Optical model, Elastic scattering

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1. Introduction

2. Optical model: Formal Theory and Basic Equations

Let's consider the Schrödinger equation described scattering of two complex nuclei

$$(E - H)\Psi = 0,$$

where H is the Hamiltonian and Ψ is the total wave-functions.

We can select open $P\Psi$ and closed $Q\Psi$ components of the total wave-function by using the projection operators P and Q, i.e.

$$\Psi = P\Psi + Q\Psi.$$

The wave functions of open and closed channels are orthogonal to each other

$$\langle P\Psi | Q\Psi \rangle = 0.$$

The projection operators must obey the equations

$$P + Q = 1,$$
$$P^\dagger = P, Q^\dagger = Q,$$
$$P^2 = P, Q^2 = Q,$$
$$PQ = QP = 0.$$

By using these projection operators we rewrite the Schrödinger equation in the form

$$(E - H)\Psi \equiv (E - H)(P\Psi + Q\Psi) = 0.$$

Operating from the left side by P and Q operators we get, respectively,

$$(E - PHP)P\Psi = PHQ\Psi = (PHQ)Q\Psi,$$
$$(E - QHQ)Q\Psi = QHP\Psi = (QHP)P\Psi.$$

Here we use early pointed properties of projection operators. Formal solution of the last equation in the operator form is

$$Q\Psi = \frac{1}{(E - QHQ + i\varepsilon)}(QHP)P\Psi,$$

where ε is infinitesimal. Using this solution we obtain a Schrödinger equation for open channels

$$(E - H_{\rm eff})P\Psi = (E - H_{\rm eff})\Psi_P = 0,$$

where

$$H_{\rm eff} = (PHP) + (PHQ)\frac{1}{(E - QHQ + i\varepsilon)}(QHP) \equiv H_{PP} + H_{PQ}\frac{1}{(E - H_{QQ} + i\varepsilon)}H_{QP}.$$

is the effective Hamiltonian of open channels. The second term of describes the influence of coupling between open and closed channels on the open channels.

Lets closed channels can be constricted by the discrete i and continuous states α. The corresponding wave functions is

$$Q\Psi = \sum_i a_i\psi_{Qi} + \sum_\alpha \int d\mathcal{E}\ a(\alpha, \mathcal{E})\psi_{Q\alpha}(\mathcal{E}),$$

where a_i and $a(\alpha, \mathcal{E})$ are the amplitudes,

$$H_{QQ}\psi_{Qi} = E_{Qi}\psi_{Qi},$$
$$H_{QQ}\psi_{Q\alpha}(\mathcal{E}) = \mathcal{E}\psi_{Q\alpha}(\mathcal{E}),$$

are the corresponding Schrödinger equations,

$$\begin{aligned} \langle\psi_{Qi}|\psi_{Qi'}\rangle &= \delta_{ii'}, \\ \langle\psi_{Q\alpha}(\mathcal{E})|\psi_{Q\alpha'}(\mathcal{E}')\rangle &= \delta_{\alpha\alpha'}\,\delta(\mathcal{E}-\mathcal{E}'), \\ \langle\psi_{Qi}|\psi_{Q\alpha}(\mathcal{E})\rangle &= 0. \end{aligned}$$

Then the effective Hamiltonian for Schrödinger equation of open channels is

$$\begin{aligned} H_{\text{eff}} &= H_{PP} + H_{PQ}\frac{1}{(E - H_{QQ} + i\varepsilon)}H_{QP} \\ &= H_{PP} + \sum_i \frac{H_{PQ}\psi_{Qi}\rangle\langle\psi_{Qi}H_{QP}}{(E - E_{Qi} + i\varepsilon)} + \sum_\alpha \int d\mathcal{E}\frac{H_{PQ}\psi_{Q\alpha}(\mathcal{E})\rangle\langle\psi_{Q\alpha}(\mathcal{E})H_{QP}}{(E - \mathcal{E} + i\varepsilon)}. \end{aligned}$$

Taking into account that

$$\int d\mathcal{E}\frac{f(\mathcal{E})}{(E-\mathcal{E}+i\varepsilon)} = \text{p.v.}\int d\mathcal{E}\frac{f(\mathcal{E})}{(E-\mathcal{E})} - i\pi\int d\mathcal{E}\,\delta(E-\mathcal{E})\,f(\mathcal{E}) = \text{p.v.}\int d\mathcal{E}\frac{f(\mathcal{E})}{(E-\mathcal{E})} - i\pi f(E),$$

we find real and imaginary part of the effective Hamiltonian

$$H_{\text{eff}} = H_{\text{eff}}^{Re} + iH_{\text{eff}}^{Im},$$

where p.v. indicates the principal value of the integral and

$$
\begin{aligned}
H_{\text{eff}}^{Re} &= H_{PP} + \sum_i \frac{H_{PQ}\psi_{Qi}\rangle\langle\psi_{Qi}H_{QP}}{(E - E_{Qi} + i\varepsilon)} + \sum_\alpha \text{p.v.} \int d\mathcal{E} \frac{H_{PQ}\psi_{Q\alpha}(\mathcal{E})\rangle\langle\psi_{Q\alpha}(\mathcal{E})H_{QP}}{(E - \mathcal{E} + i\varepsilon)}, \\
H_{\text{eff}}^{Im} &= -\pi \sum_\alpha H_{PQ}\psi_{Q\alpha}(E)\rangle\langle\psi_{Q\alpha}(E)H_{QP}.
\end{aligned}
$$

The consequences of this expression for effective Hamiltonian:

- The effective Hamiltonian is complex.

- The effective Hamiltonian is nonlocal (or momentum dependence), because of coupling with closed channels Q $H_{\text{eff}}^{Re} f(r)\rangle = H_{PP} f(r)\rangle + \sum_i \frac{H_{PQ}\psi_{Qi}(r)\rangle\langle\psi_{Qi}(r')H_{QP}|f(r')\rangle}{(E-E_{Qi}+i\varepsilon)} + + \sum_\alpha \text{p.v.} \int d\mathcal{E} \frac{H_{PQ}\psi_{Q\alpha}(\mathcal{E},r)\rangle\langle\psi_{Q\alpha}(\mathcal{E},r')H_{QP}|f(r')\rangle}{(E-\mathcal{E}+i\varepsilon)}$. Note that H_{PP} is local.

- The effective Hamiltonian is energy-dependent, i. e. H_{eff}^{Re} is the function of E.

- The effective Hamiltonian has resonant nature, see resonances at $E = E_{Qi}$.

- The effective Hamiltonian depends on the particular model space. By choosing different model spaces for P and Q operators we get different effective potential.

Lets $H = H(R,\{r\})$ is the total hamiltonian of system of complex colliding nuclei and $\Psi(R,\{r\})$ is the total wave-function, which can be expanded as the sum on the eigenstates of each nuclei (open channels, i.e. related to P projection)

$$\Psi(R,r) = \sum_n \psi_n(R)\varphi_n(\{r\}).$$

where $\psi_n(R)$ is the wave-function described the relative motion of two nuclei at channel n with distance between mass centers R, $\varphi_n(\{r\}) = \phi_{1n}(r_1)\phi_{2n}(r_2)$ is the wave function of intrinsic states of corresponding nuclei $\phi_{1n}(r_1)$ and $\phi_{2n}(r_2)$ at channel n, and r_1 and r_2 are intrinsic coordinate.

Note that at $R \to \infty$

$$H\varphi_n(\{r\}) = \varepsilon_n\varphi_n(\{r\}),$$

where ε_n is eigenenergy of nuclei at channel n.

The Schrödinger equation is

$$(H - E)\Psi(R,r) = 0.$$

Multiplying the Schrödinger equation

$$(H - E)\Psi(R, r) = 0.$$

on $\varphi_n^*(\{r\})$ and taking the integral on intrinsic coordinate we get

$$\int d\{r\}\ \varphi_n^*(\{r\})(H(R,\{r\}) - E)\Psi(R, r) = \int d\{r\}\ \varphi_n^*(\{r\})(H(R,\{r\}) - E)\sum_m \psi_m(R)\varphi_m(\{r\})$$
$$= (h_{nn}(R) - E)\psi_n(R) + \sum_{m,\ m\neq n} h_{nm}(R)\psi_m(R) = 0,$$

where

$$h_{nm}(R) = \int d\{r\}\varphi_n^*(\{r\})H(R,\{r\})\varphi_m(\{r\}) = \delta_{nm}\frac{-\hbar^2\Delta_R}{2\mu_n} + V_{nm}^{Nucl}(R) + V_{nm}^{Coul}(R) + iW_{nn}(R)\delta_{nm} + \delta_{nm}\varepsilon_n,$$

$V_{nm}^{Nucl}(R)$ and $V_{nm}^{Coul}(R)$ are nuclear and Coulomb part of nucleus-nucleus matrix elements, μ_n is the reduced mass in channel n, and $W_{nn}(R)$ is imaginary part of potential related to the coupling to the closed channels Q.

We consider spinless colliding nuclei, therefore we present $\psi_n(R) = \sum_{LM} \frac{\xi_{nL}(R)}{R} Y_{LM}(\Omega)$. Multiplying this system of equations on $Y_{LM}(\Omega)$ and taking the integral on angle Ω we have system of coupled-channel equations

$$\left[\frac{-\hbar^2}{2\mu_n}\frac{\partial^2}{\partial R^2} + \frac{\hbar^2 L(L+1)}{2\mu_n R^2} + V_{nn}^{Nucl}(R) + V_{nn}^{Coul}(R) + iW_{nn}(R) + \varepsilon_n - E\right]\xi_{nL}(R)$$
$$= -\sum_{m,m\neq n}\left[V_{nm}^{Nucl}(R) + V_{nm}^{Coul}(R)\right]\xi_{mL}(R).$$

Often, the Woods-Saxon parametrization

$$V_{nn}^{Nucl}(R) = -\frac{V_n^0}{1+\exp[(R-R_0)/d]}, \qquad R_0 = r_0\left(A_{1n}^{1/3} + A_{2n}^{1/3}\right),$$

is adopted for the nuclear potential and Coulomb potential for spherical nuclei is

$$V_{nn}^{Coul}(R) = \begin{cases} \frac{e^2 Z_{1n} Z_{2n}}{R}, & \text{if } R \geq R_{0C}, \\ \frac{e^2 Z_{1n} Z_{2n}}{R_{0C}}\left[\frac{3}{2} - \frac{R^2}{2R_{0C}^2}\right], & \text{if } R < R_{0C}. \end{cases}$$

Note that non-diagonal terms of nuclear and Coulomb matrix elements $V_{nm}^{Nucl}(R)$ and $V_{nm}^{Coul}(R)$ depend on the nuclear model and reaction type.

The imaginary potential usually has volume and surface parts

$$W_{nn}(R) = W_{nn}^{\text{vol}}(R) + W_{nn}^{\text{surf}}(R),$$

where

$$\begin{aligned} W_{nn}^{\text{vol}}(R) &= \tfrac{W_n^0}{1+\exp[(R-R_{0W})/d_W]}, & R_{0W} &= r_{0W}\left(A_{1n}^{1/3} + A_{2n}^{1/3}\right), \\ W_{nn}^{\text{surf}}(R) &= \tfrac{d}{dR}\tfrac{W_n^{0S}}{1+\exp[(R-R_{0WS})/d_{WS}]}, & R_{0WS} &= r_{0WS}\left(A_{1n}^{1/3} + A_{2n}^{1/3}\right). \end{aligned}$$

The solutions of coupled-channel equations

$$\left[\frac{-\hbar^2}{2\mu_n}\frac{\partial^2}{\partial R^2}+\frac{\hbar^2 L(L+1)}{2\mu_n R^2}+V_{nn}^{Nucl}(R)+V_{nn}^{Coul}(R)+iW_{nn}(R)+\varepsilon_n-E\right]\xi_{nL}(R)=$$
$$-\sum_{m,m\neq n}\left[V_{nm}^{Nucl}(R)+V_{nm}^{Coul}(R)\right]\xi_{mL}(R).$$

are matched the following boundary conditions

$$\begin{aligned}\xi_{nL}(R)|_{R=0} &= 0,\\ \xi_{nL}(R)|_{R\to\infty} &= \frac{i}{2}\left[\delta_{nn_0}H_L^-(\eta_{n_0},k_{n_0}R)-\left(\frac{v_{n_0}}{v_n}\right)^{1/2}S_{nL}H_L^+(\eta_n,k_nR)\right].\end{aligned}$$

Here n_0 is the incident channel (note that $\varepsilon_{n_0}=0$), $k_n=\sqrt{2\mu_n(E-\varepsilon_n)/\hbar^2}$, $\eta=\mu_n Z_{1n}Z_{2n}e^2/(\hbar^2 k_n)$, S_{nL} is the S-matrix, and $H_L^\pm(\eta,k_nR)$ are the Coulomb functions $H_L^\pm(\eta,x)=G_L(\eta,x)\pm iF_L(\eta,x)$ with asymptotics

$$\begin{aligned}F_L(\eta,x)|_{x\to 0} &= 0,\\ G_L(\eta,x)|_{x\to 0}=H_L^\pm(\eta,x)|_{x\to 0} &= +\infty,\\ H_L^\pm(\eta,x)|_{x\to\infty} &= \exp\left[\pm i\left(x-\eta\ln(2x)-\frac{\pi L}{2}+\sigma_L\right)\right],\\ F_L(\eta,x)|_{x\to\infty} &= \sin\left[i\left(x-\eta\ln(2x)-\frac{\pi L}{2}+\sigma_L\right)\right],\\ G_L(\eta,x)|_{x\to\infty} &= \cos\left[i\left(x-\eta\ln(2x)-\frac{\pi L}{2}+\sigma_L\right)\right],\\ \exp(i\sigma_L) &= \left[\frac{\Gamma(1+L+i\eta)}{\Gamma(1+L-i\eta)}\right]^{1/2},\sigma_L=\arg\Gamma(1+L+i\eta)\end{aligned}$$

3. Optical model: Elastic scattering, Numerical solutions

Lets consider for simplicity one channel and omit index n for simplicity. The system of coupled-channel equations are reduced to single Schrodinger equation

$$\left[\frac{-\hbar^2}{2\mu}\frac{\partial^2}{\partial R^2} + \frac{\hbar^2 L(L+1)}{2\mu R^2} + V^{Nucl}(R) + V^{Coul}(R) + iW(R) - E\right]\xi_L(R) = 0.$$

The boundary condition

$$\begin{aligned} \xi_L(R)|_{R=0} &= 0, \\ \xi_L(R)|_{R\to\infty} &= \frac{i}{2}\left[H_L^-(\eta, kR) - S_L H_L^+(\eta, kR)\right] = F_L(\eta, kR) + C_L[G_L(\eta, kR) + iF_L(\eta, kR)], \end{aligned}$$

where $C_L = \frac{-i}{2}(S_L - 1)$.

