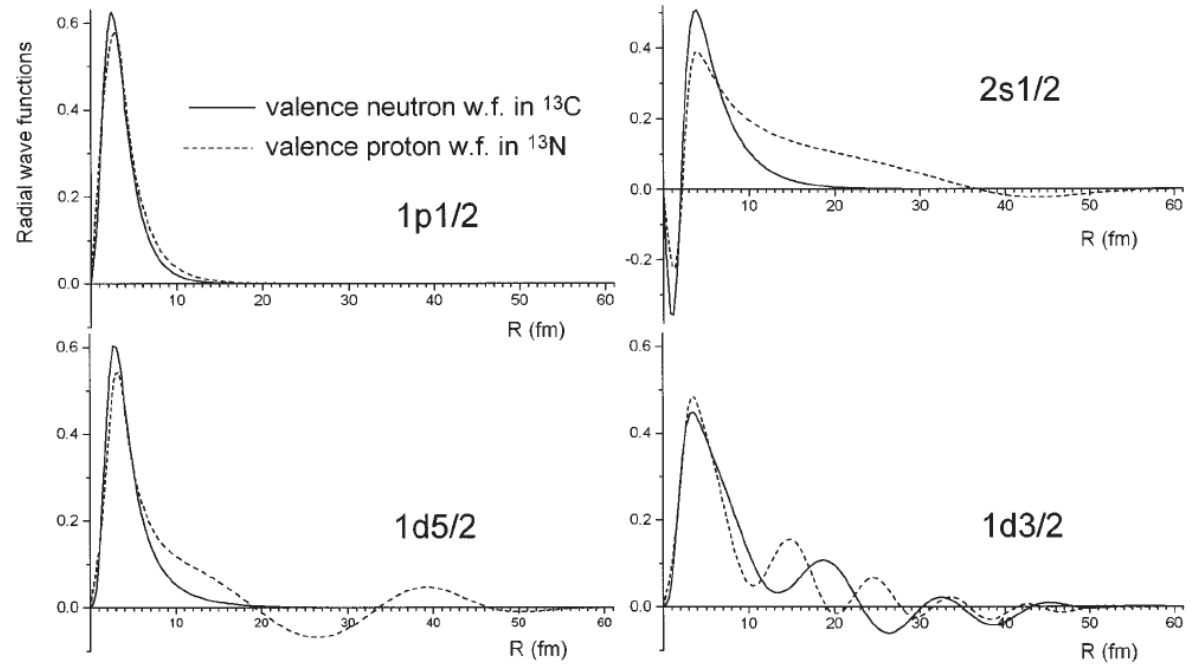
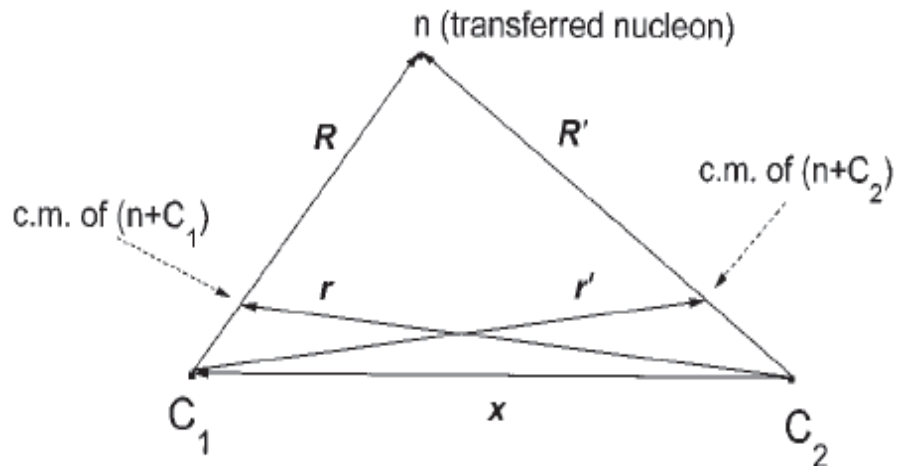


TABLE III. Parameters of the core-core optical potential $V_{CC}(\Pi j E_{c.m.}; r)$, where “ j ” specifies channel.

$$V_{CC}(\Pi j E_{c.m.}; r) = \frac{1 - (-)^\Pi}{2} \frac{V^- + \hat{V}^- E_{c.m.}}{1 + \exp[(r - 1.42 \times 2A^{1/3})/0.22]} + \frac{1 + (-)^\Pi}{2} \frac{V^+ + \hat{V}^+ E_{c.m.}}{1 + \exp[(r - 1.25 \times 2A^{1/3})/0.51]}$$

$$+ i W_j(E_{c.m.}) \left\{ \frac{1 - (-)^\Pi}{2} \frac{1}{1 + \exp[(r - 1.44 \times 2A^{1/3})/1.03]} + \frac{1 + (-)^\Pi}{2} \frac{1}{1 + \exp[(r - 1.32 \times 2A^{1/3})/0.96]} \right\},$$

$$W_j(E_{c.m.}) = \begin{cases} -0.7(1 - 1/[1 + \exp\{1.5487(E_{c.m.} - 5.7754)\}]), & \text{for } j=1 \text{ (elastic channel)} \\ -0.94 - 0.2 E_{c.m.}, & \text{for } E_{c.m.} < 9 \text{ MeV} \\ -2.74, & \text{for } 9 \text{ MeV} < E_{c.m.} \end{cases} \quad \text{for } j \geq 2 \text{ (inelastic channel).}$$

(unit of r ; in fm, unit of energy; in MeV, $A=12$).

Π	$E_{c.m.}$ (MeV)	V^Π (MeV)	\hat{V}^Π (MeV)
-	<6	-22.2	0.0
	>6	-27.0	0.8
+	<7.8	-19.76	0.0
	>8.3	-24.0	0.8

TABLE II. CFP values of single-particle states used in the calculations.

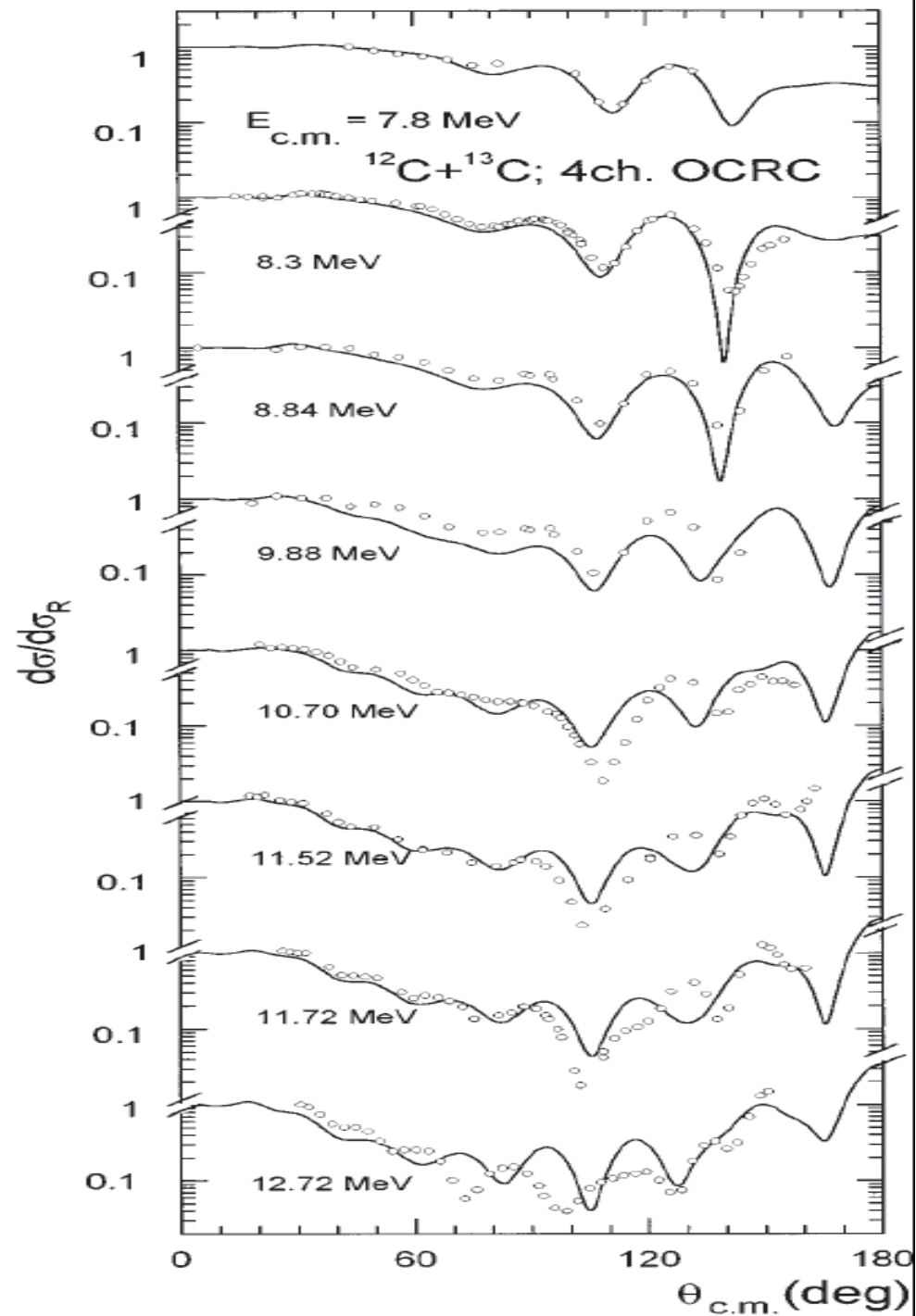
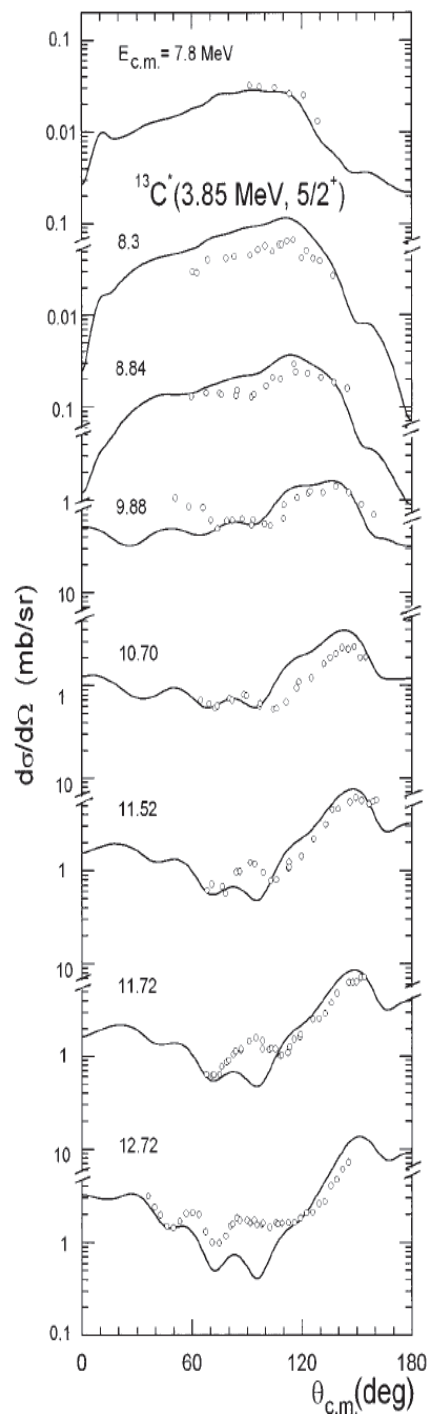
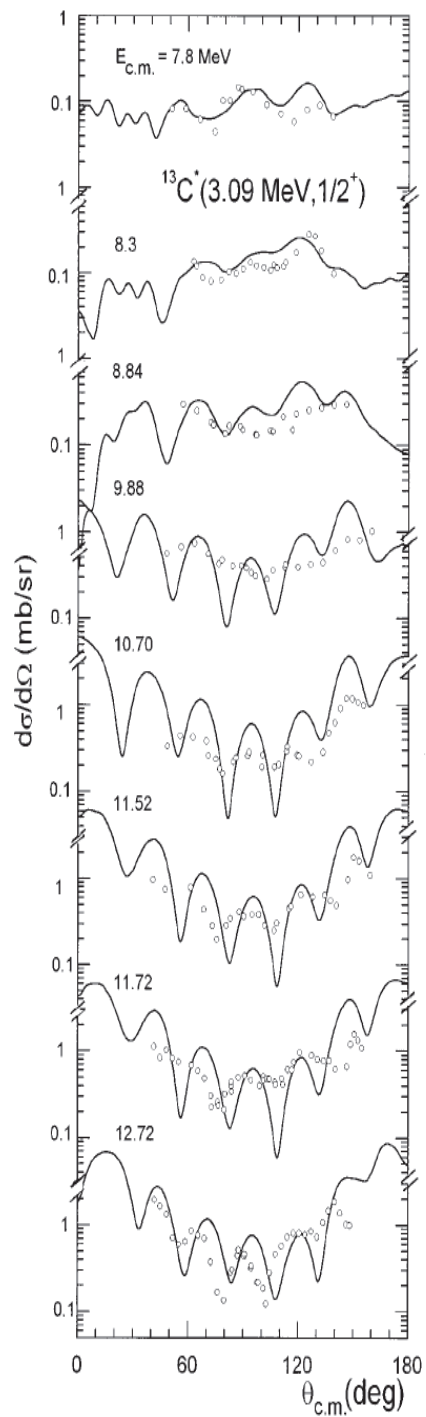
Nucleus	$1p_{\frac{1}{2}}$	$2s_{\frac{1}{2}}$	$1d_{\frac{5}{2}}$	$1d_{\frac{3}{2}}$
^{13}C	0.78	0.80	0.92	0.90
^{13}N	0.78	0.80	0.92	0.90