The total scattering amplitude is given as the sum of Coulomb scattering amplitude $f_C(\vartheta)$ related to long-range Coulomb interaction and contribution to the amplitude induced by short-range interaction related to nuclear forced between colliding nuclei

$$F(\vartheta) = f_C(\vartheta) + \frac{1}{k}\sum_{L=0}^{\infty} \exp{(2i\sigma_L)}(2L+1)C_L P_L(\cos\vartheta),$$

where

$$f_C(\vartheta) = -\eta\frac{\exp{[-i\eta\ln{[\sin^2(\vartheta/2)]} + 2i\sigma_0]}}{2k\sin^2(\vartheta/2)]}$$

is exact Coulomb scattering amplitude.

The differential scattering cross section is

$$\frac{d\sigma(\vartheta)}{d\vartheta} = |F(\vartheta)|^2.$$

The Rutherford differential cross section (when neglecting both nuclear and imaginary potentials) is

$$\frac{d\sigma_{\rm R}(\vartheta)}{d\vartheta} = \left(\frac{Z_1 Z_2 e^2}{4E}\right)^2 \frac{1}{\sin^4 \vartheta/2}.$$

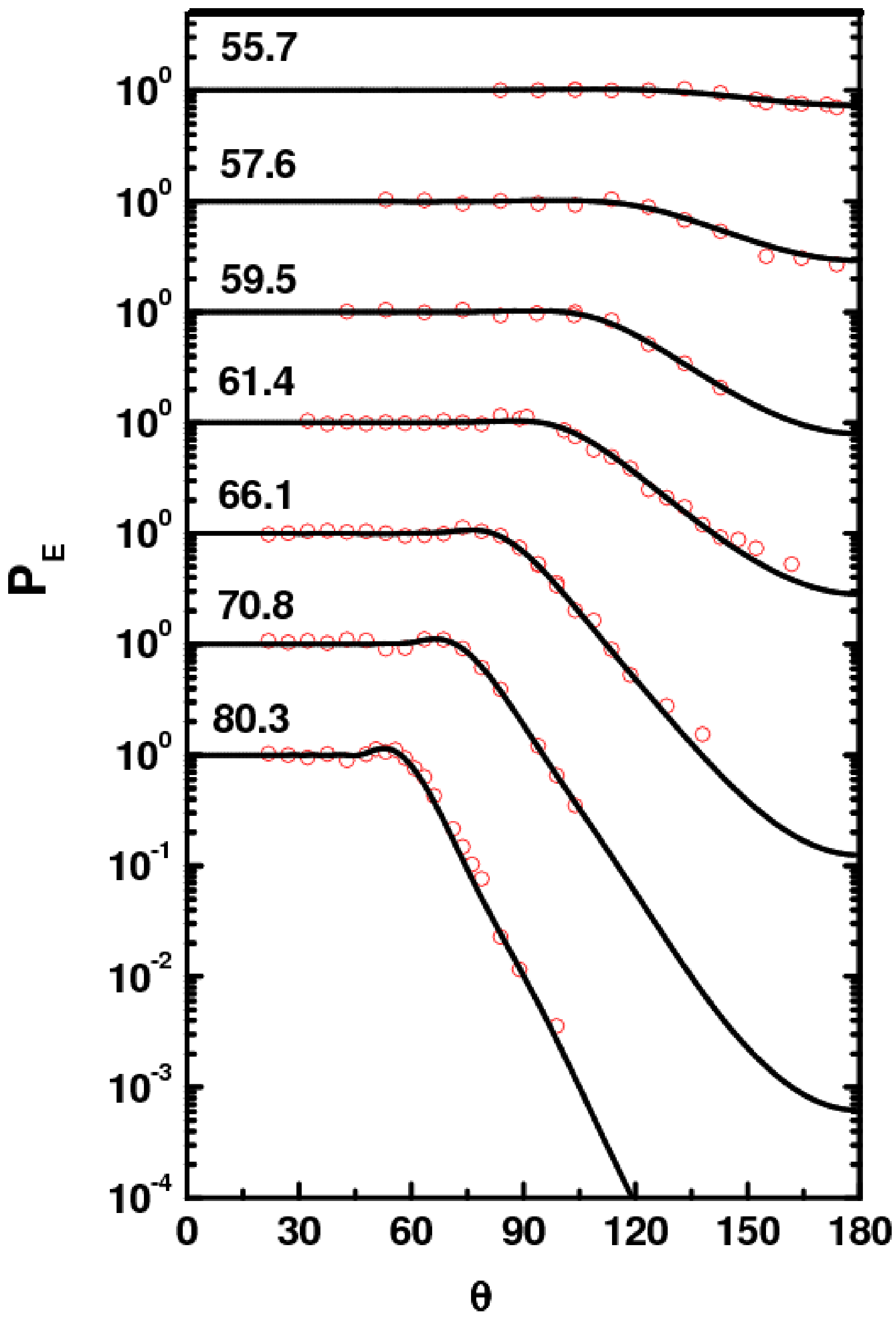
Note that

- $\sin^4 \vartheta/2|_{\vartheta=0^\circ} = 0$, therefore $\lim_{\vartheta=0^\circ} \frac{d\sigma_{\rm R}(\vartheta=0)}{d\vartheta} = \infty$;
- the cross section does not depend on signum of the charges;
- the angular distribution cross section does not depend on the energy;
- $\frac{d\sigma_{\rm R}(\vartheta)}{d\vartheta} \propto \frac{1}{E^2}$.

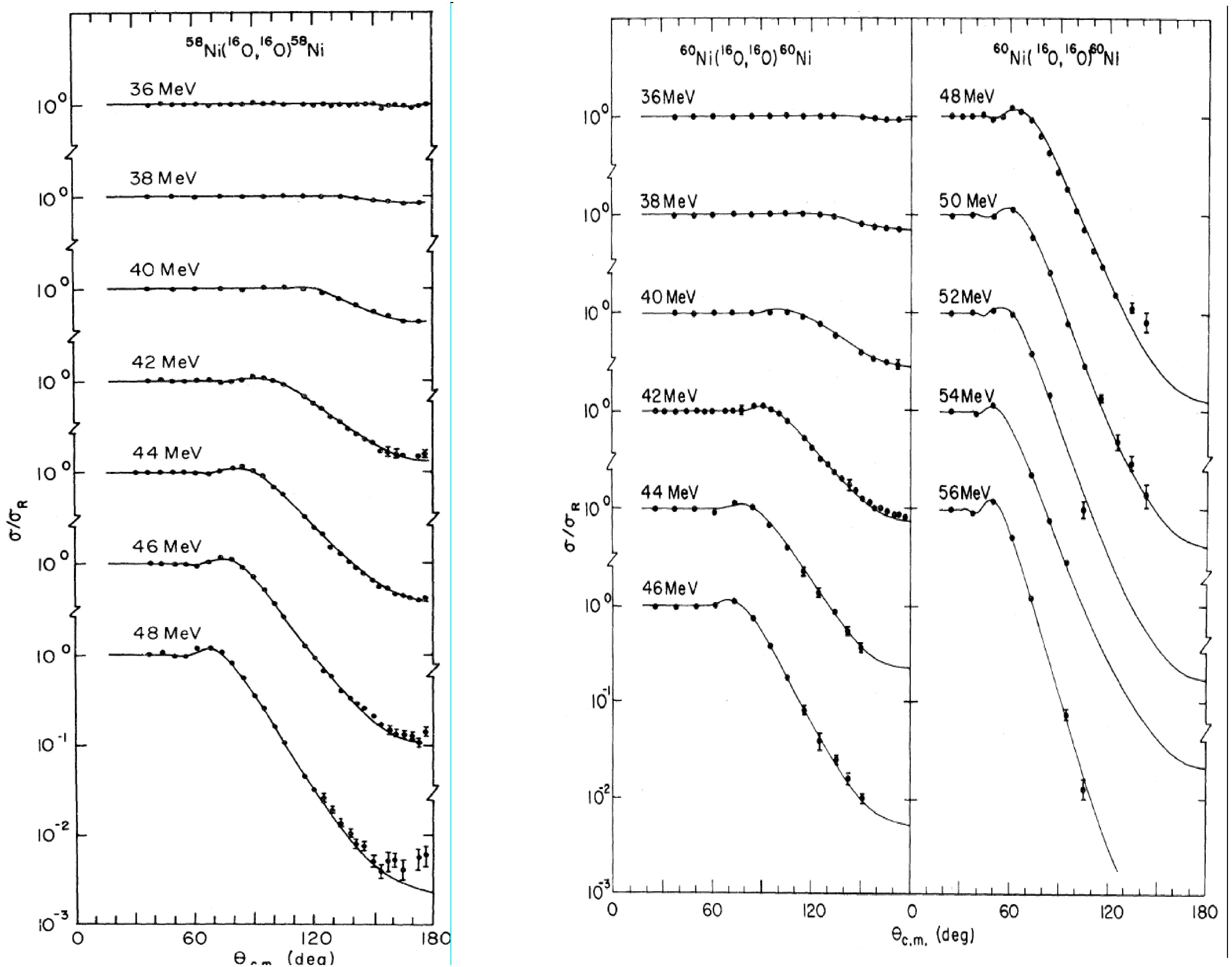
Therefore, often the results of elastic scattering is presented as

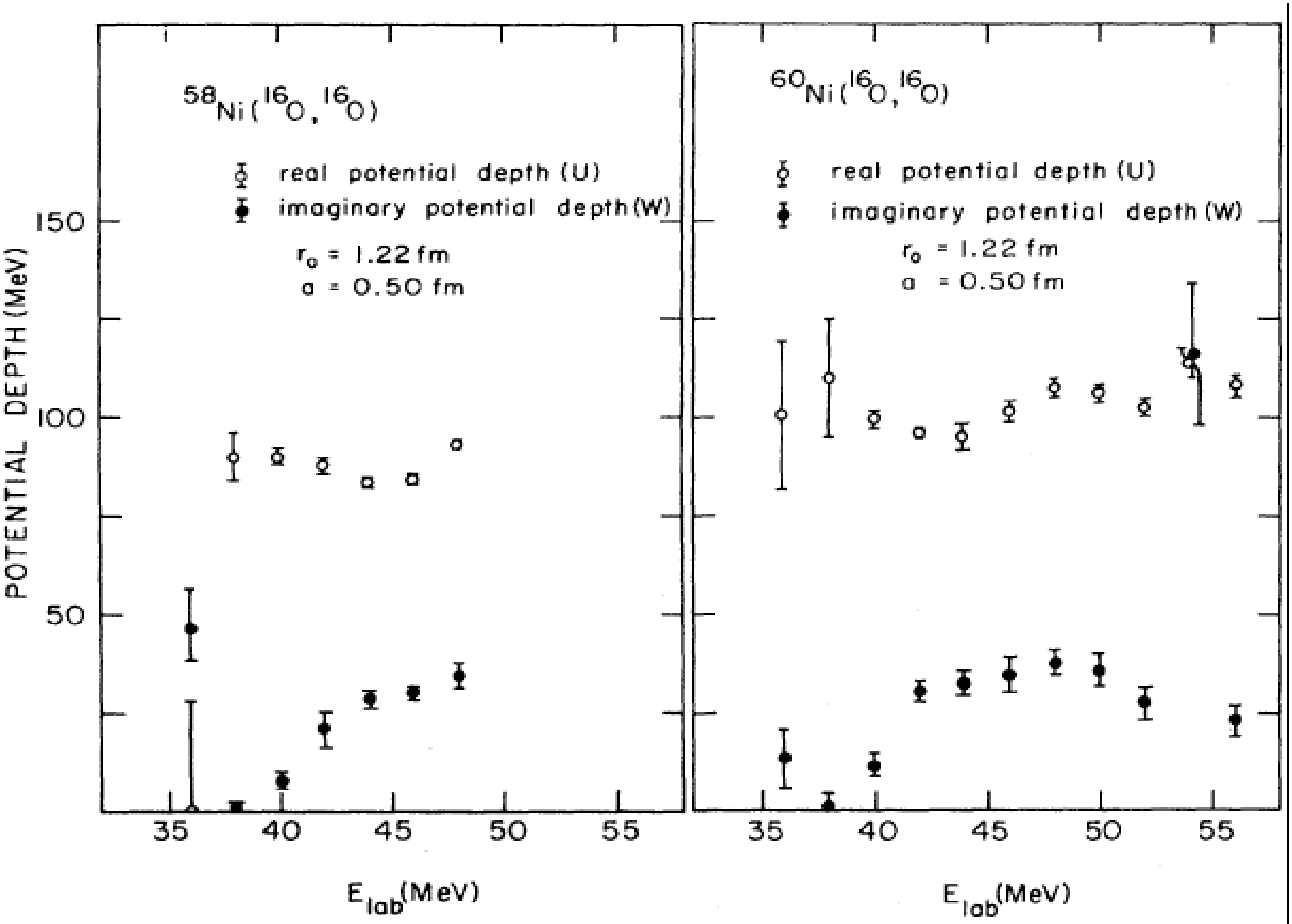
$$\frac{\frac{d\sigma(\vartheta)}{d\vartheta}}{\frac{d\sigma_{\rm R}(\vartheta)}{d\vartheta}} = \frac{|F(\vartheta)|^2}{\left(\frac{Z_1 Z_2 e^2}{4E}\right)^2 \frac{1}{\sin^4 \vartheta/2}}.$$

Elastic scattering: $^{12}C+^{208}Pb$: W. Y. So, et al., Phys. Rev. C 77, 024609 (2008)



^{58}Ni+^{16}O, ^{60}Ni+^{16}O: L. West, K.W. Kemper, N.R. Fletcher, Phys. Rev. C 11, 859 (1975)





$^{58}Ni(^{16}O,^{16}O)$
real potential depth (U)
imaginary potential depth (W)
r_o = 1.22 fm
a = 0.50 fm
$^{60}Ni(^{16}O,^{16}O)$
real potential depth (U)
imaginary potential depth (W)
r_o = 1.22 fm
a = 0.50 fm
POTENTIAL DEPTH (MeV)
150
100
50
35
40
45
50
55
E_{lab}(MeV)

The radial Schr"odinger equation can be written in the form

$$\frac{d^2}{dR^2}\xi(r) = A(R)\xi(R).$$

We introduce the auxiliary function

$$\zeta(R) = \xi(R) - \frac{h^2}{12}A(R)\xi(R),$$

where h is step of finite difference algorithm. For function ζ there is Noumerov algorithm based on the finite difference formula for three consecutive points on a mesh with step h,

$$\zeta_{i+1}(R+h) = \left[2 + \frac{h^2 A_i(R)}{1-(h^2/12)A_i(R)}\right]\zeta_i(R) - \zeta_{i-1}(R-h).$$

The boundary conditions at $R = 0$ is $\xi(R) = 0$.

Due to this starting point R_i, which is close to $R = 0$ is

$$\begin{aligned} \zeta_0(R_i) &= 0, & \zeta_1(R_i+h) &= c, \text{ for } L \neq 1., \\ \zeta_0(R_i) &= -0.2c, & \zeta_1(R_i+h) &= c, \text{ for } L = 1. \end{aligned}$$

Asymptotic at $R = R_m$ for scattering $E > 0$ is

$$\xi(R_m) = F_L(\eta, kR_m) + C_L[G_L(\eta, kR_m) + iF_L(\eta, kR_m)].$$

The short-range potential at the matching point R_m is negligible.

Using this boundary condition we determine the C_L.

Substituting the value of C_L into

$$
\begin{aligned}
F(\vartheta) &= f_C(\vartheta) + \frac{1}{k}\sum_{L=0}^{\infty} \exp(2i\sigma_L)(2L+1)C_L P_L(\cos\vartheta) \\
&= -\eta\frac{\exp\left[-i\eta\ln\left[\sin^2(\vartheta/2)\right] + 2i\sigma_0\right]}{2k\sin^2(\vartheta/2)]} + \frac{1}{k}\sum_{L=0}^{\infty} \exp(2i\sigma_L)(2L+1)C_L P_L(\cos\vartheta), \\
\sigma(\vartheta) &= |F(\vartheta)|^2.
\end{aligned}
$$

we find the total scattering amplitude and cross section.

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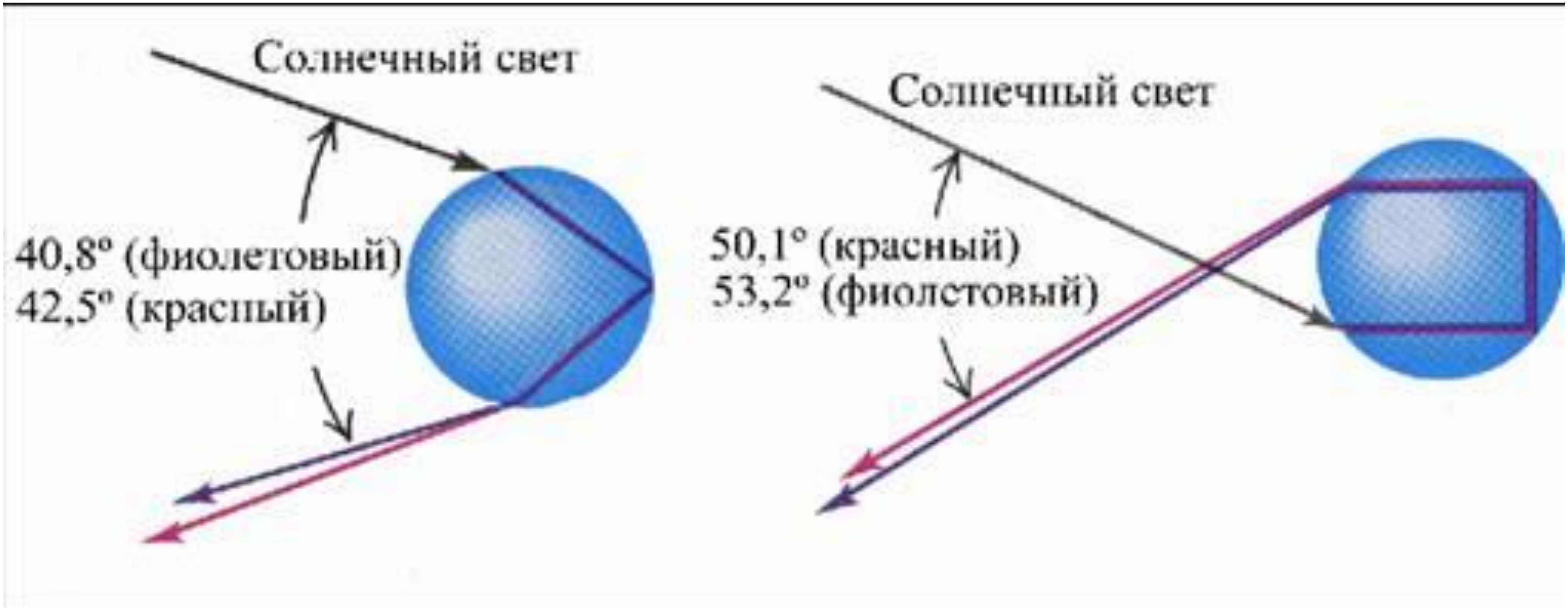
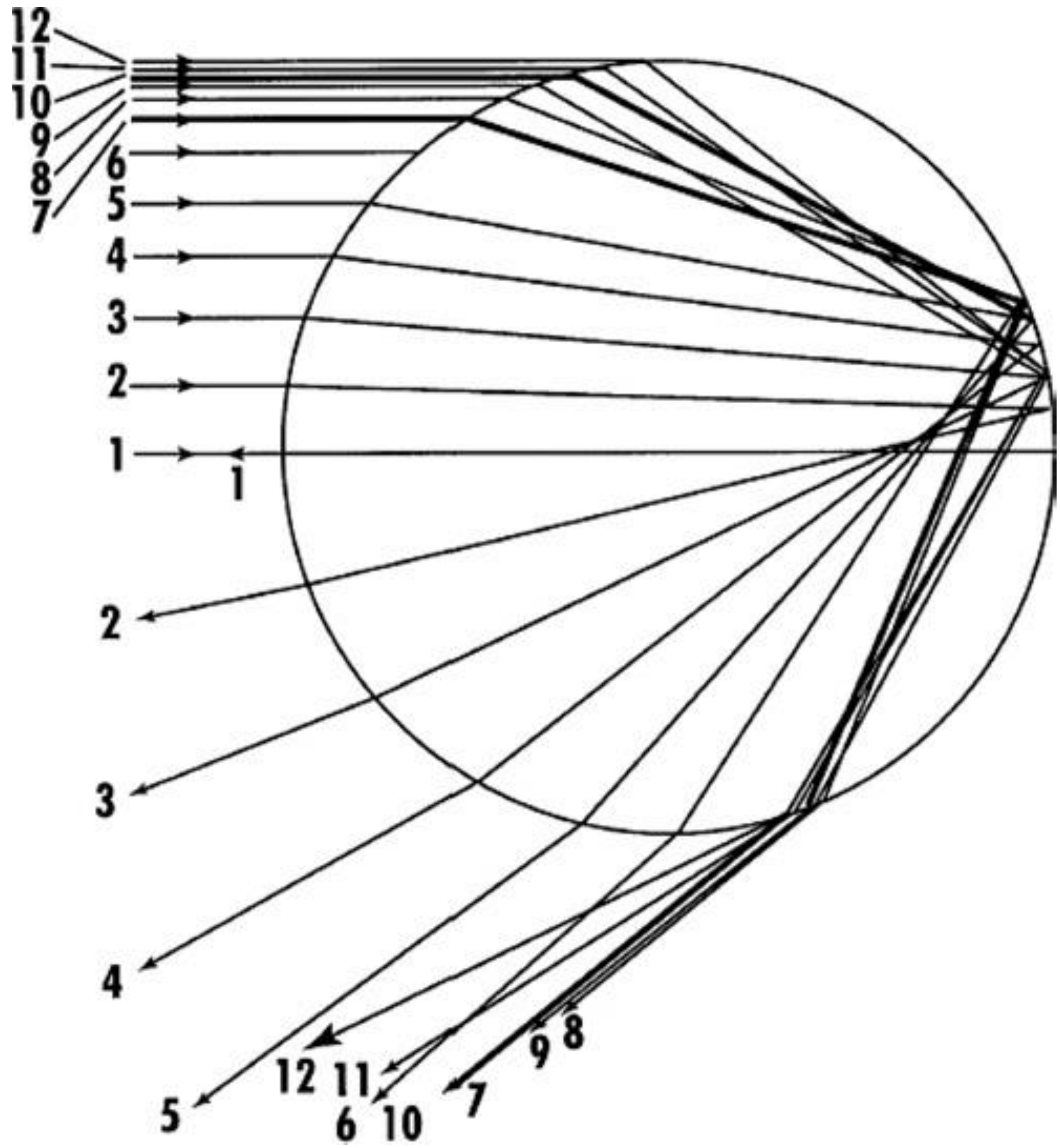
Rainbow



Солнечный свет
40,8° (фиолетовый)
42,5° (красный)
Солнечный свет
50,1° (красный)
53,2° (фиолстовый)