TABLE I. Parameters of the potential $V_{nc}(R)$ between valence particle and core nucleus.

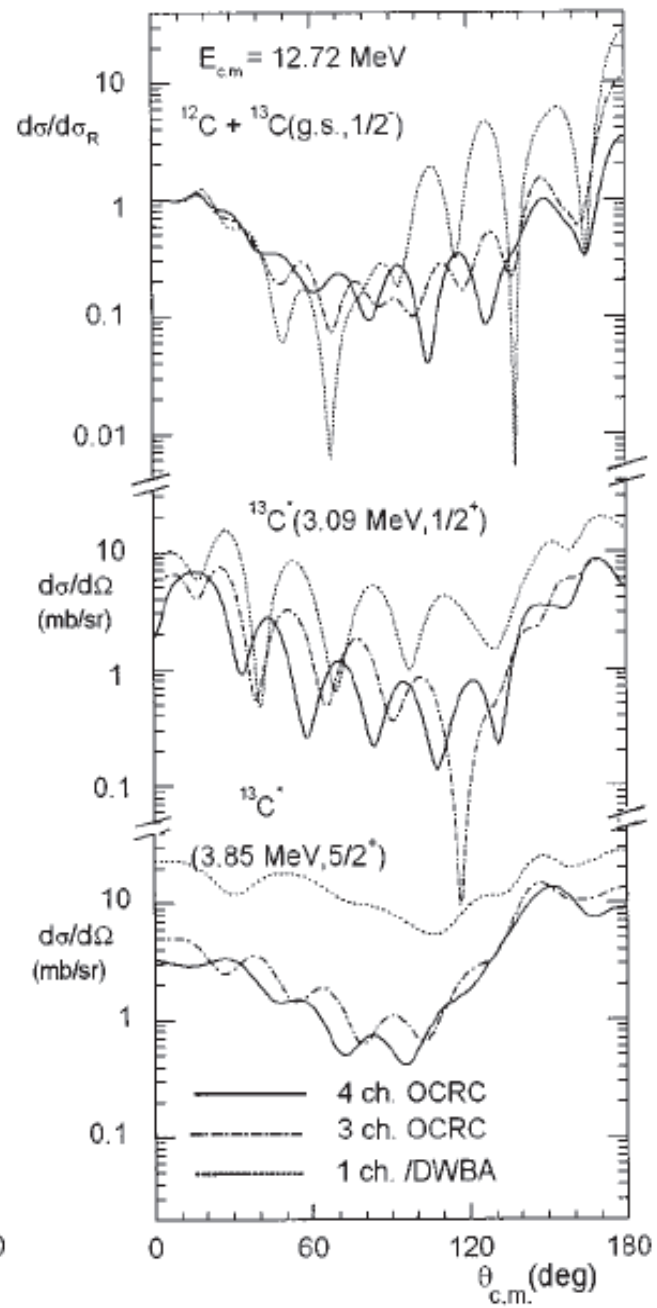
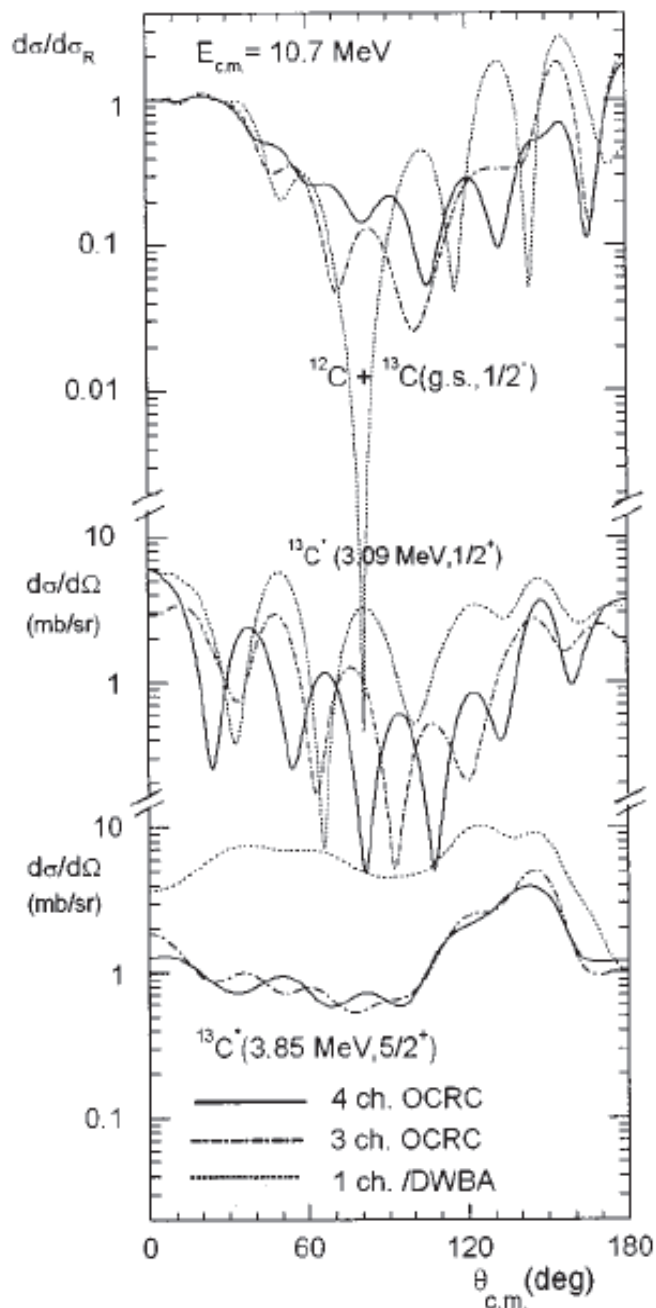
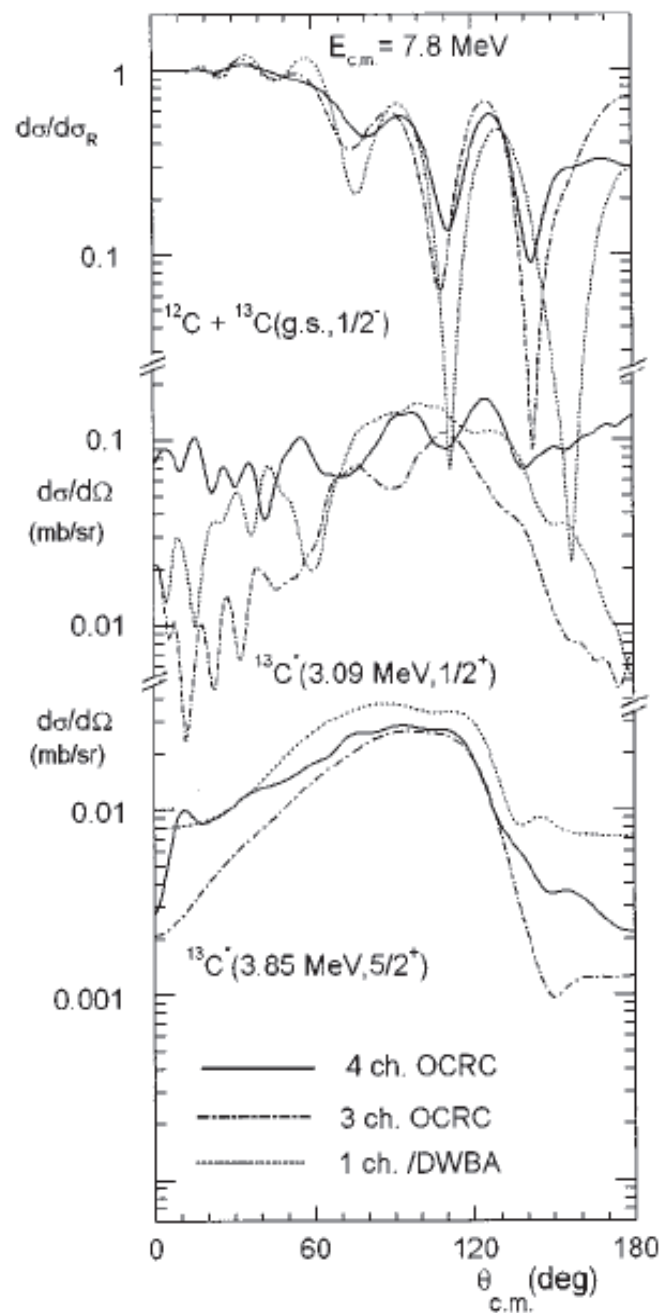
$$V_{nc}(R) = \frac{1 + (-)^\pi}{2} \frac{V^+}{1 + \exp[(R - R^+)/a^+]} + \frac{1 - (-)^\pi}{2} \frac{V^-}{1 + \exp[(R - R^-)/a^-]} + (\not{S}) \frac{1}{R} \frac{d}{dR} \frac{V'}{1 + \exp[(R - R')/a']} + V_{\text{Coul.}}(R_C)$$

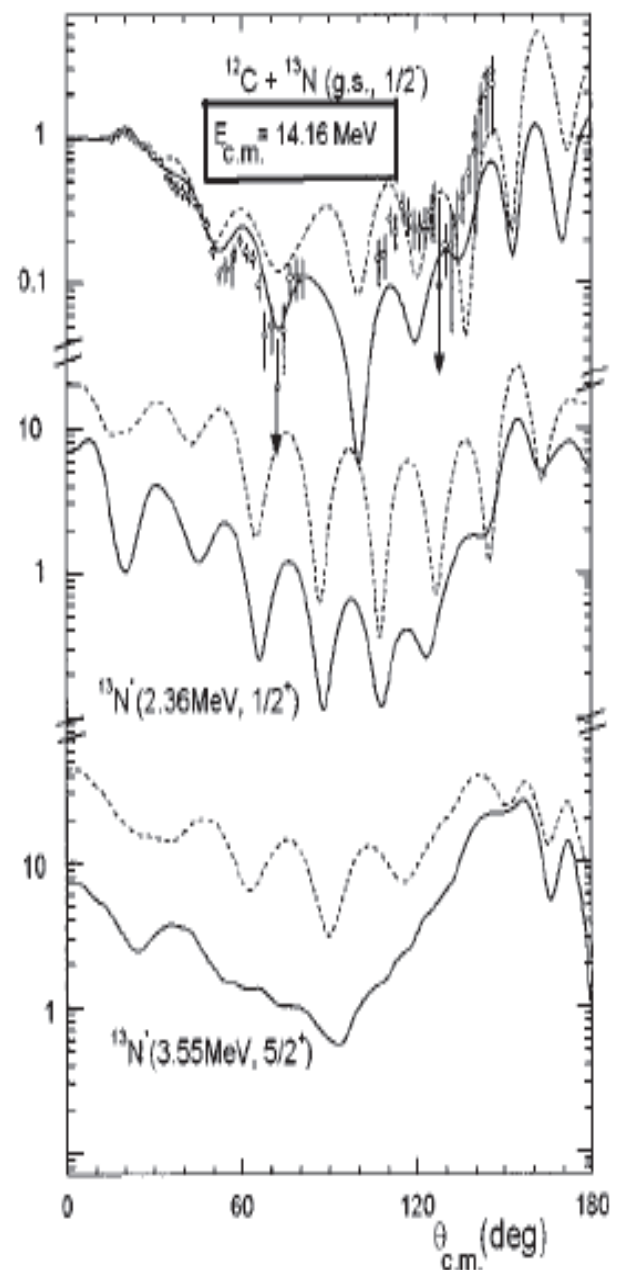
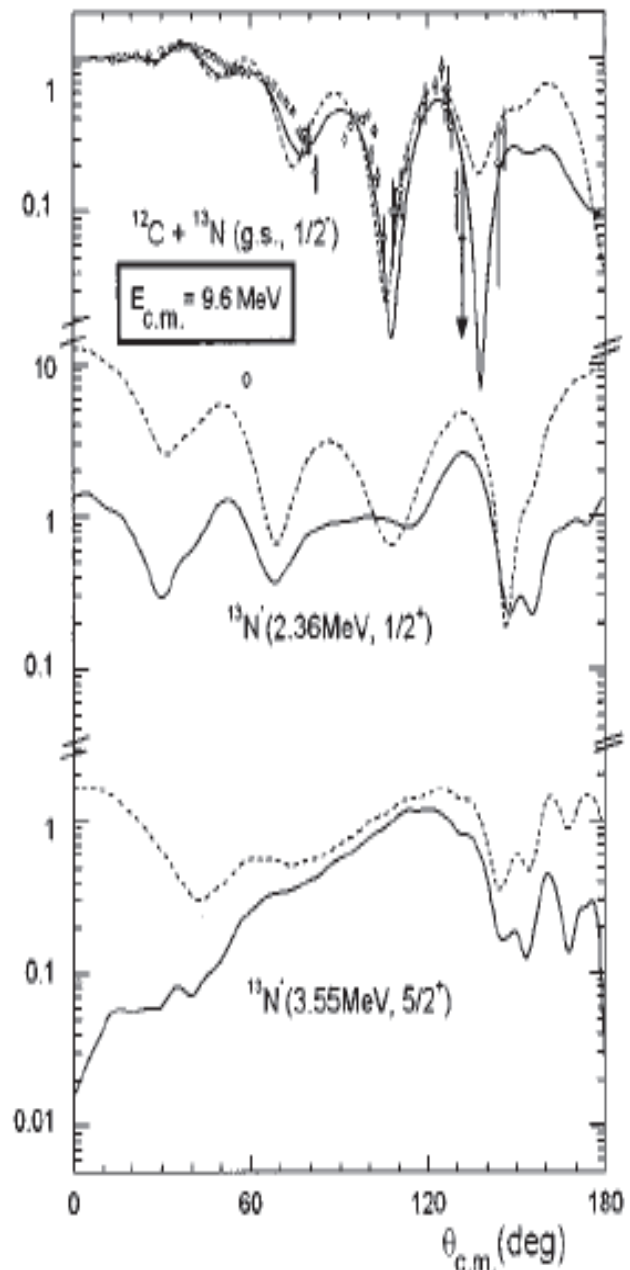
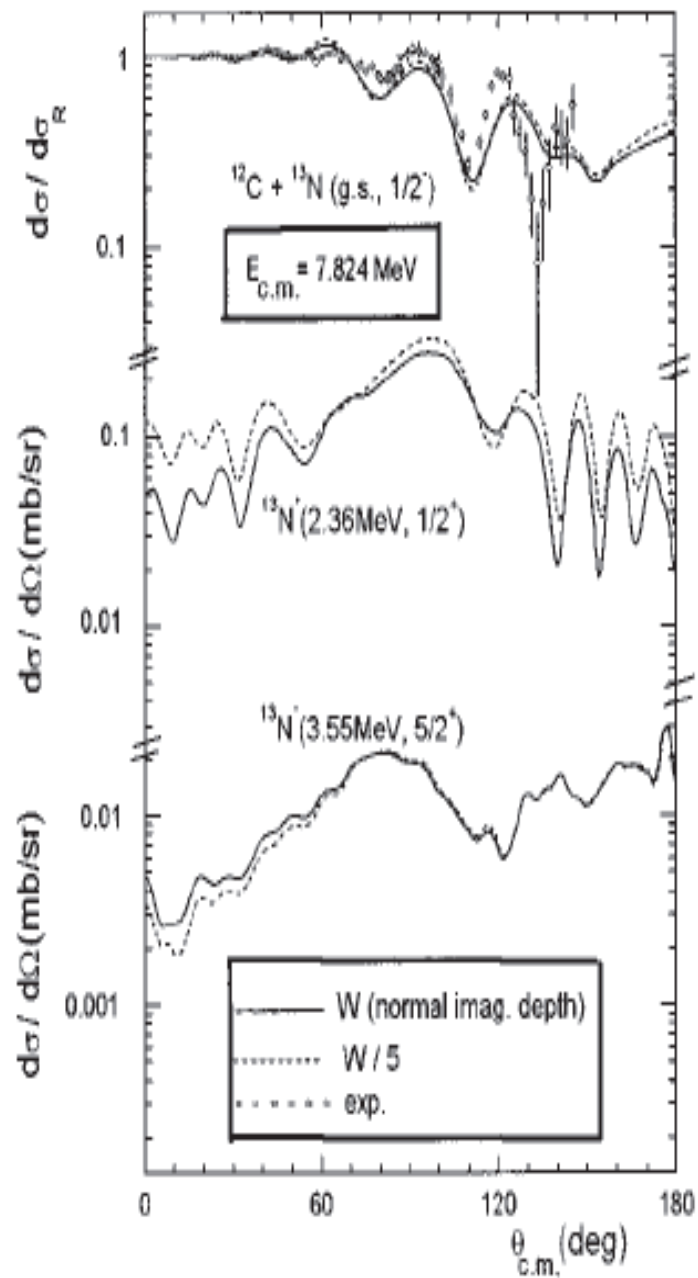
with $R^+ = 2r_0^+ A^{1/3}$, $R^- = 2r_0^- A^{1/3}$, $R' = 2r_0' A^{1/3}$, and $R_C = 2r_C A^{1/3}$

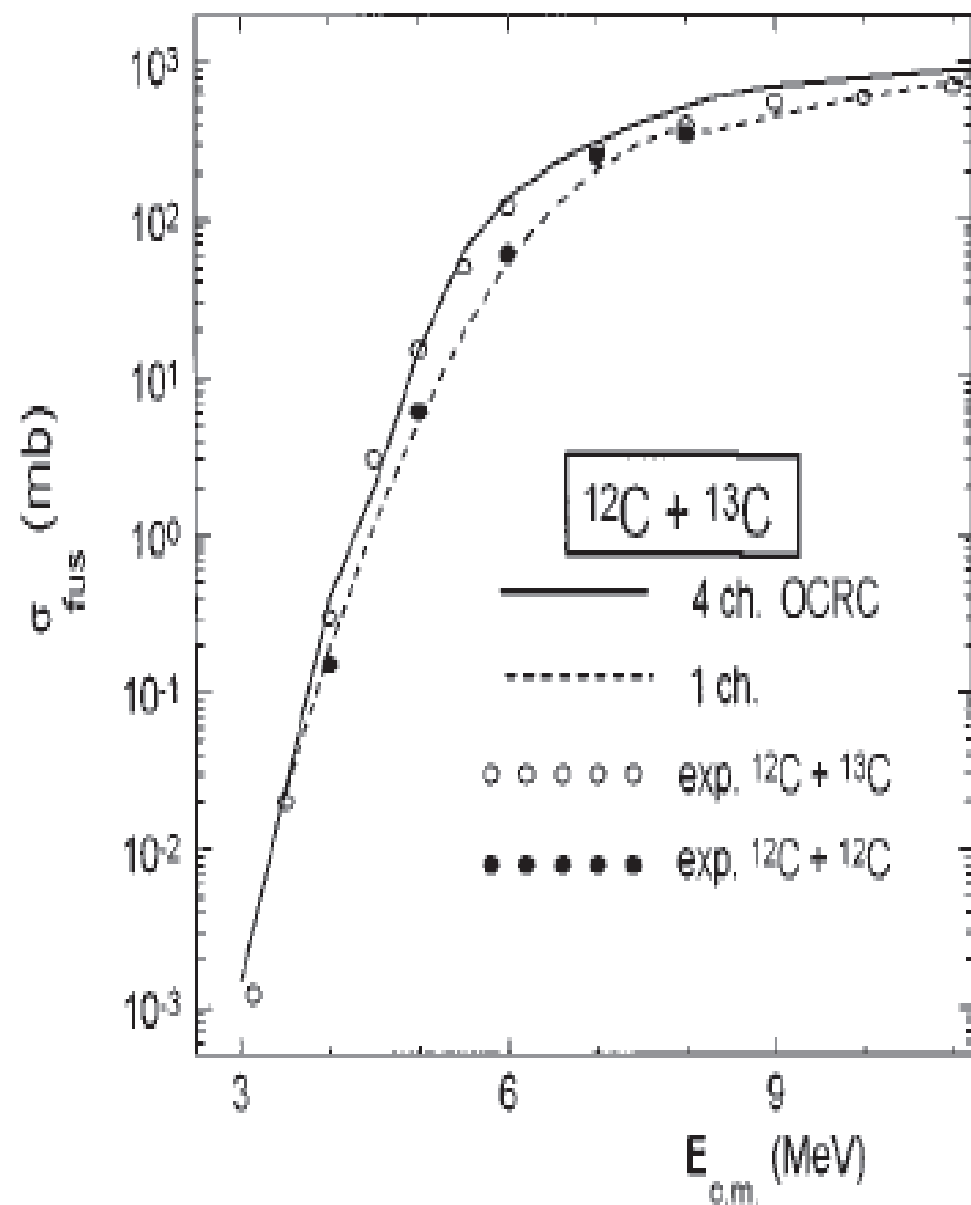
System	$\pi = -$			$\pi = +$			(\not{S})			r_C (fm)
	r_0^- (fm)	a^- (fm)	V^- (MeV)	r_0^+ (fm)	a^+ (fm)	V^+ (MeV)	r_0' (fm)	a' (fm)	V' (MeV)	
$n + ^{12}\text{C}$	1.16	0.705	-57.4	1.16	0.728	-73.62	1.16	0.705	-29.56	
$p + ^{12}\text{C}$	1.193	0.700	-48.27	1.25	0.700	-59.39	1.25	0.700	-28.18	1.25