12
11
10
9
8
7
6
5
4
3
2
1
1
2
3
4
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12
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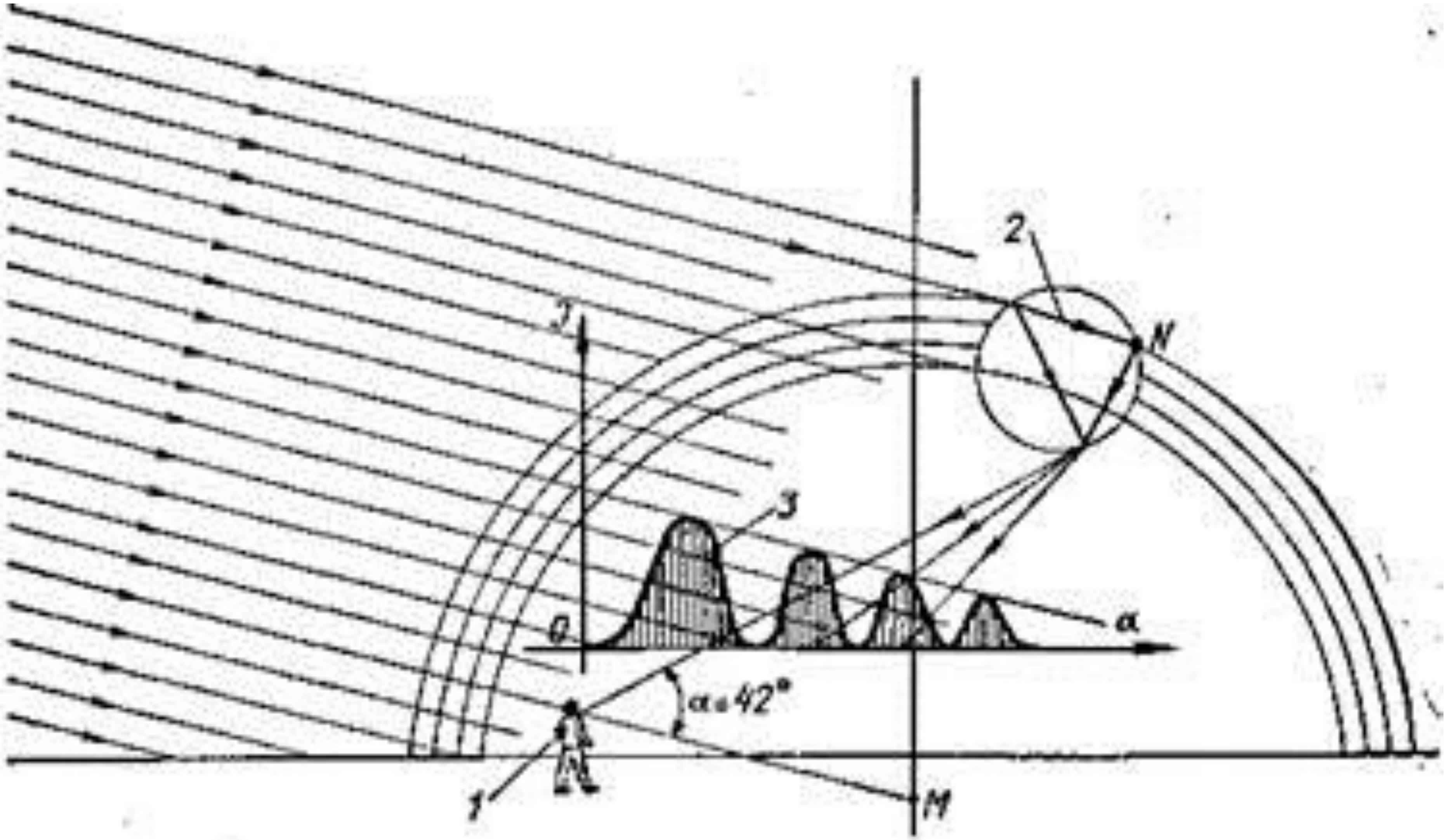
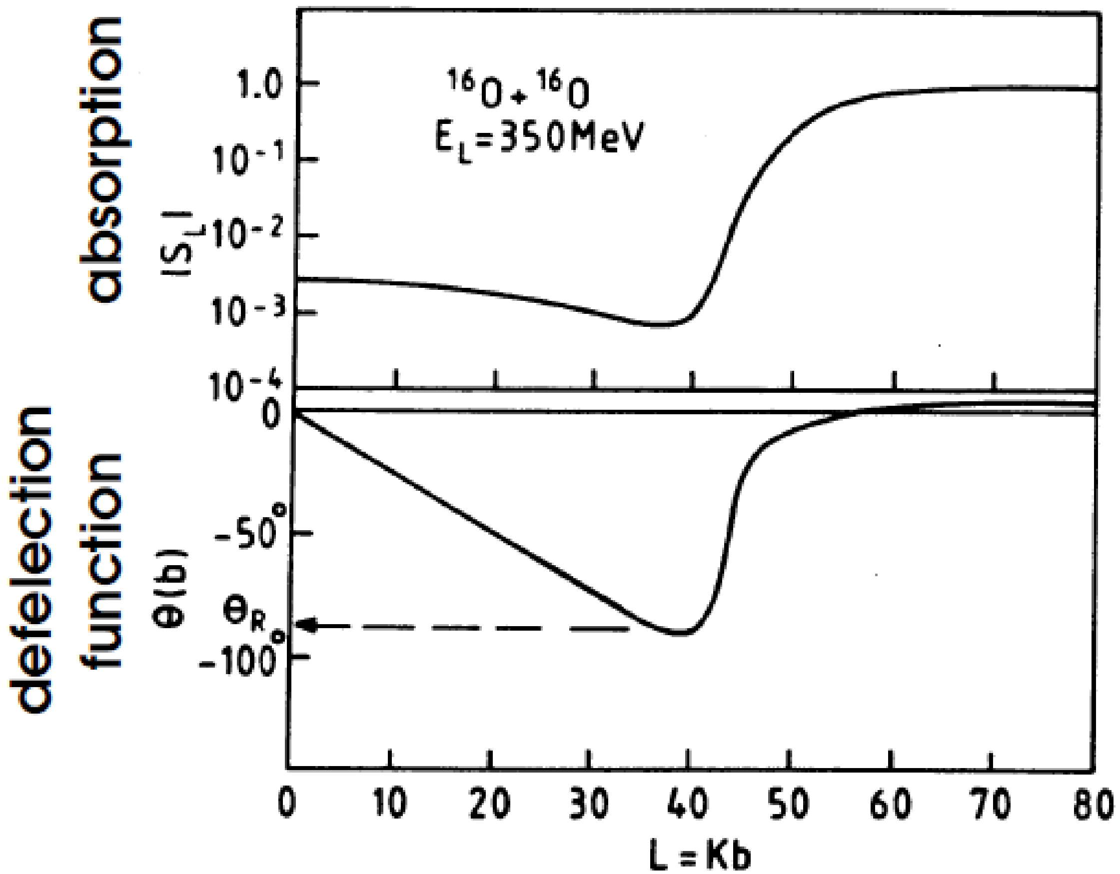
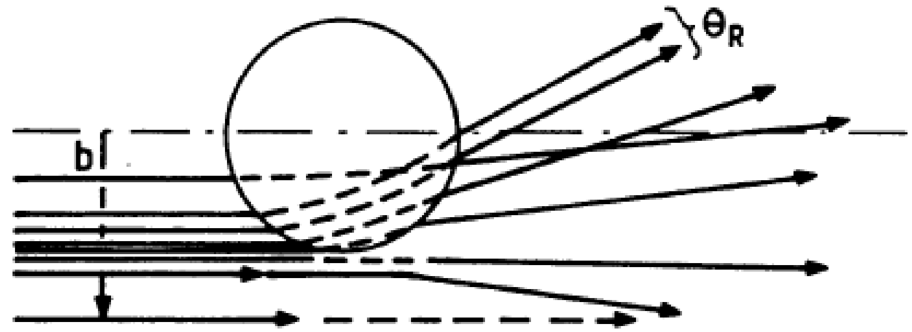


Рис. 6. Так образуется радуга (схема): *1* — наблюдатель, *2* — ход и преломление лучей в капле, *3* — график изменения яркости света *I* по углам α при выходе лучей из капли




Nuclear rainbow in elastic scattering





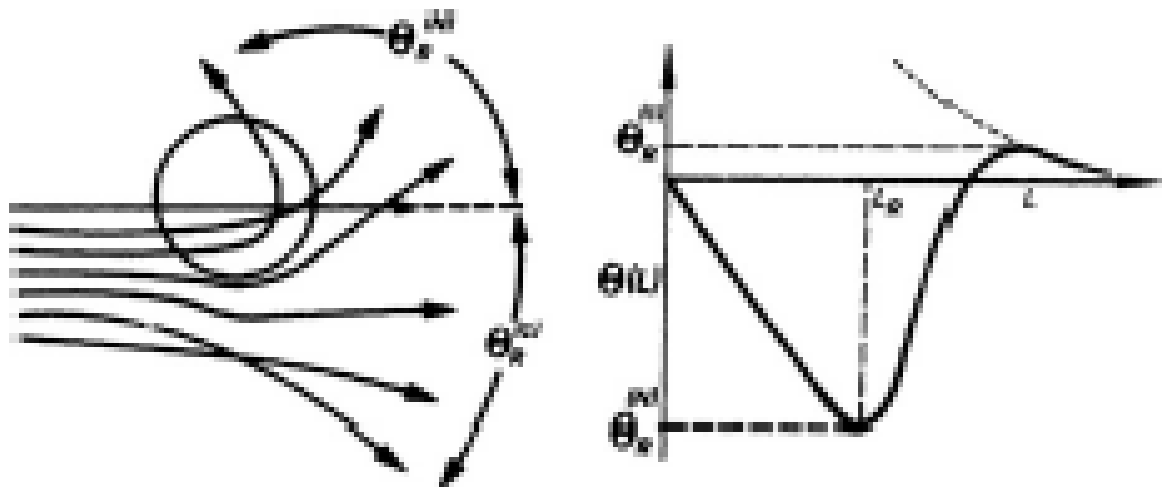


Figure 9. Left: Classical trajectories of the nuclear wave scattered elastically by a short-range attractive nuclear potential and a long-range repulsive Coulomb potential which lead to the nuclear (N) and Coulomb (C) rainbows, respectively. Right: The corresponding deflection function. Illustration taken from [25].

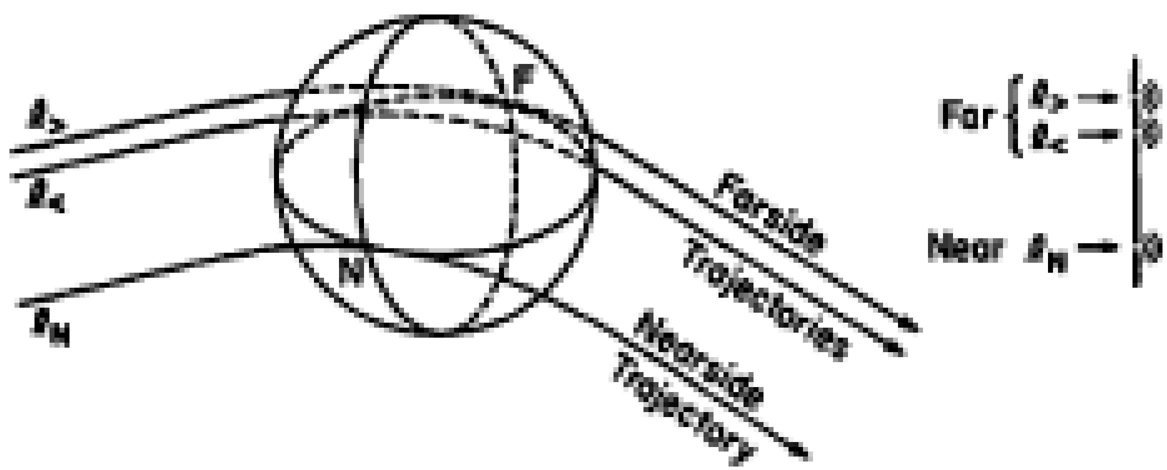
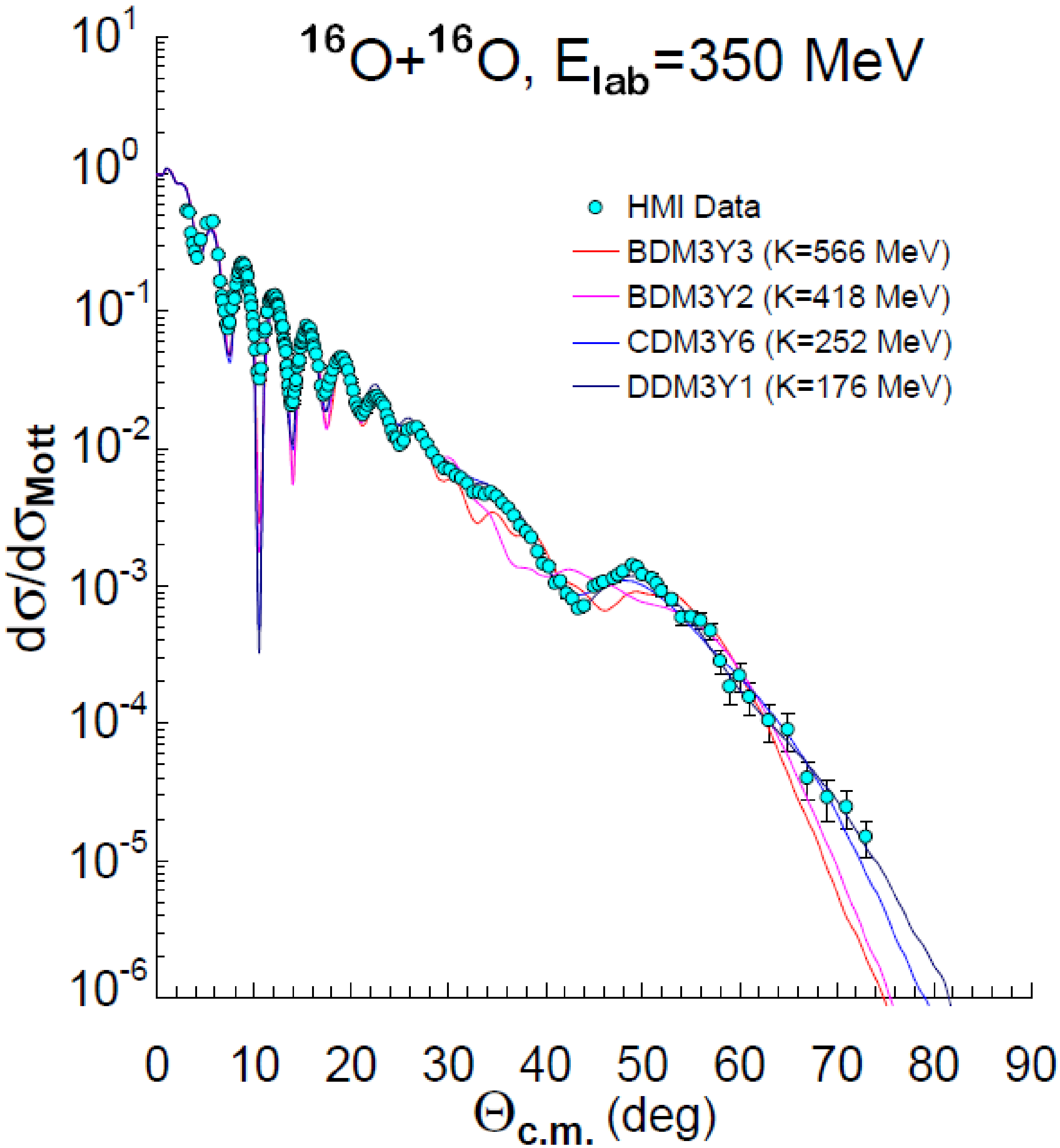


Figure 10. Schematic representation of three trajectories being deflected to the same scattering angle. The right-hand part shows the analogy to a three-slit interference pattern. Illustration taken from [31].