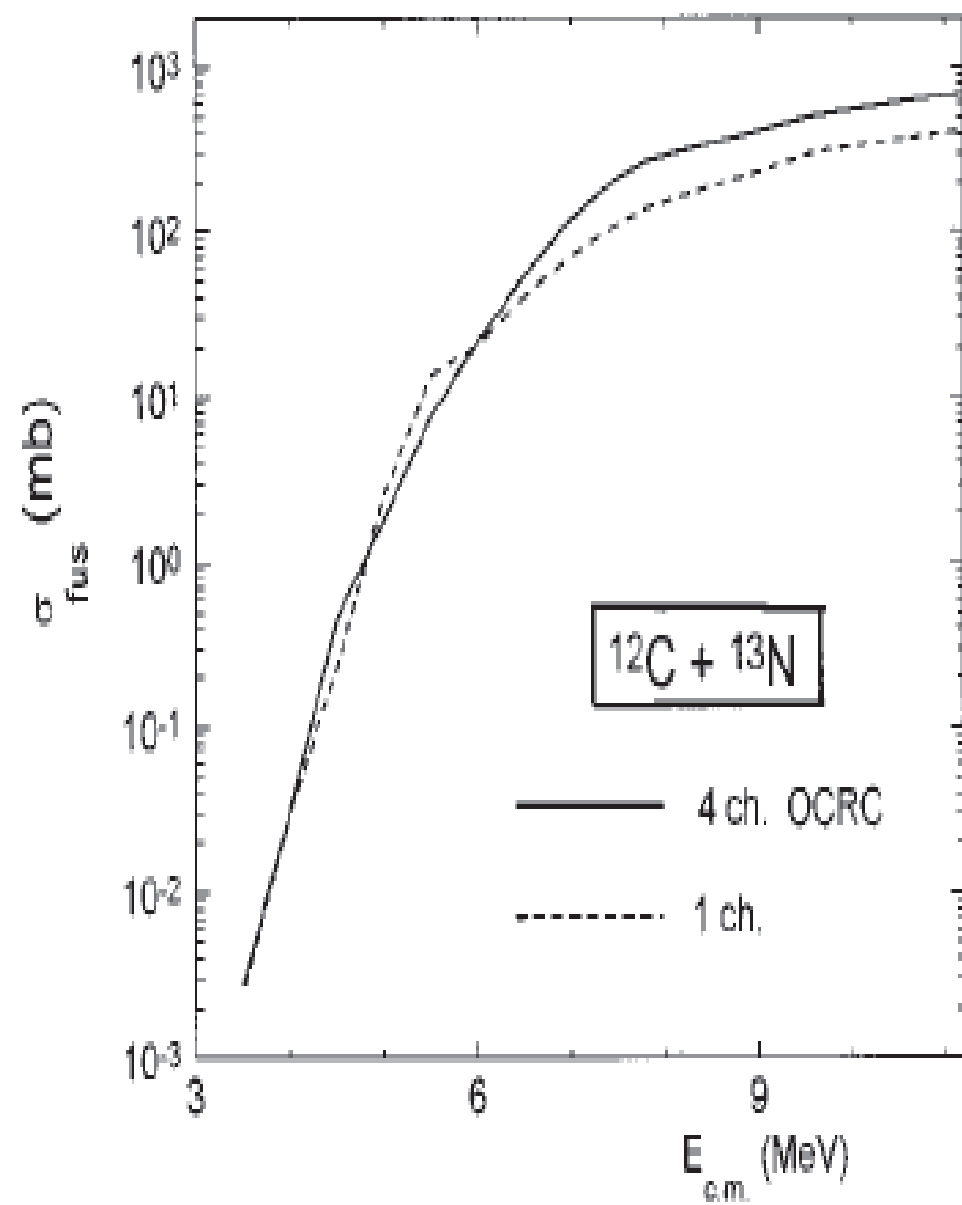
Comparison between DWBA, OCRC 3 and 4 channels calculations.







(a)



(b)

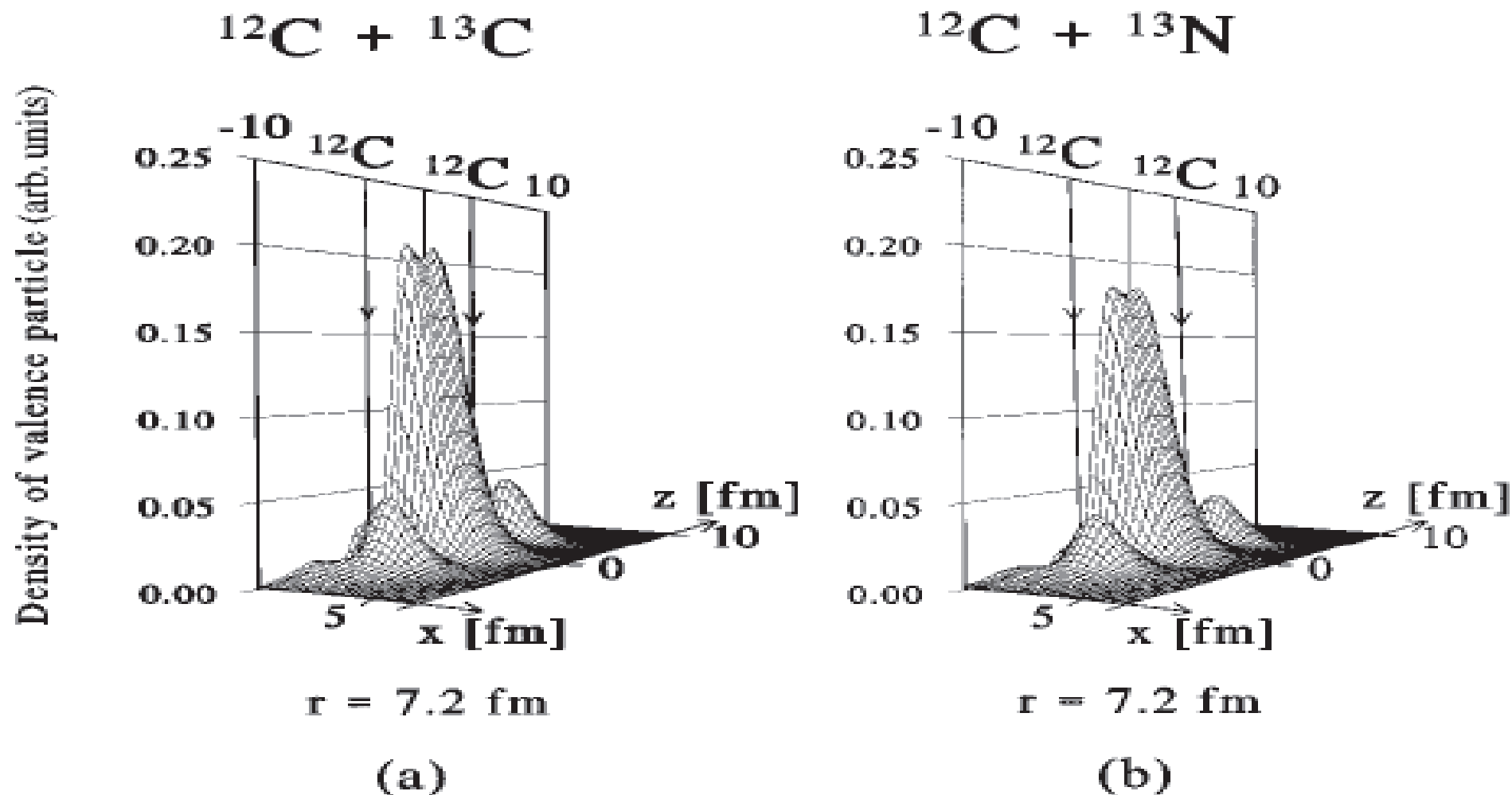
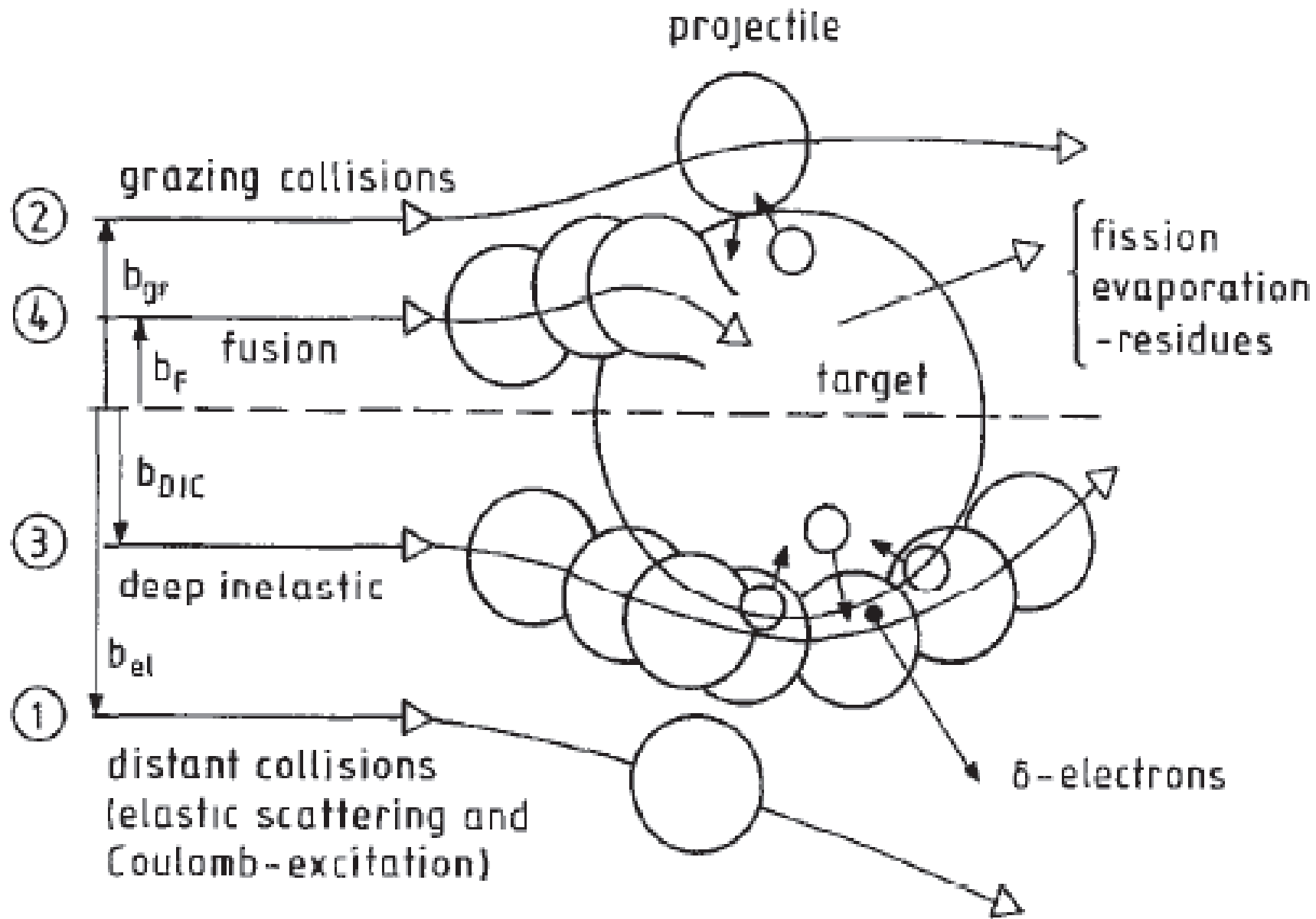
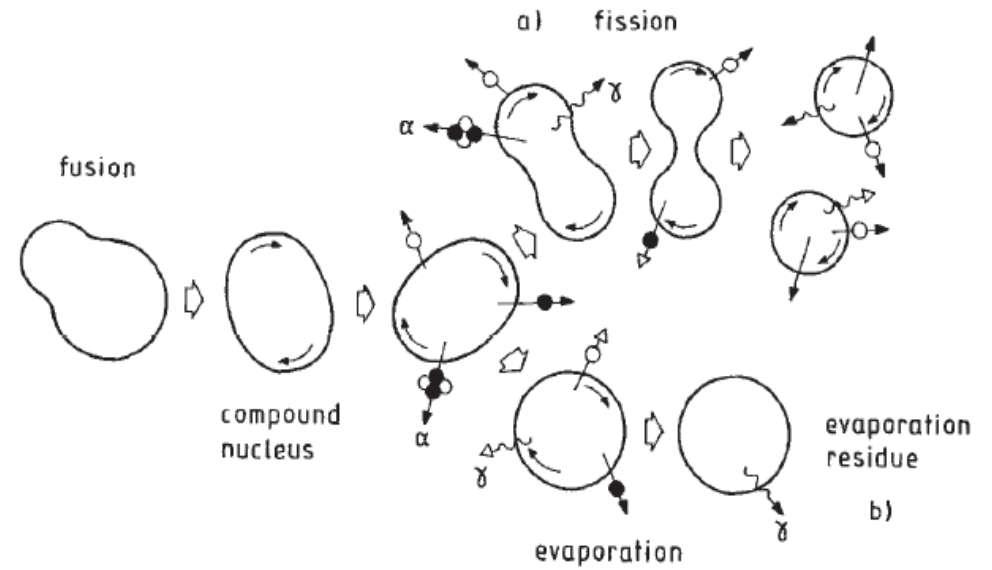
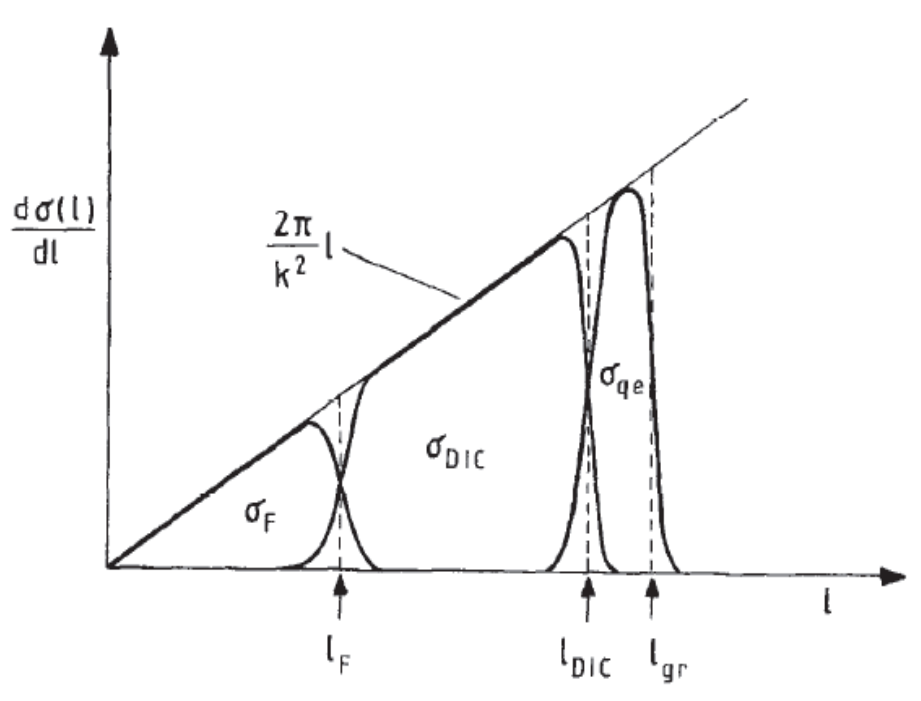


FIG. 18. Density distributions of the valence particles in the ground RMO (rotating molecular orbital: see the text) states at the internuclear distance $r = 7.8 \text{ fm}$ (a) in the $^{12}\text{C} + ^{13}\text{C}$ system and (b) in the $^{12}\text{C} + ^{13}\text{N}$ system. The positions of the core nuclei for the respective systems are marked by arrows.

5. Deep-inelastic collisions (DIC). Di-nuclear system. Fast fission.



$$\sigma_{fus}(E) = \frac{\pi^2 \hbar^2}{2M_N A_{12} E} \sum_{\ell=0}^{\infty} (2\ell + 1) T_{\ell}(E) = \frac{\pi^2 \hbar^2}{2M_N A_{12} E} \sum_{\ell=0}^{\infty} (2\ell + 1) \frac{1}{1 + \exp[-2\pi(E - Barrier)/(\hbar\omega_{Barrier})]}$$



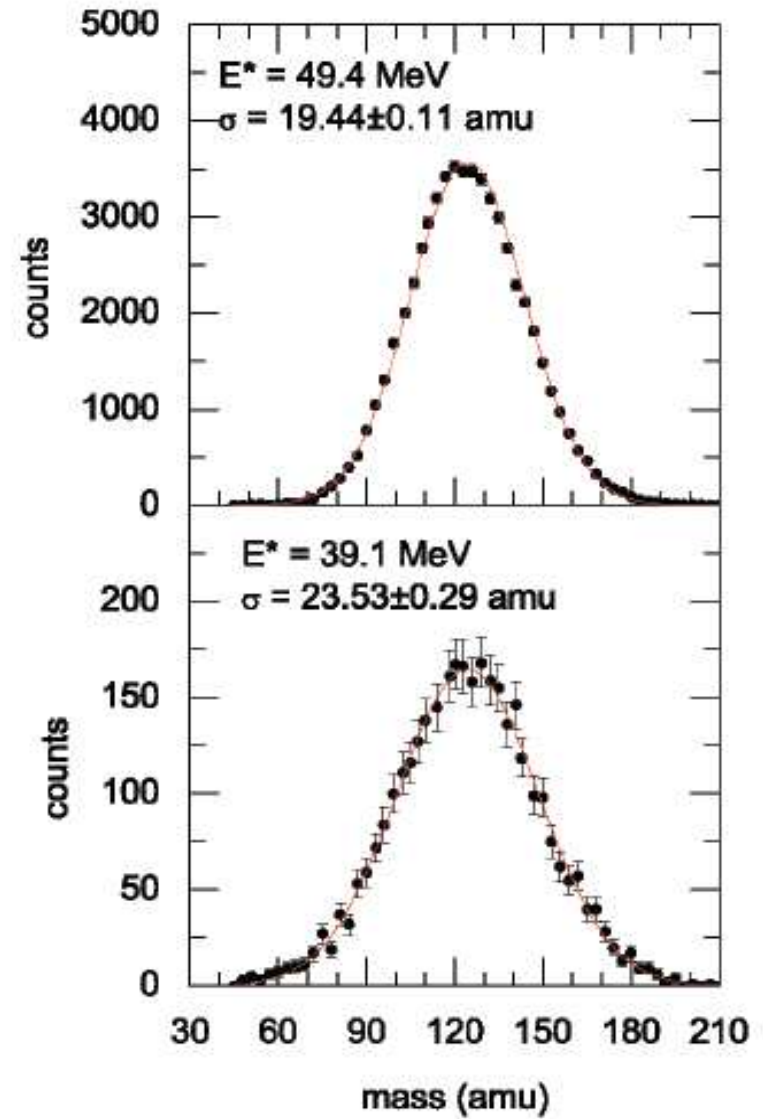
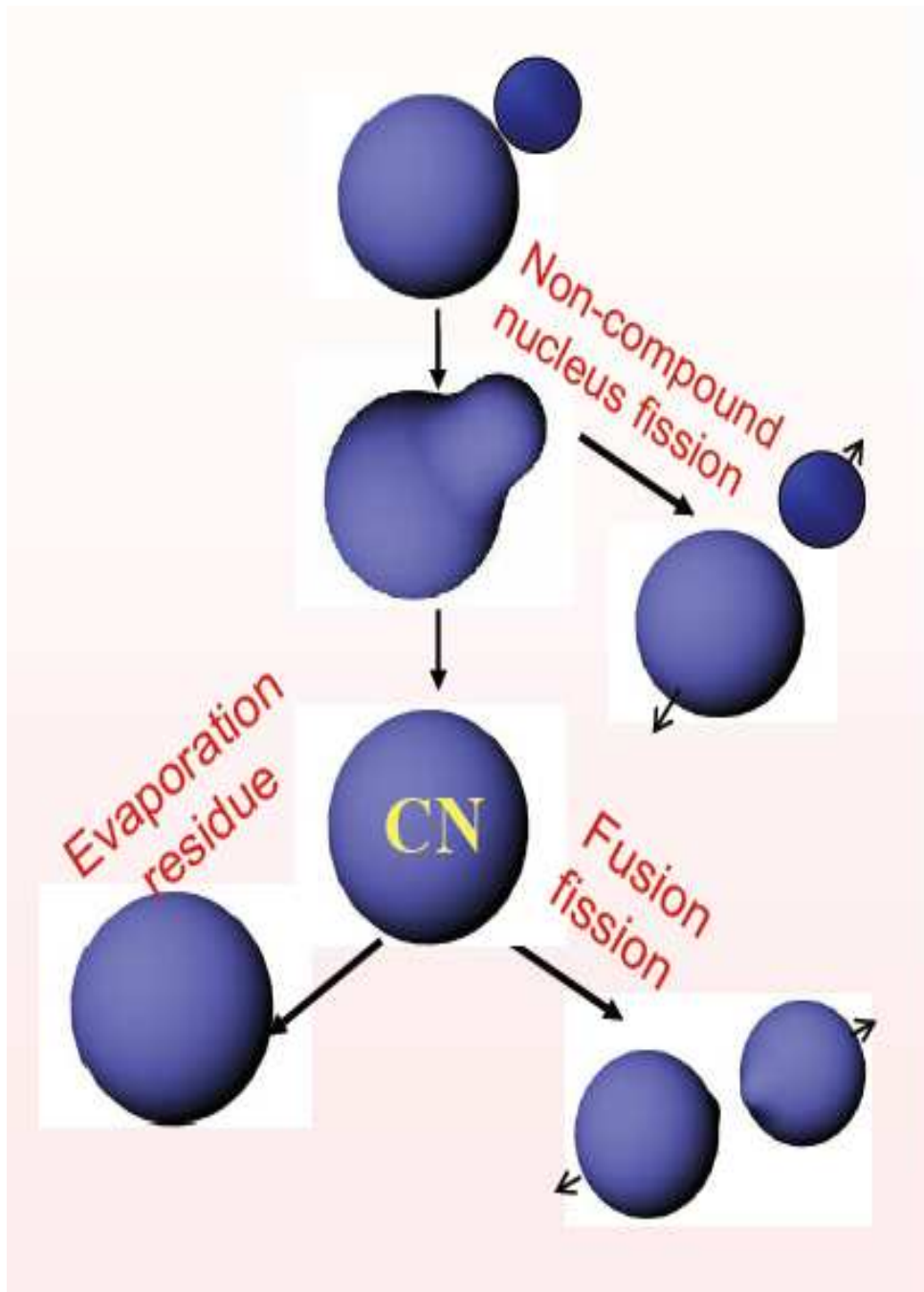
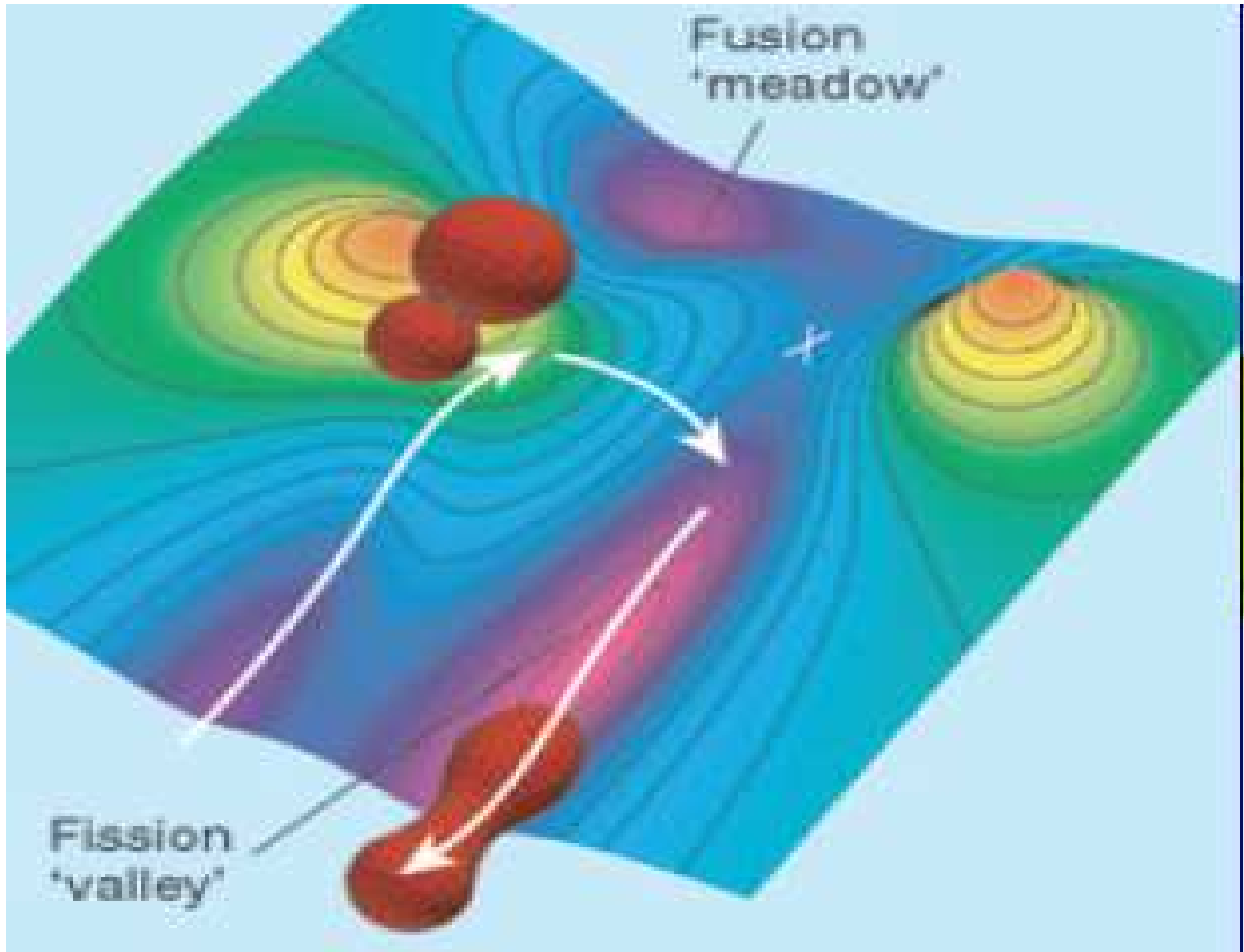