$^{16}O+^{16}O$, E_{lab}=350 MeV
HMI Data
BDM3Y3 (K=566 MeV)
BDM3Y2 (K=418 MeV)
CDM3Y6 (K=252 MeV)
DDM3Y1 (K=176 MeV)
$d\sigma/d\sigma_{Mott}$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0 10 20 30 40 50 60 70 80 90
$\Theta_{c.m.}$ (deg)

Rainbow Θ=55°

The nucleus-nucleus potential $V(R) = V_N(R) + V_{COUL}(R) + V_l(R)$

The energy density approximation

The interaction energy between nuclei at the center-to-center distance R

$$V_N(R) = E_{12}(R) - E_1 - E_2.$$

Here

$$E_{12}(R) = \int \varepsilon\left[\rho_{1p}(\vec{r}) + \rho_{2p}(\vec{r}, R),\ \ \rho_{1n}(\vec{r}) + \rho_{2n}(\vec{r}, R)\right] d\vec{r}$$

is the energy of the interacting nuclei at finite distance R,

$$E_{1(2)} = \int \varepsilon\left[\rho_{1(2)p}(\vec{r}),\ \ \rho_{1(2)n}(\vec{r})\right] d\vec{r}$$

is the energies of the non-interacting nuclei,

$$\varepsilon\left[\rho_p(\vec{r}), \rho_n(\vec{r})\right] = \tau\left[\rho_p(\vec{r})\right] + \tau\left[\rho_n(\vec{r})\right] + v_{Sk}\left[\rho_p(\vec{r}), \rho_n(\vec{r})\right]$$

is the energy density functional, $\rho_{1(2)n}(\vec{r})$ and $\rho_{1(2)p}(\vec{r})$ – neutron and proton density of 1(2) nucleus.

Potential evaluated in Energy Density Functional has two contributions:

- intrinsic kinetic energy of nucleons $V_T(R)$,

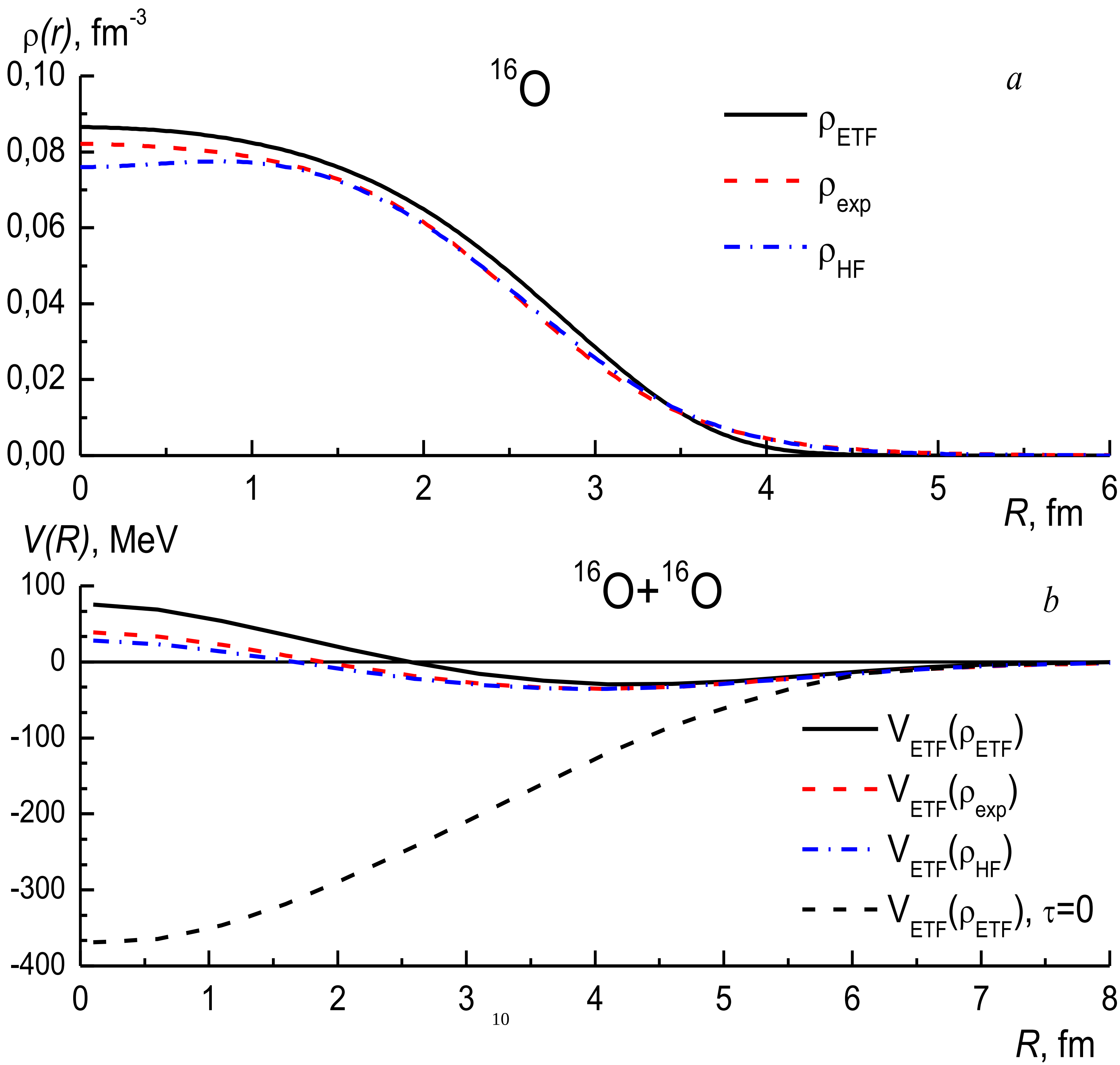
- nucleon-nucleon interaction $V_{nn}(R)$

$$V_N(R) = V_T(R) + V_{nn}(R),$$

where

$$V_T(R) = \int \tau\left[\rho_{1p}(\vec{r}) + \rho_{2p}(\vec{r},R), \rho_{1n}(\vec{r}) + \rho_{2n}(\vec{r},R)\right]d\vec{r} - \\ - \int \tau\left[\rho_{1p}(\vec{r}), \rho_{1n}(\vec{r})\right]d\vec{r} - \int \tau\left[\rho_{2p}(\vec{r}), \rho_{2n}(\vec{r})\right]d\vec{r},$$

$$V_{nn}(R) = \int v_{Sk}\left[\rho_{1p}(\vec{r}) + \rho_{2p}(\vec{r},R), \rho_{1n}(\vec{r}) + \rho_{2n}(\vec{r},R)\right]d\vec{r} - \\ - \int v_{Sk}\left[\rho_{1p}(\vec{r}), \rho_{1n}(\vec{r})\right]d\vec{r} - \int v_{Sk}\left[\rho_{2p}(\vec{r}), \rho_{2n}(\vec{r})\right]d\vec{r}$$



ρ(r), fm^-3
0,10
0,08
0,06
0,04
0,02
0,00
16O
a
ρETF
ρexp
ρHF
0
1
2
3
4
5
6
R, fm
V(R), MeV
100
0
-100
-200
-300
-400
16O+16O
b
VETF(ρETF)
VETF(ρexp)
VETF(ρHF)
VETF(ρETF), τ=0
0
1
2
3
4
5
6
7
8
R, fm

Standard double-folding approximation

The nucleus-nucleus potential in double-folding approximation

$$V_{DF}(R) = \int d\vec{r}_1 d\vec{r}_2 \rho_1(\vec{r}_1) V_{nn}(\vec{r}_1, \vec{r}_2, E) \rho_2(R, \vec{r}_2),$$

$V_{nn}(\vec{r}_1, \vec{r}_2, E)$ – the nucleon-nucleon interaction.

The modified double-folding approximation (taken into account the intrinsic kinetic energy of nucleons)

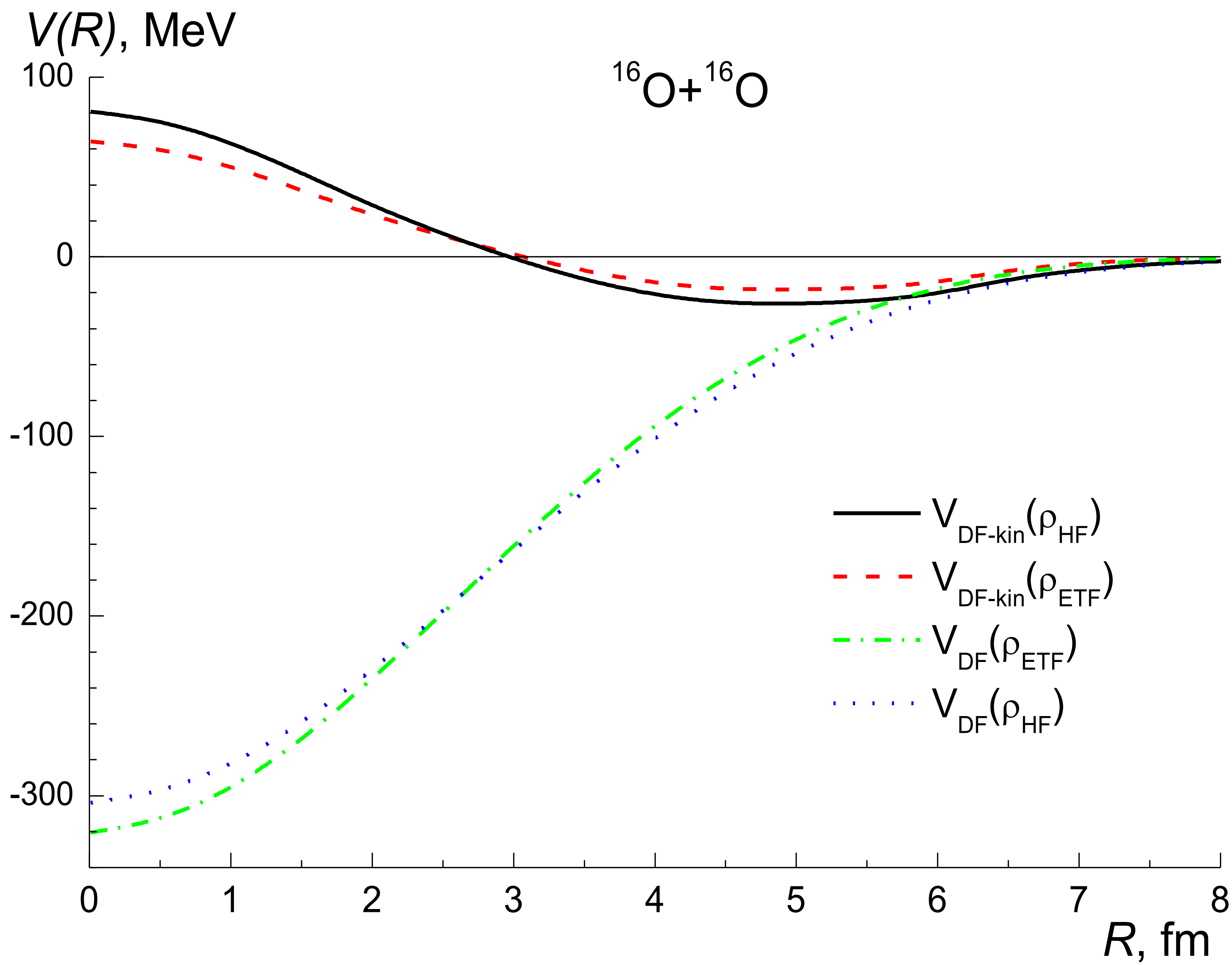
$$V_{Df-kin}(R) = V_T(R) + V_{DF}(R).$$

Here

$$V_T(R) = \int \tau\left[\rho_{1p}(\vec{r}) + \rho_{2p}(\vec{r}, R), \rho_{1n}(\vec{r}) + \rho_{2n}(\vec{r}, R)\right] d\vec{r} - \int \tau\left[\rho_{1p}(\vec{r}), \rho_{1n}(\vec{r})\right] d\vec{r} - \int \tau\left[\rho_{2p}(\vec{r}), \rho_{2n}(\vec{r})\right] d\vec{r},$$

- kinetic energy of nucleons

$V_{DF}(R)$ - double-folding potential, which connect with nucleon-nucleon interaction.



V(R), MeV
16O+16O
100
0
-100
-200
-300
0
1
2
3
4
5
6
7
8
R, fm
VDF-kin(ρHF)
VDF-kin(ρETF)
VDF(ρETF)
VDF(ρHF)

Therefore:

Nucleus-nucleus potential has core induced by intrinsic kinetic energy of nucleons in colliding nuclei.