Fig. 2. Mass distributions at two excitation energies for the system $^{19}\text{F} + ^{232}\text{Th}$. The Gaussian fits are shown by the solid lines.



DIC are going through the formation of di-nuclear system.

Due to high angular momentum the colliding nuclei cannot form a compound nucleus, however nucleons exchange strongly between nucleon during di-nuclear system life-time.

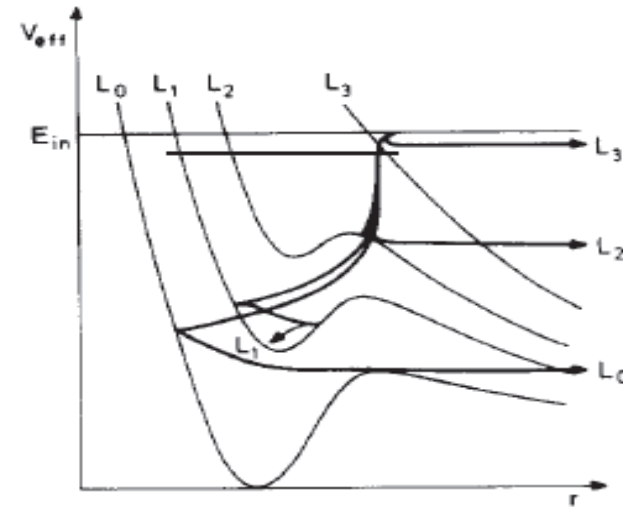
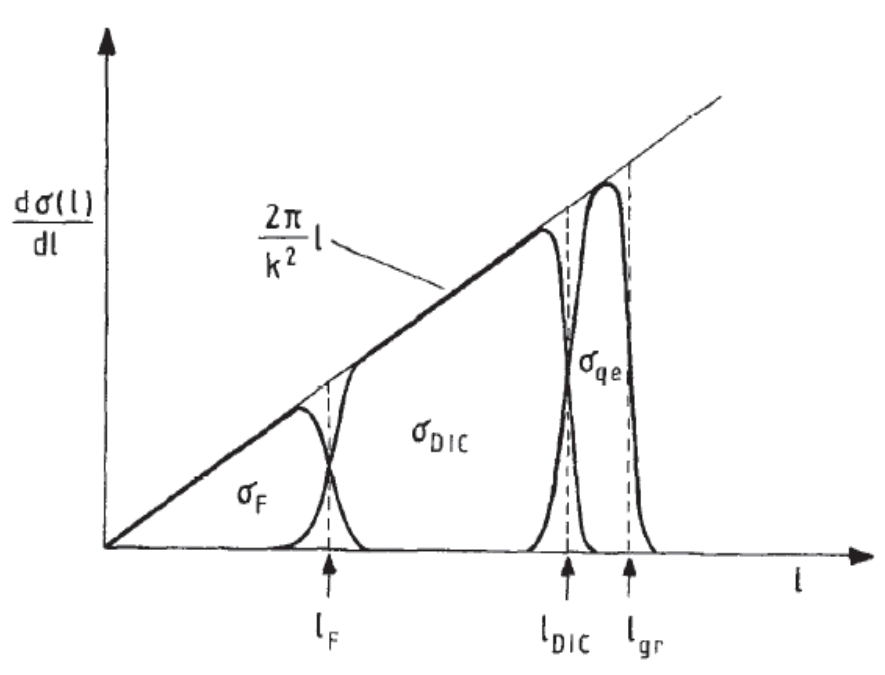
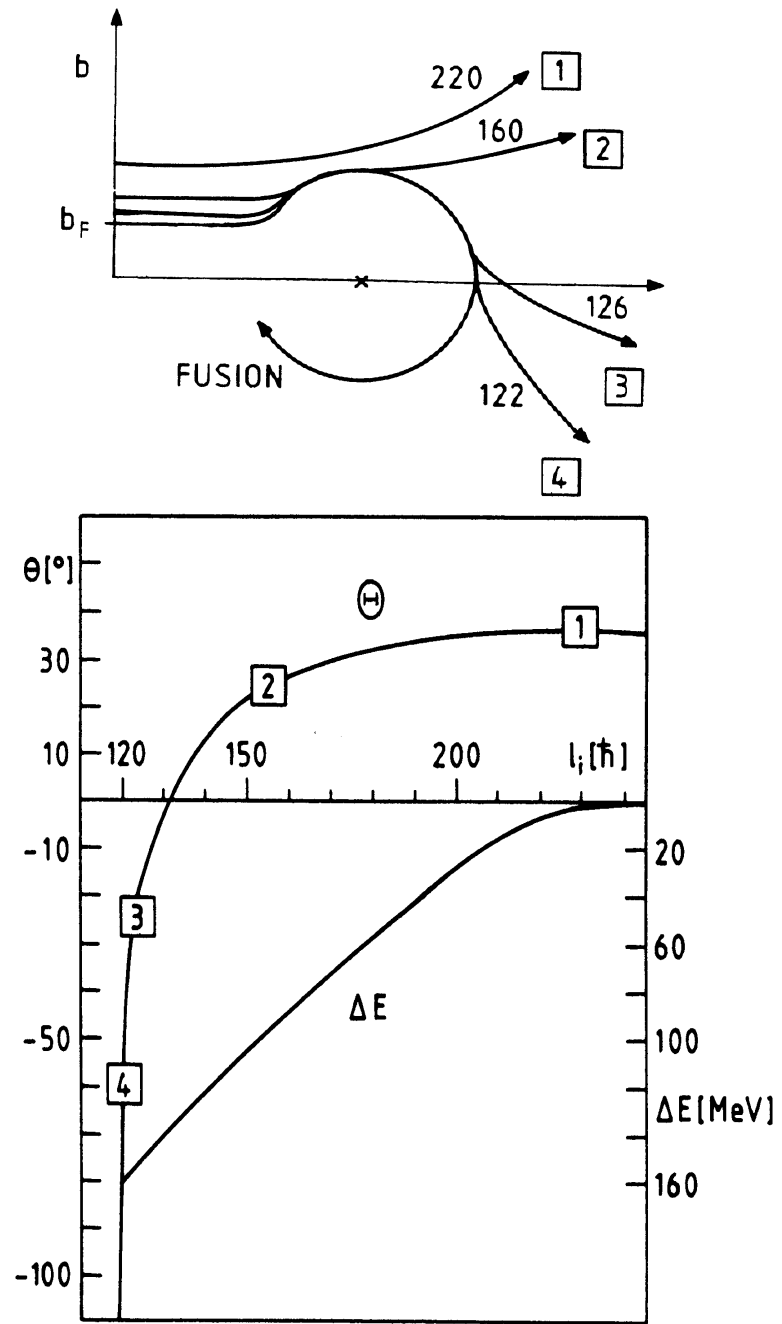
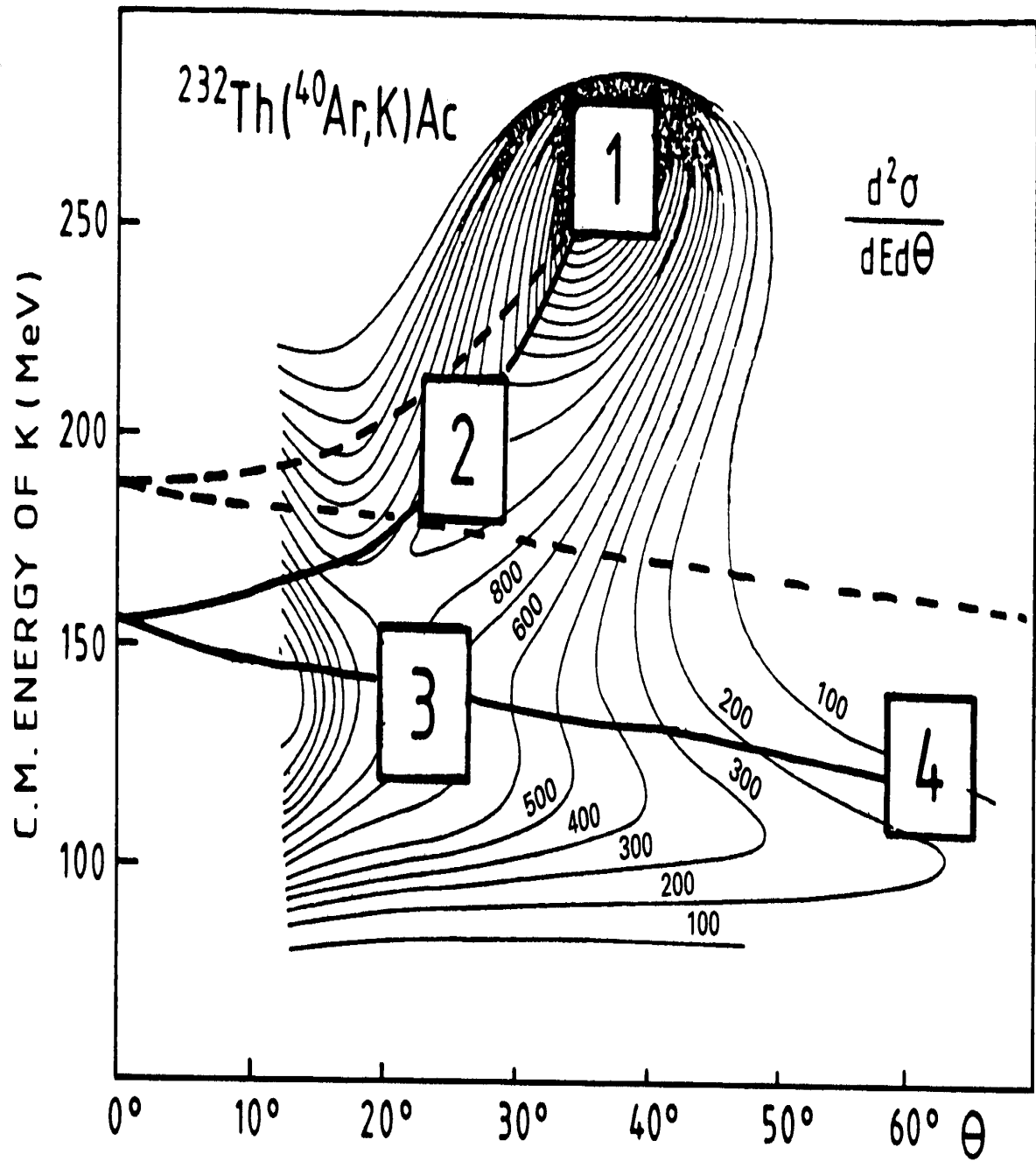


Fig. 1. Effective potentials and trajectories for different angular momenta L . Trajectory L_1 contributes to fusion cross section, trajectory L_2 contributes to deep inelastic cross section (schematic).



De Broglie wave length is $\lambda = \frac{h}{mv} = 2\pi \sqrt{\frac{\hbar^2}{2\mu E}} = 2\pi \sqrt{\frac{\hbar^2}{2[M_N A_1 A_2 / (A_1 + A_2)] E}} \ll R_{nucleus}$.

Example: $^{40}\text{Ar} + ^{232}\text{Th}$, $E_{lab} = 379 \text{ MeV}$, $E_{cm} = [232 / (232 + 40)] 379 = 323 \text{ MeV}$,

$\lambda = 2 \cdot 3.1415 \cdot \sqrt{20.748 / (232 \cdot 40 / (232 + 40) \cdot 323)} = 0.27 \text{ fm}$.

Therefore for the DIC: Trajectory of collisions can be obtained by solving newtonian classic equations of motion, which include the friction force proportional to the velocities:

$$\frac{d\mu\dot{R}}{dt} - \mu r \dot{\vartheta}^2 + \frac{dV}{dR} + K_R \dot{R} = 0,$$

$$\frac{d\mu R^2 \dot{\vartheta}}{dt} + K_\vartheta R^2 \dot{\vartheta} = 0.$$

Here $K_R(R)$ is the radial friction coefficient and $K_\vartheta(R)$ is the tangential friction coefficient.

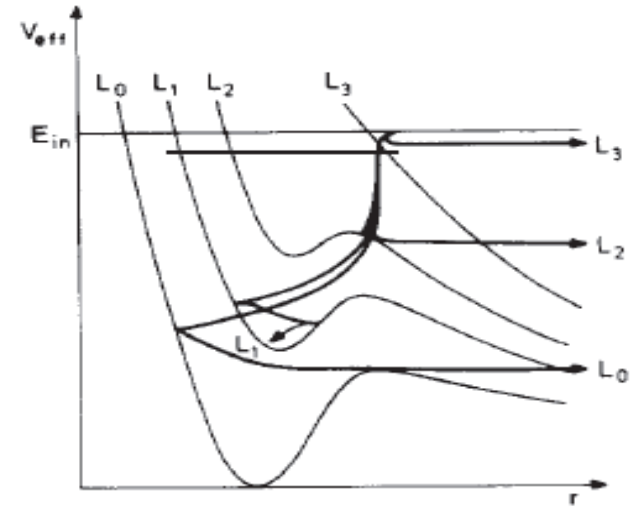
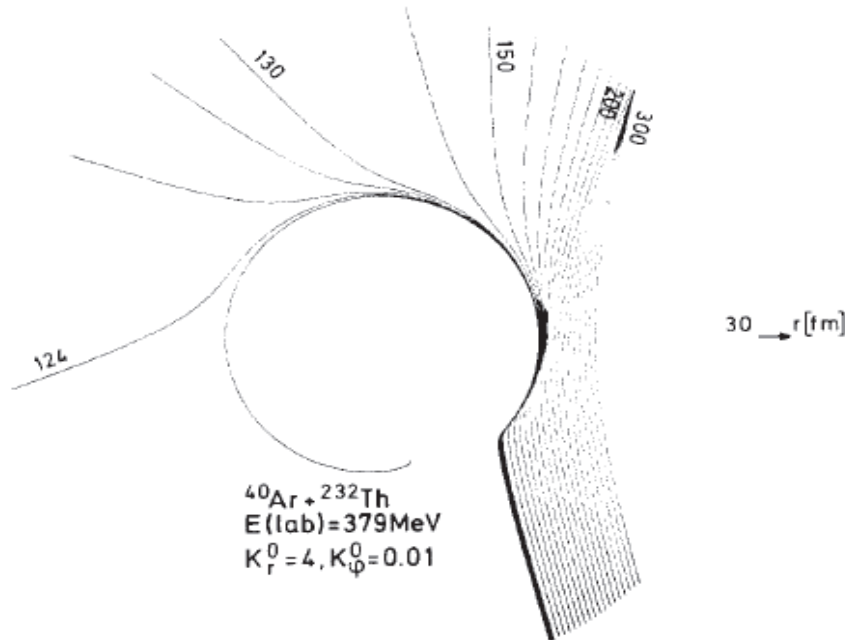
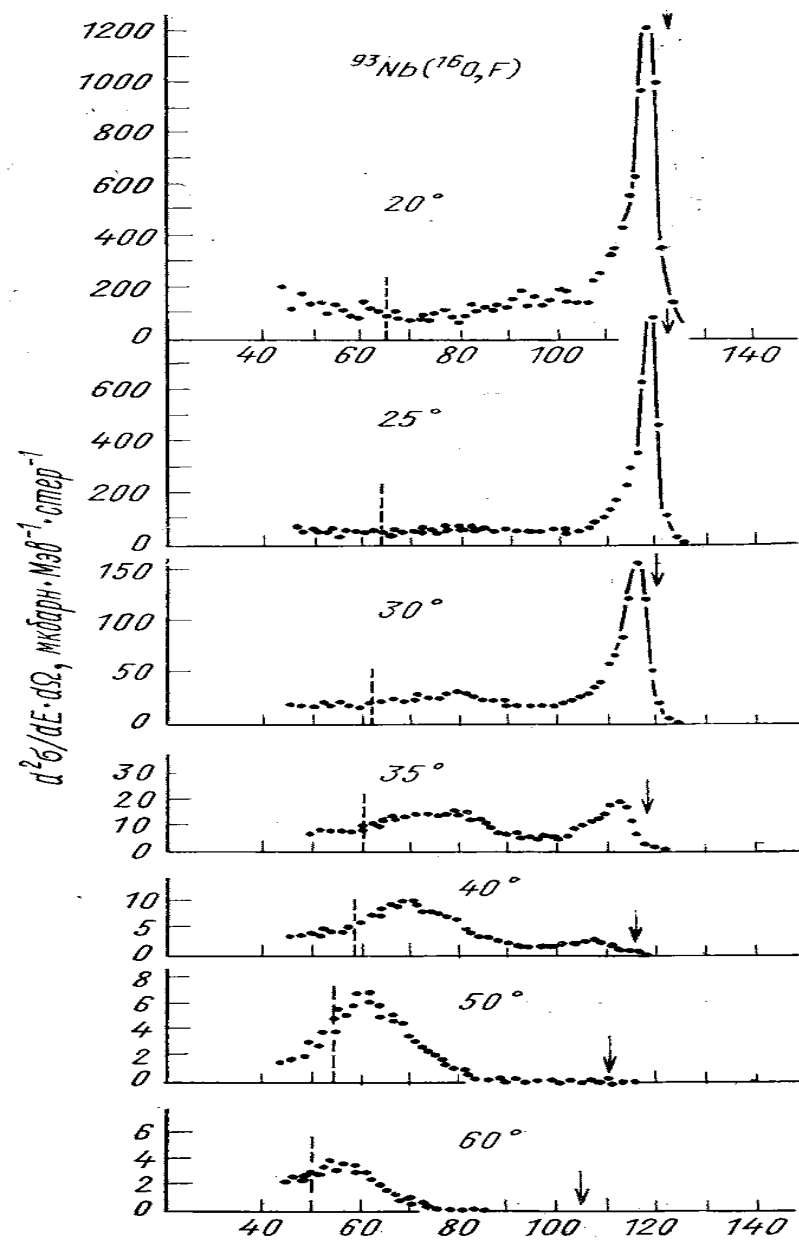
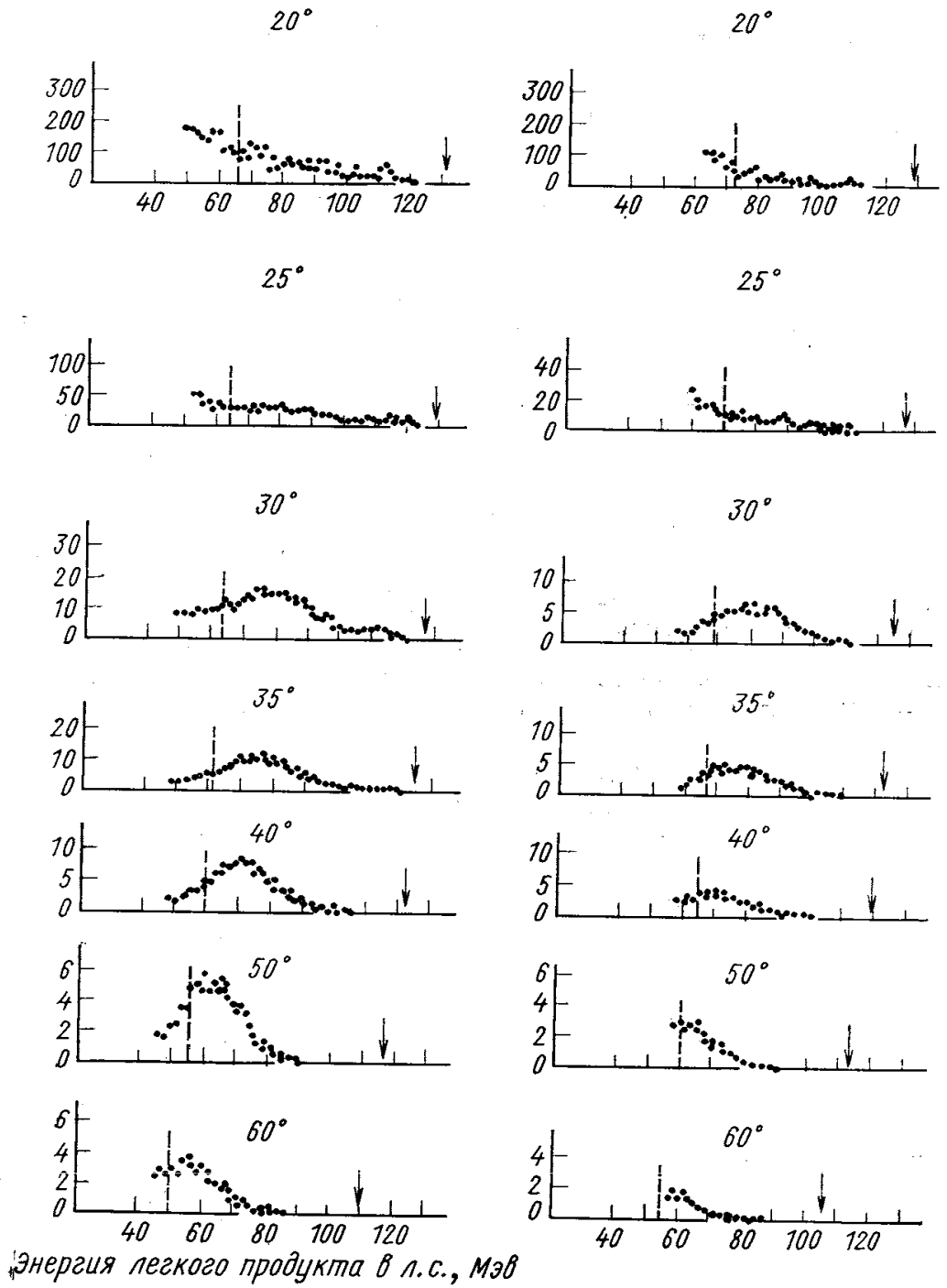