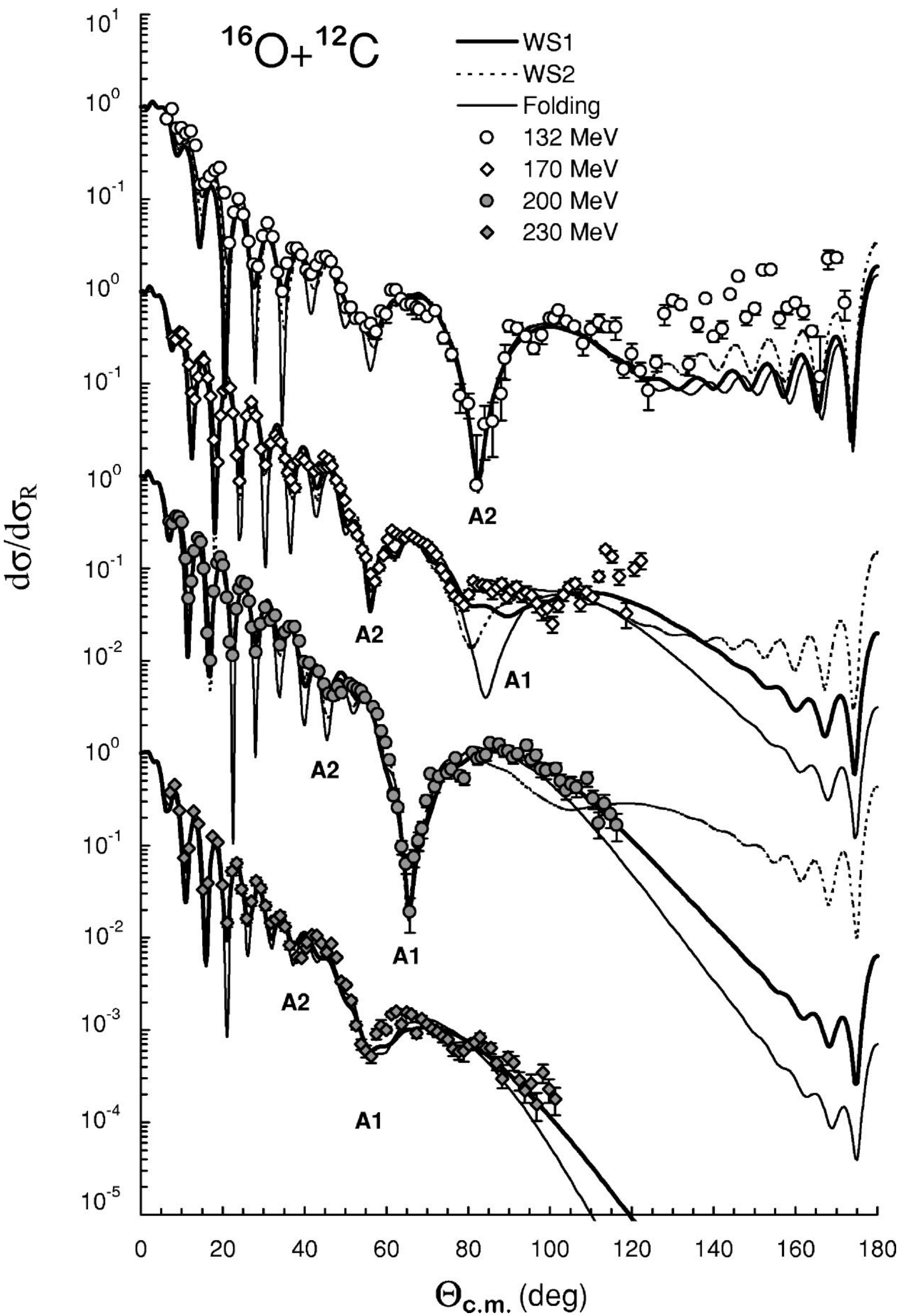
Ogloblin et al. **, PHYSICAL REVIEW C 62, 044601(2000)**

$$U(R)=V_C(R)-V\left[1+\exp\left(\frac{R-R_V}{a_V}\right)\right]^{-1}-iW\left[1+\exp\left(\frac{R-R_W}{a_W}\right)\right]^{-1},$$

where

$$R_{V,W}=r_{V,W}(16^{1/3}+12^{1/3}).$$

E_{lab} (MeV)	Potential	V (MeV)	r_V (fm)	a_V (fm)
132	Folding	0.798 [a]		
	WS1	282.2	0.586	0.978
	WS2	361.8	0.653	0.807
170	Folding	0.787 [a]		
	WS1	255.5	0.629	0.967
	WS2	333.4	0.604	0.920
200	Folding	0.771 [a]		
	WS1	216.3	0.683	0.927
	WS2	314.7	0.625	0.908
230	Folding	0.774 [a]		
	WS1	179.9	0.763	0.836
260	Folding	0.772 [a]		
	WS1	174.3	0.769	0.831
350	Folding	0.770 [a]		
480	Folding	0.770 [a]		
608	Folding	0.750 [a]		
	WS1	158.2	0.703	0.931
1503	Folding	0.813 [a]		
	WS1	77.99	0.886	0.767



PHENOMENOLOGICAL PARAMETERIZATIONS OF POTENTIAL WITH REPULSIVE CORE

The real part of the nucleus-nucleus potential

$$v(R) = v_C(R) + v_N(R) + v_l(R).$$

The Coulomb part

$$v_C(R) = \begin{cases} \dfrac{Z_1 Z_2 e^2}{R}, & R \geq R_C \\ \dfrac{Z_1 Z_2 e^2}{R_C}\left[\dfrac{3}{2} - \dfrac{R^2}{2R_C^2}\right], & R < R_C \end{cases}$$

The centrifugal part

$$v_l(R) = \frac{\hbar^2 l(l+1)}{2M\left[A_1A_2/(A_1+A_2)\right]R^2}.$$

Here $A_{1,2}$ - the number of nucleons,

$Z_{1,2}$ - the number of protons in corresponding nuclei,

e - the charge of proton,

M - the mass of nucleon,

$R_C = r_C(A_1^{1/3} + A_2^{1/3})$ and

l - the orbital momentum value.

The imaginary part of the nucleus-nucleus potential consists of

volume and surface parts

$$W(R) = -\frac{W_0}{1+\exp\left[R - r_w(A_1^{1/3} + A_2^{1/3})/d_w\right]} - \frac{W_s \exp\left[R - r_s(A_1^{1/3} + A_2^{1/3})/d_s\right]}{d_s\left\{1+\exp\left[R - r_s(A_1^{1/3} + A_2^{1/3})/d_s\right]\right\}^2}$$

Parameterizations of nuclear part:

Type A.

$$V_N(R) = \frac{-V_0}{1+\exp\left[R - r_0(A_1^{1/3} + A_2^{1/3})/d_0\right]} + V_{core}(R).$$

$V_{\mathrm{core}}(R)$ is the core potential

$$V_{core}(R) = C_{core} v(R,a),$$

where

$$v(R,a) = \begin{cases} \dfrac{4\pi a^3}{3} - \pi R a^2 + \dfrac{\pi R^3}{12}, & R < 2a \\ 0, & R \geq 2a \end{cases}$$

Here C_{core} and a are fitting parameters, which depend on the collision energy.

Parameterizations of nuclear part:

Type B.

$$v_N(R)=\begin{cases}\dfrac{-V_0}{1+\exp\left[R-r_0(A_1^{1/3}+A_2^{1/3})/d_0\right]}, & R\geq R_m\\ b_0+b_1s+b_2s^2+b_3s^3+b_4s^4, & R<R_m\end{cases}$$

The nuclear part of potential and the derivative of potential should be continuous at the matching point R_m, therefore

$$b_0=\frac{-V_0}{1+\exp\left[R_m-r_0(A_1^{1/3}+A_2^{1/3})/d_0\right]},$$

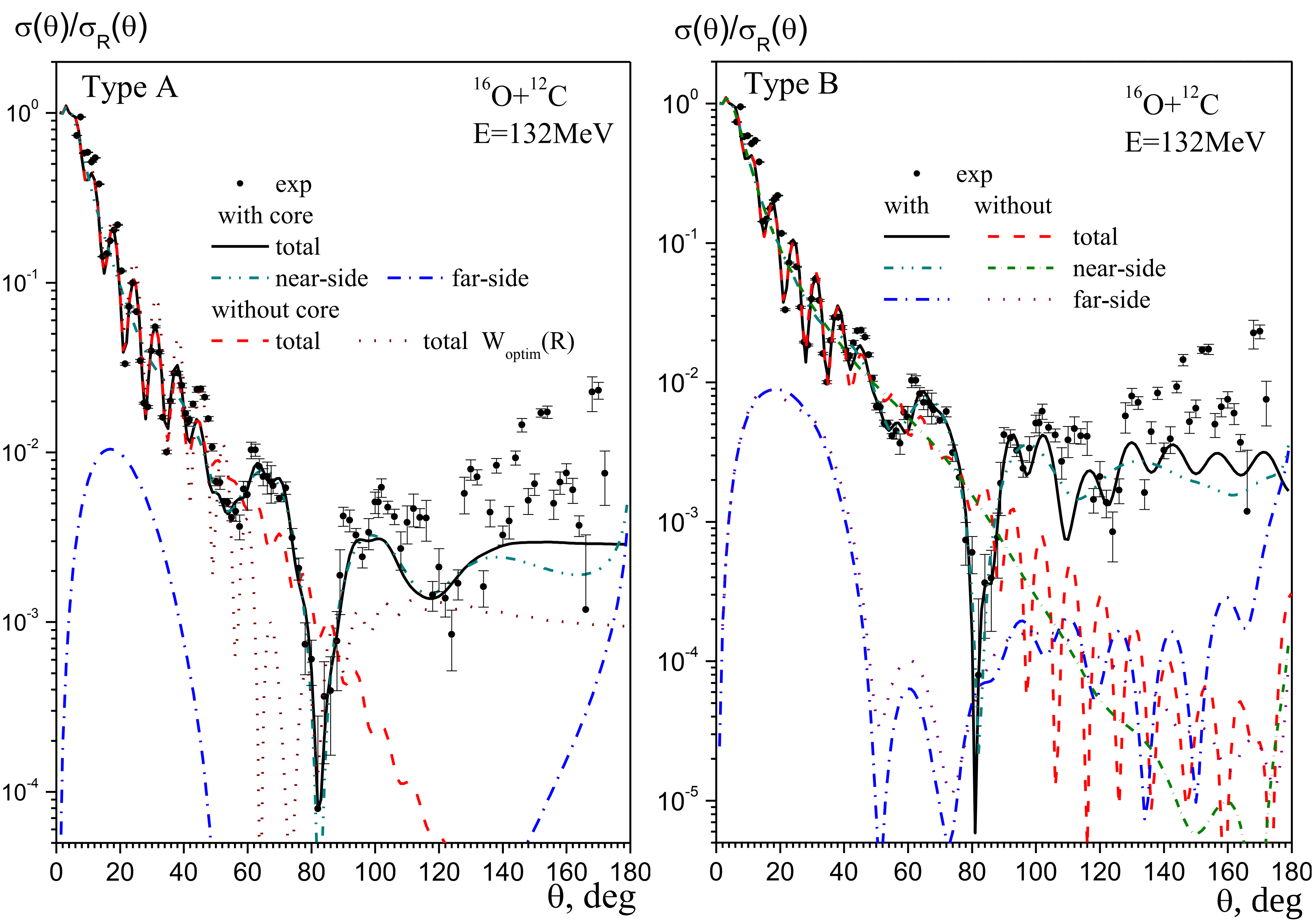
$$b_1=\frac{V_0\exp\left[R_m-r_0(A_1^{1/3}+A_2^{1/3})/d_0\right]}{d_0\left\{1+\exp\left[R_m-r_0(A_1^{1/3}+A_2^{1/3})/d_0\right]\right\}^2}.$$

RESULTS AND DISCUSSIONS

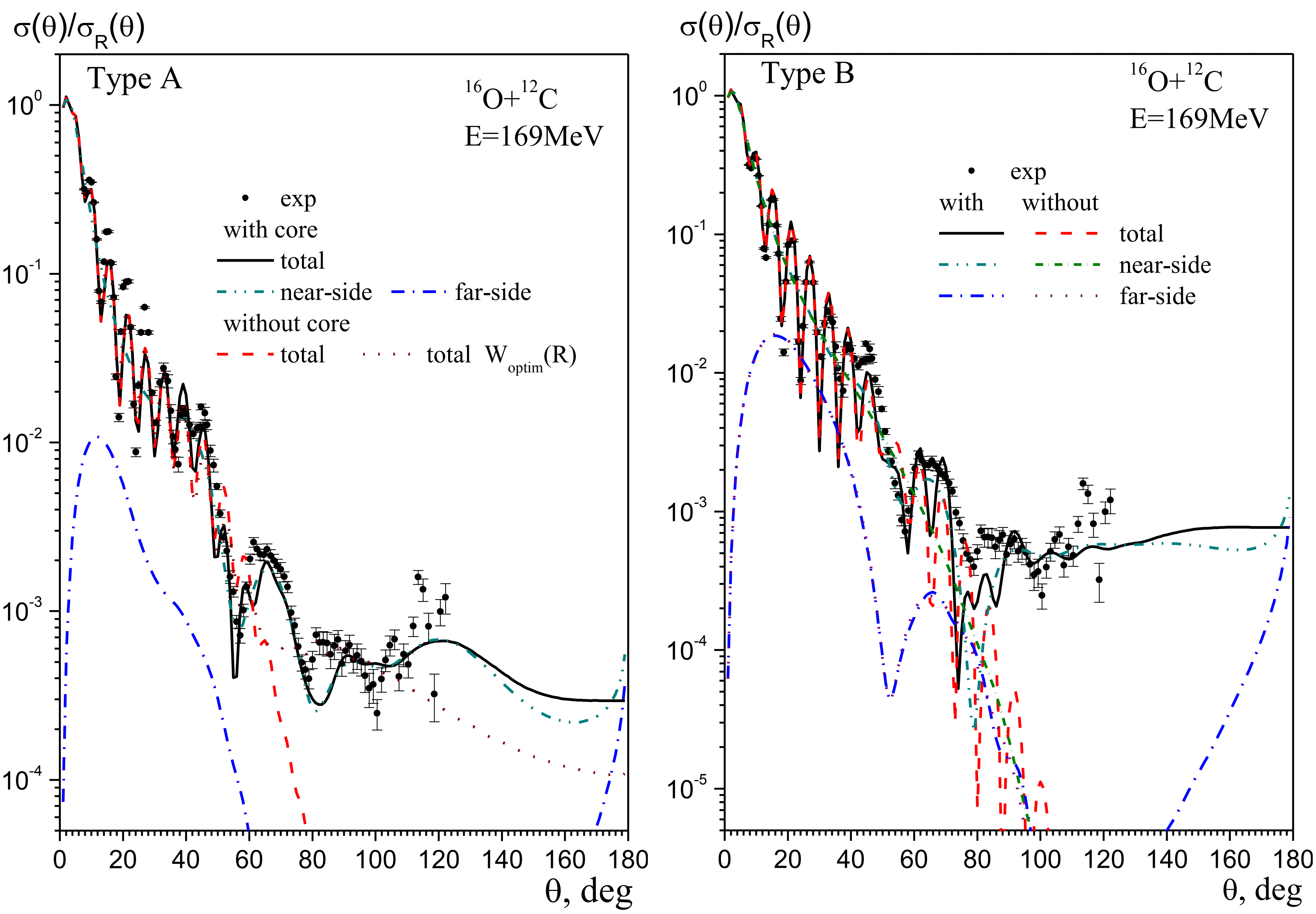
We find parameters of the potential by fitting the data for the $^{16}O+^{12}C$, $^{16}O+^{16}O$, $^{12}C+^{12}C$ elastic scattering for different collision energy. We minimize

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{(\sigma_{calc}(\theta_i)-\sigma_{\exp}(\theta_i))^2}{\delta\sigma_{\exp}(\theta_i)}.$$

We have assumed for all data points $\delta\sigma_{\exp}(\theta_i) = 0.1\sigma_{\exp}(\theta_i)$.