Fig. 1. Effective potentials and trajectories for different angular momenta L . Trajectory L_1 contributes to fusion cross section, trajectory L_2 contributes to deep inelastic cross section (schematic).





Энергия легкого продукта в л.с., Мэв

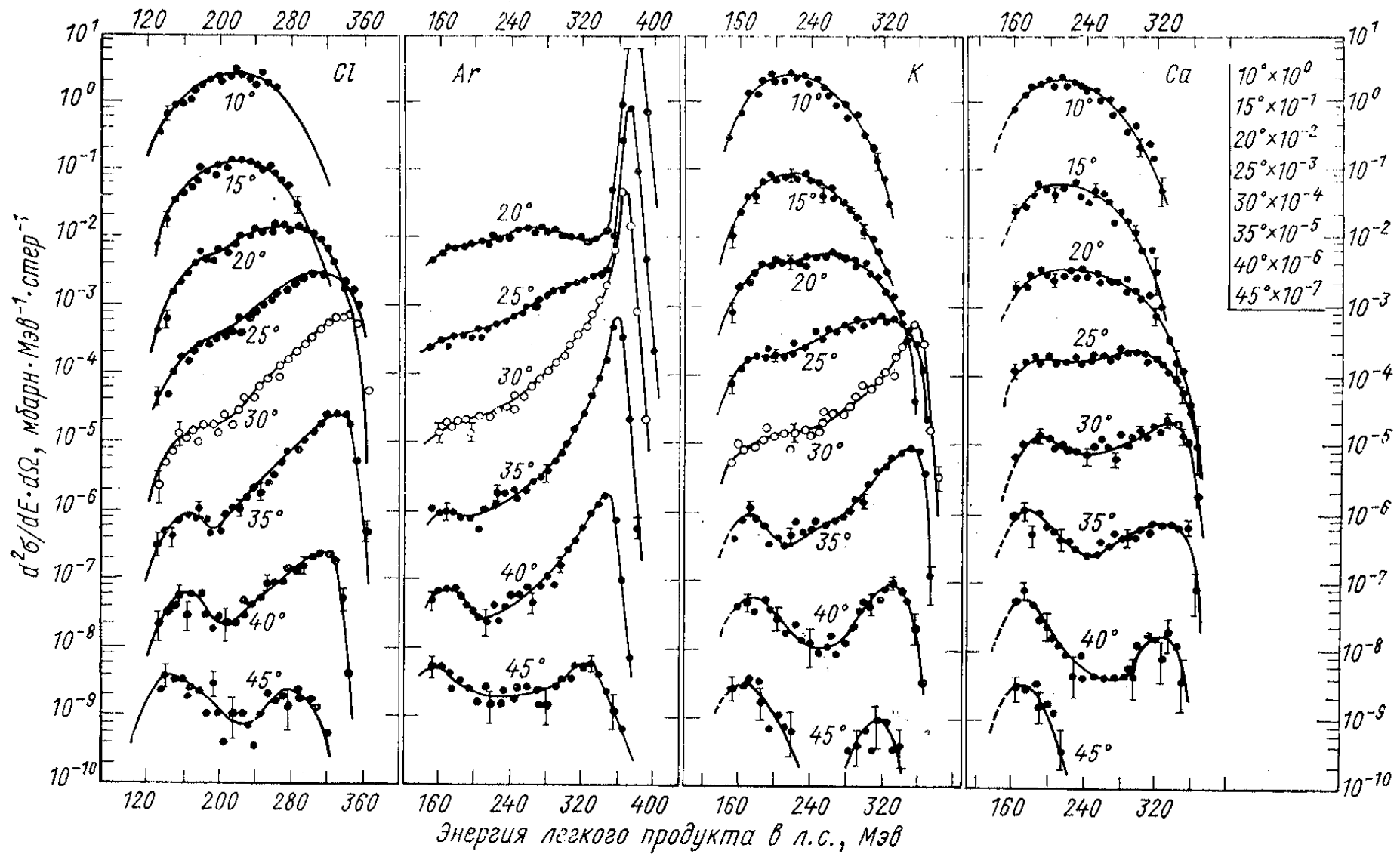
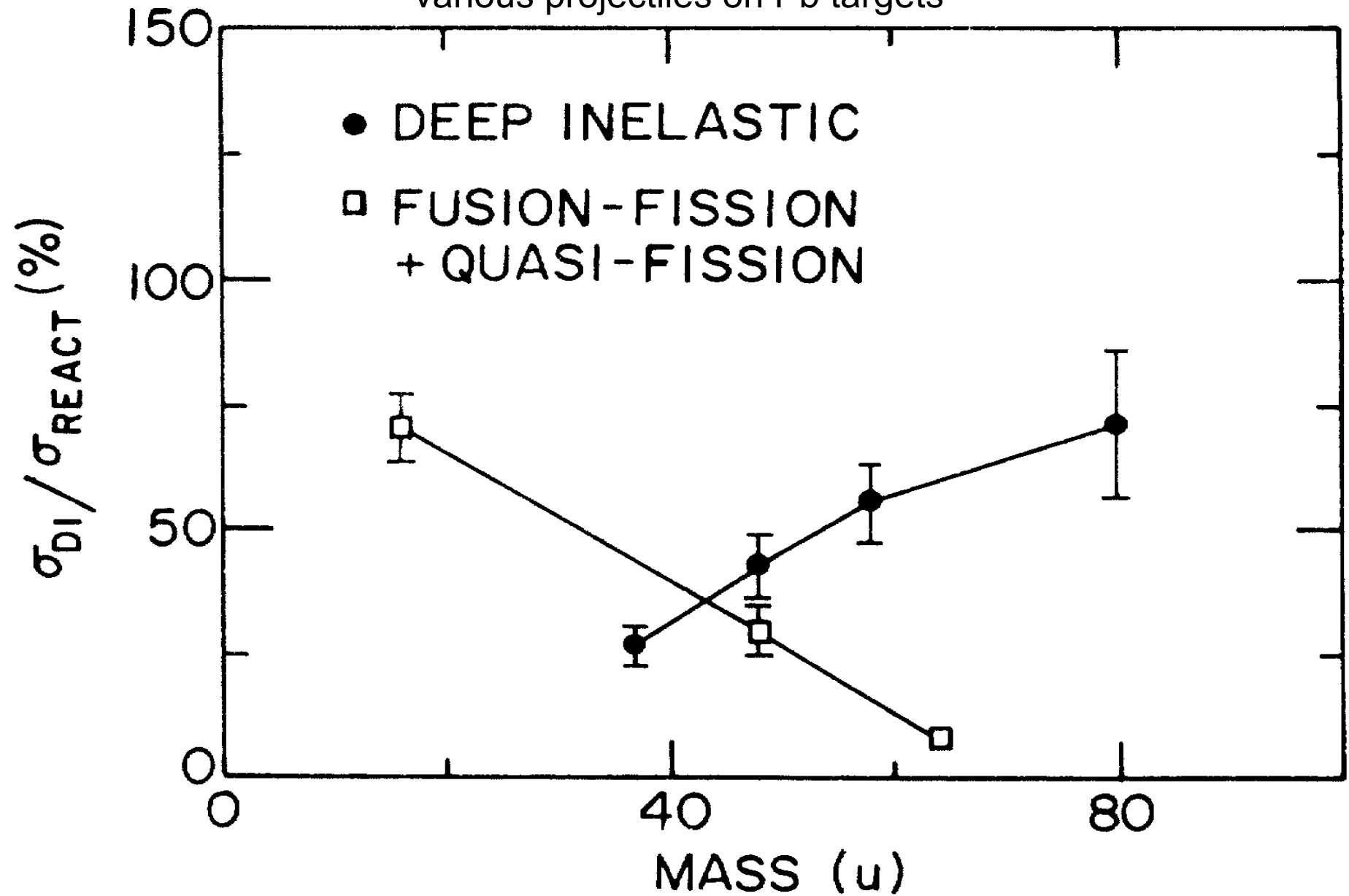


Рис. 5. Энергетические спектры Cl, Ar, K, Ca из облучения ^{232}Th ионами ^{40}Ar с энергией 388 Мэв. Учтены потери энергии в тонкой мишени для ионов ^{40}Ar и продуктов реакции. Данные для каждого угла умножены на коэффициенты, указанные справа вверху [30]

Contribution of reactions of various type into total reactions for various projectiles on Pb targets



6. Conclusion

The transfer phenomena are very common for nuclear reactions.

There are

- sub-barrier single-nucleon and few-nucleon transfer;
- quasi-elastic single-nucleon and few-nucleon transfer;
- multi-nucleon transfer at DIC.

The transfer process depends on collision energy and angular momentum.

Thanks for your attention!