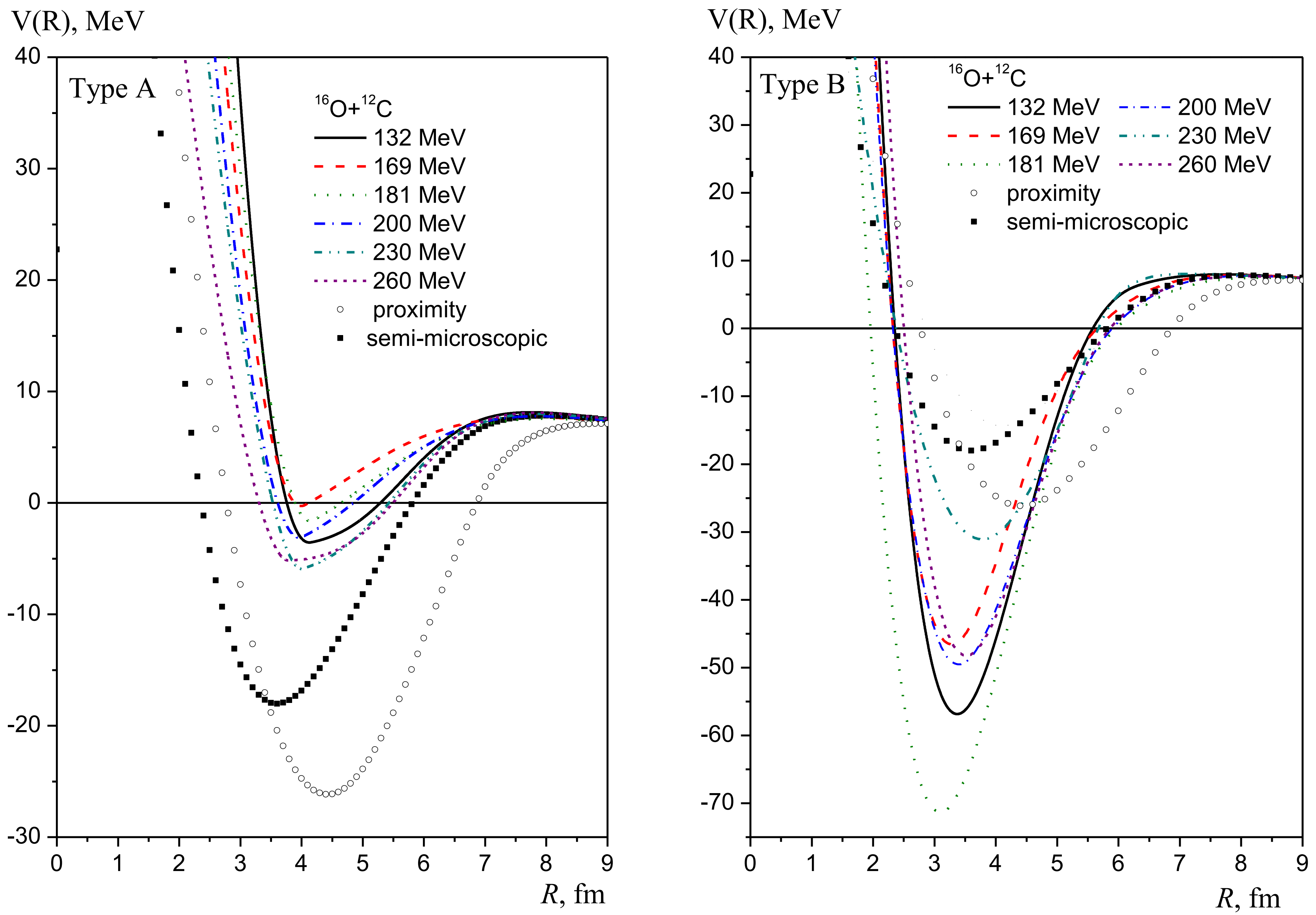
	type A	type B	Ogloblin et al.
χ^2	17.82	16.2	21.1

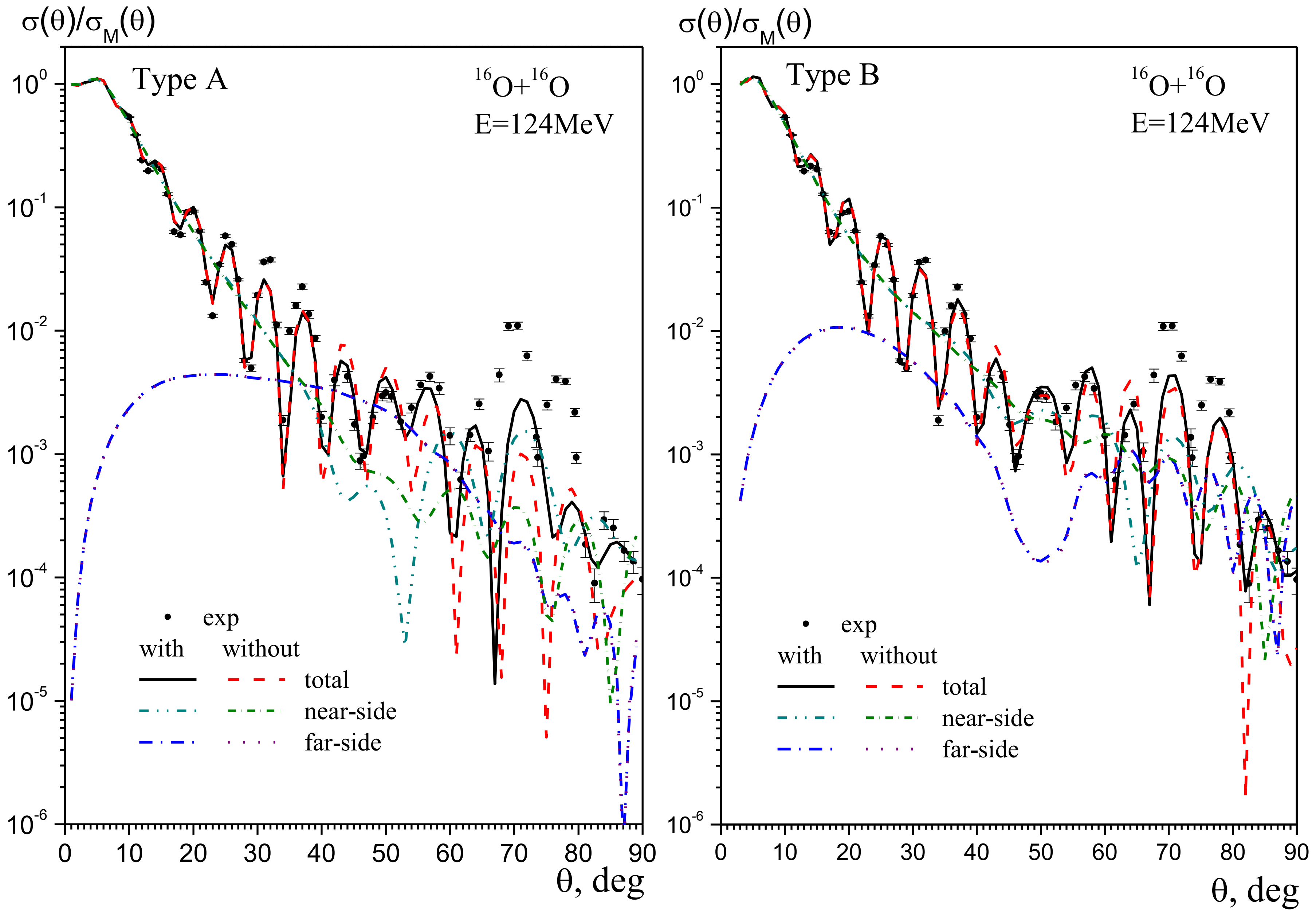


	type A	type B	Ogloblin et al.
χ^2	13.45	16.3	14.1

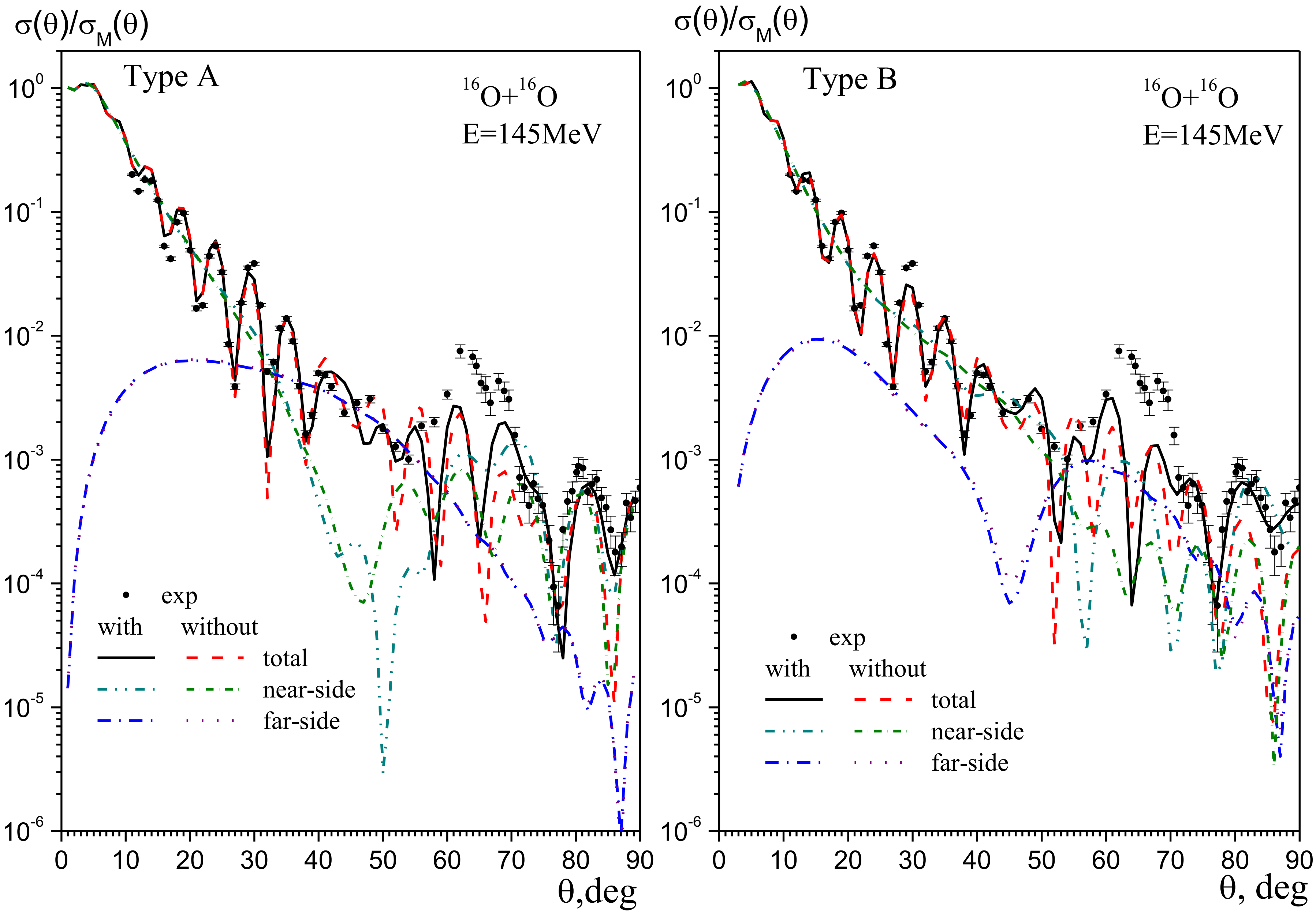
For the system $^{16}O+^{12}C$.

E_{lab} (MeV)	132	169	181	200	230	260
χ^2(type A)	17.82	13.45	20.14	14.89	9.34	11.58
χ^2 **(**type B)	16.2	16.3	21.2	15.0	10.1	5.5
χ^2 (Ogloblin et al)	21.1	14.1	-	10.7	9.2	5.6


V(R), MeV
Type A
^{16}O+^{12}C
132 MeV
169 MeV
181 MeV
200 MeV
230 MeV
260 MeV
proximity
semi-microscopic
40
30
20
10
0
-10
-20
-30
0
1
2
3
4
5
6
7
8
9
R, fm
V(R), MeV
Type B
^{16}O+^{12}C
132 MeV
169 MeV
181 MeV
200 MeV
230 MeV
260 MeV
proximity
semi-microscopic
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
0
1
2
3
4
5
6
7
8
9
R, fm



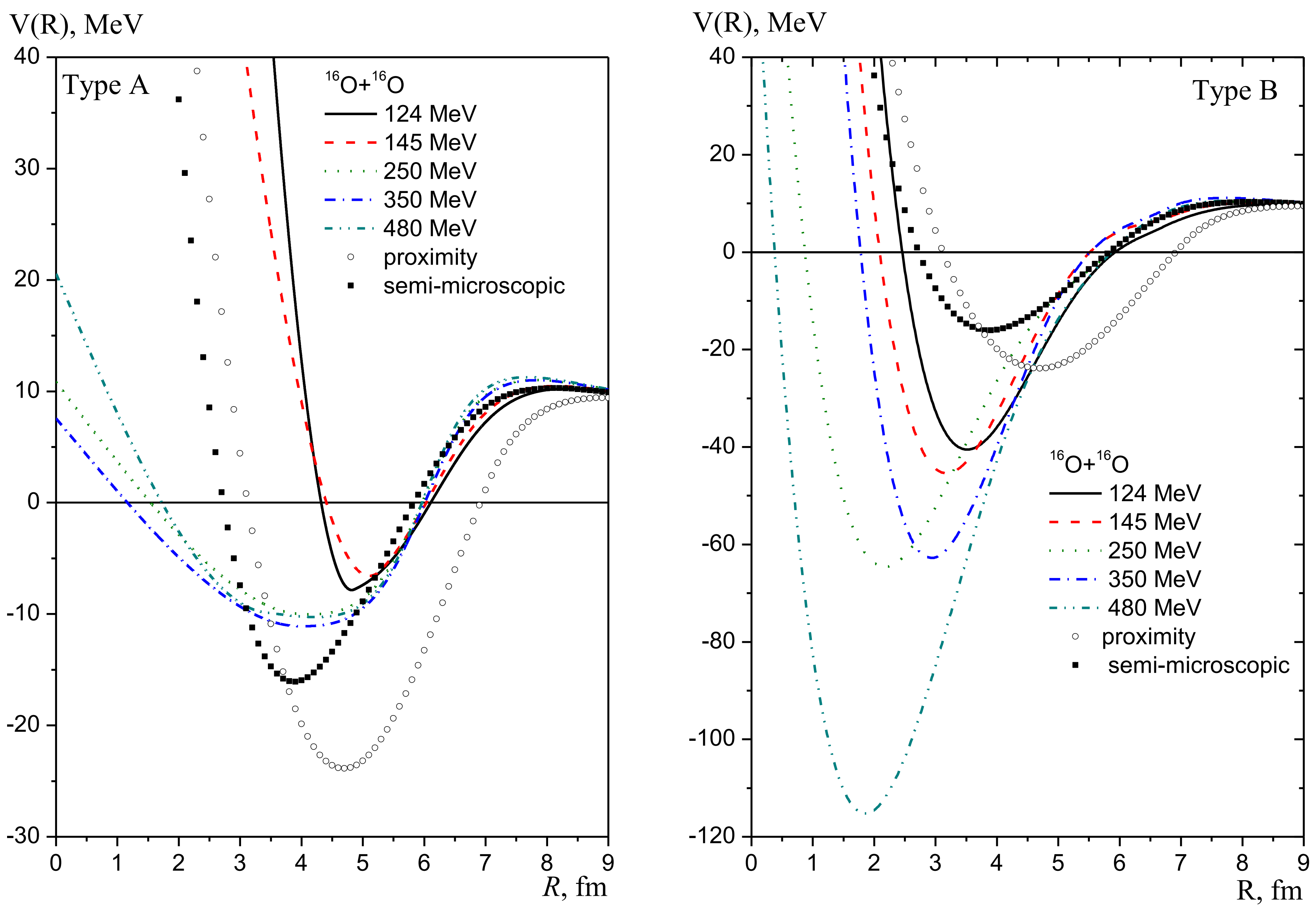
	type A	type B	Khoa et al.
χ^2	16.9	11.01	8.4(12.9)

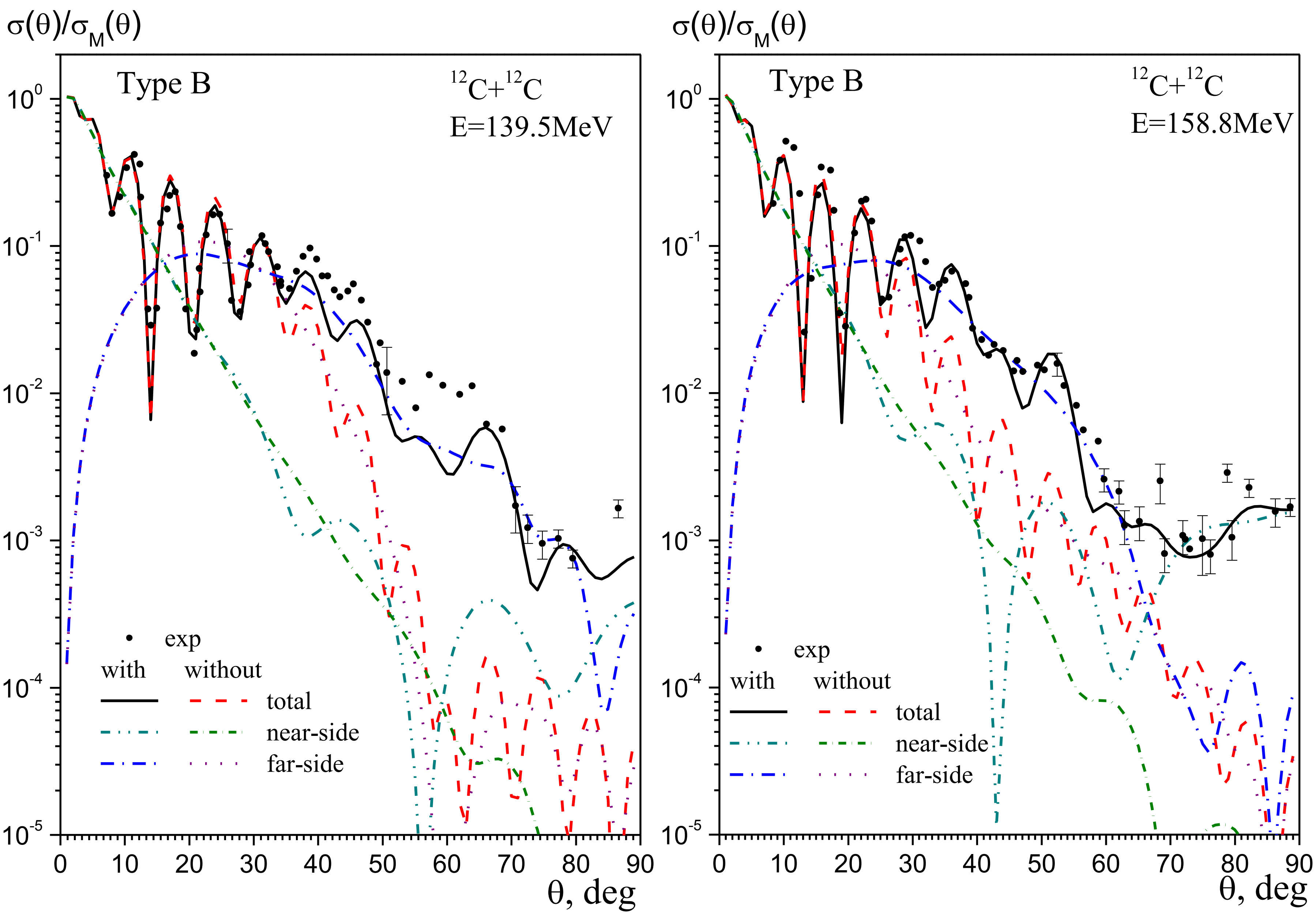


	type A	type B	Khoa et al.
χ^2	17.8	14.42	9.0

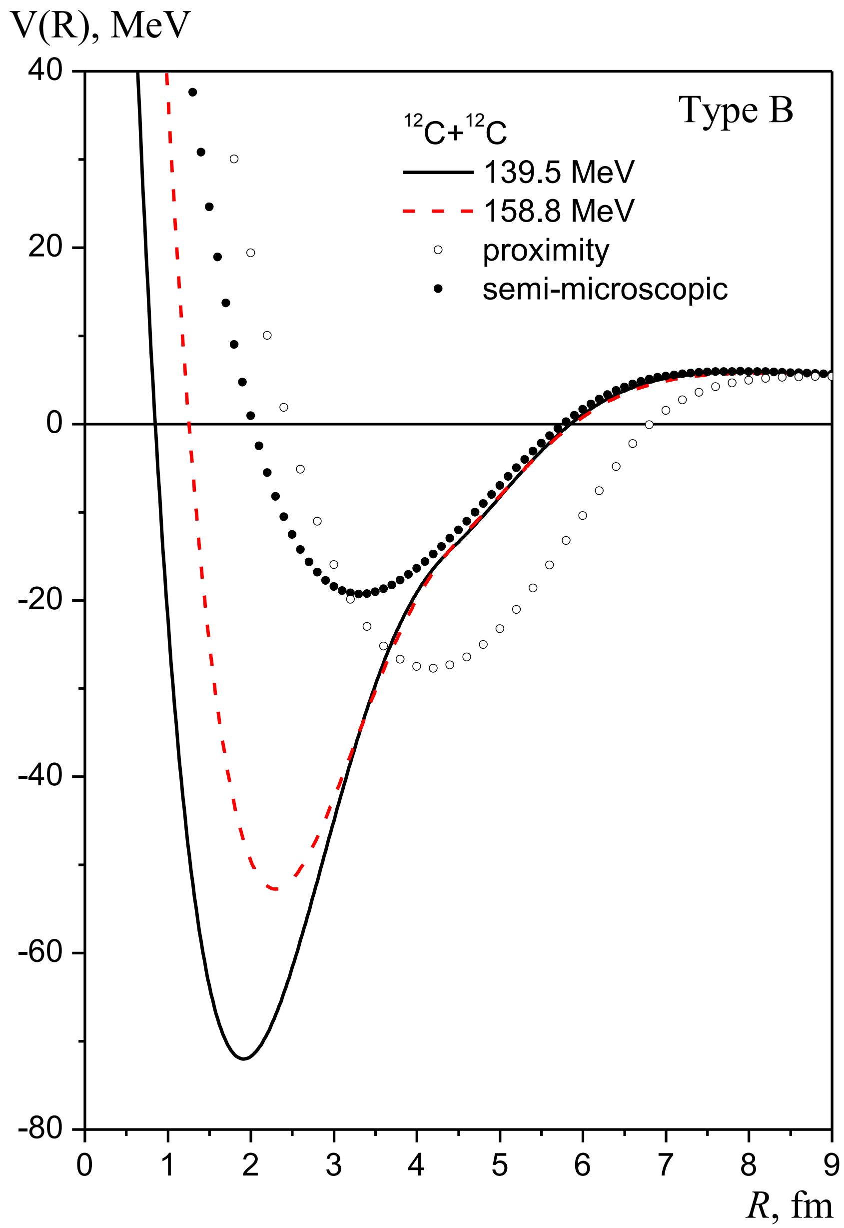
For the system $^{16}O+^{16}O$.

E_{lab} (MeV)	124	145	250	350	480
χ^2(type A)	16.9	17.8	10.85	5.88	6.15
χ^2(type B)	11.01	14.42	7.86	7.79	5.66
χ^2(Khoa et al.)	8.4(12.9)	9.0	7.6(9.9)	4.2(5.1)	4.7(5.8)


V(R), MeV
Type A
^{16}O+^{16}O
124 MeV
145 MeV
250 MeV
350 MeV
480 MeV
proximity
semi-microscopic
R, fm
V(R), MeV
Type B
R, fm



E_{lab} (MeV)	139.5	158.8
χ^2	8.9	7.72


V(R), MeV
Type B
12C+12C
139.5 MeV
158.8 MeV
proximity
semi-microscopic
40
20
0
-20
-40
-60
-80
0
1
2
3
4
5
6
7
8
9
R, fm

CONCLUSIONS

- Nucleus-nucleus potential has core induced by intrinsic kinetic energy of nucleons in colliding nuclei.
- We propose two different phenomenological parameterizations for nuclear part of potential with the repulsive core at small distances.
- It is possible to describe elastic scattering data $^{16}O+^{12}C$, $^{12}C+^{12}C$ and $^{16}O+^{16}O$ by using shallow phenomenological potential with the repulsive core.
- The elastic scattering data shows that the repulsive core of nucleus-nucleus potential takes place at distances $R \leq 2$ fm.
- The cross-section as well as both the near- and far-side cross-section components on backward angles are strongly enhanced by the repulsive core.

Thank you!