Lectures on the Physics of the Nucleus

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Overview:

- **show possible subjects of lecture**
- **find out and select, what is of interest to the audience**

A. Structure

1. Single particle models

bulk properties, nuclear matter, Fermi gas
indep. part. shell model (SM), spin-orbit, SM states, s.p. transitions
deformed SM, Nilsson model
deformation energy surfaces, Strutinsky shell corrections
pairing, quasi-particles, BCS

2. Compound nuclear model

level densities, mean free path

3. NN-interaction

2N-states, bound, scattering
realistic interactions, meson exchange, chiral interactions
effective interactions, T-, G-matrix, V_{lowk}

4. Many-body methods

SM with residual interactions, no-core SM, Monte-Carlo SM, Greenfct.-MC
Hartree-Fock, HF-Bogoliubov, Brueckner-HF
field theoretical approach, Quantumhadrodynamics (QHD), Rel. Mean field (RMF)

5. Decay modes

el.magn. decay, multipole-moments and -operators, transition rates
α-decay
β-decay, weak interaction, parity, Fermi-, Gamow-Teller transitions
fission

6. Collective motion: Rotations

dynamic and static deformations, rotations, vibrations, coupling, bands
(microscopic) HF, HFB, constrained HBF
cranking model, moment of inertia,
high spin, back bending

7. Collective motion: vibrations

harmonic vibrations, classification of modes, giant resonances (GR), exotic modes
(micr.) TDA, RPA, TDHF, GCM
strength functions, sum rules
boson expansions, Interact. Boson approximation (IBA)

B. Reactions

1. Reaction theory

scattering states, wave packet schattering, cross sections classical and qm,
Born approx. (BA), partial waves, scattering amplitude
scattering lengths, effective range theory
reactions, two-potential formula, DWBA
operator formalism, S-matrix
R-matrix approach

2. Compound nucleus scattering

resonances, overlapping resonances, fluctuations (Ericson),
CN scattering, Hauser-Feshbach formalism
average scattering, optical model

3. Optical Model

Projektion formalism (Feshbach)

complex potential, mean free path, dispersion relations
parametrizations, folding,

4. Direct reactions

DWBA, angular distributions, polarization
inelastic scattering, collective strength functions
charge exchange, β-strength functions
transfer, (1 and many nucleon), spectroscopic factors,
multistep reactions, CC, CCBA, CRC

5. Statistical reactions

pre-equilibrium reactions
exciton models

quantummech. pre-eq. theories (FKK, TUL)

6. Heavy ion collisions

semiclassical features
deep inelastic scattering, dissipation
transport approach, BUU equations
fragmentation

0. Introductory Remarks

Modell des Kerns: stark wechselwirkendes Vielteilchensystem

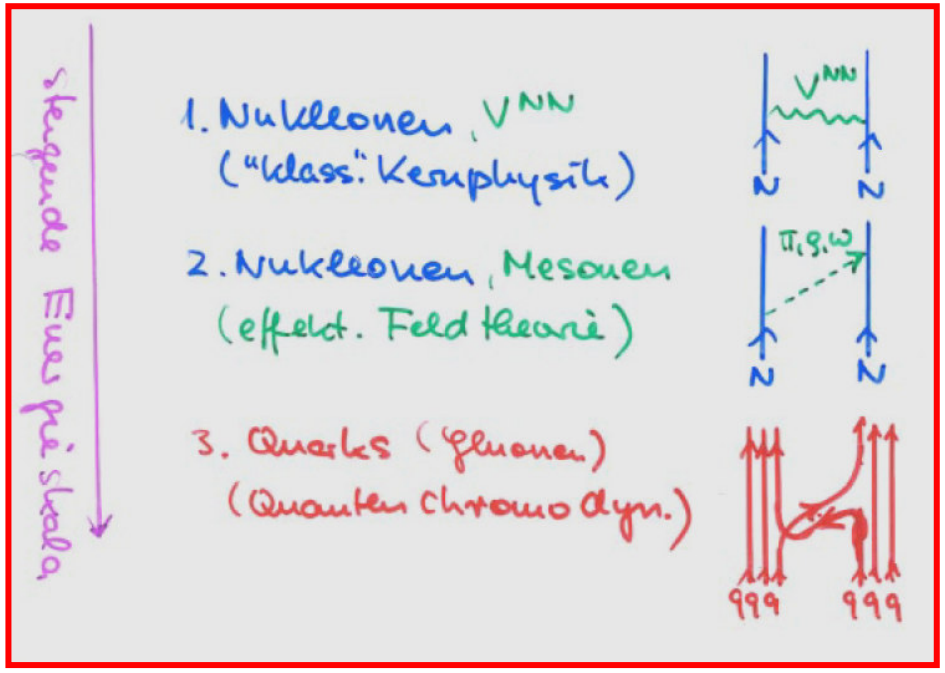
Vielteilchenaspekt

N-Teilchensystem: $1 \leq N \lesssim$ einige Hundert ↑ mesoskopisches System

Hamiltonfunktion

$$H = \sum_{i=1}^{N} T_i + \sum_{i,j=1}^{N} V_{ij}$$

spezifiziert, lösbar?

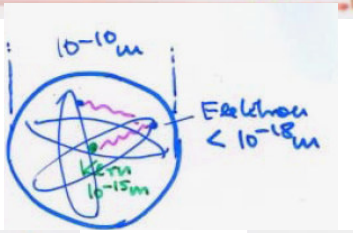


Comparison: Atom --- Nucleus

Atom

$$H = \sum_{i=1}^{Z}\left(\frac{p_i^2}{2m} - \frac{Ze^2}{r_i}\right) + \sum_{i=1}^{Z}\sum_{j=i+1}^{Z}\frac{e^2}{|\vec{r}_i - \vec{r}_j|}$$

WW mit Kern — Elektr.-Elektr.-WW

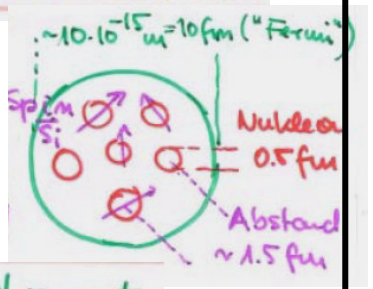


- Problem vollständig spezifiziert
  Coulomb WW bekannt
  (+ rel. Efekte; Quantenelektrodynamik; el. + magn. Momente des Kerns, ...)
- Zentralfeld dominiert, Elektr.-Elektr.-WW schwach
  (in gewissem Sinne, + mittleres Feld der Elektronen)
  → Periodensystem, Schalenmod.

Kern

$$H = \sum_{i=1}^{A}\frac{p_i^2}{2m} + \sum_{i=1}^{A}\sum_{j=i+1}^{A} V_{ij}^{NN}(\vec{r}_i - \vec{r}_j, \vec{s}_i, \vec{s}_j)$$

Nukl.- Nukl.-WW

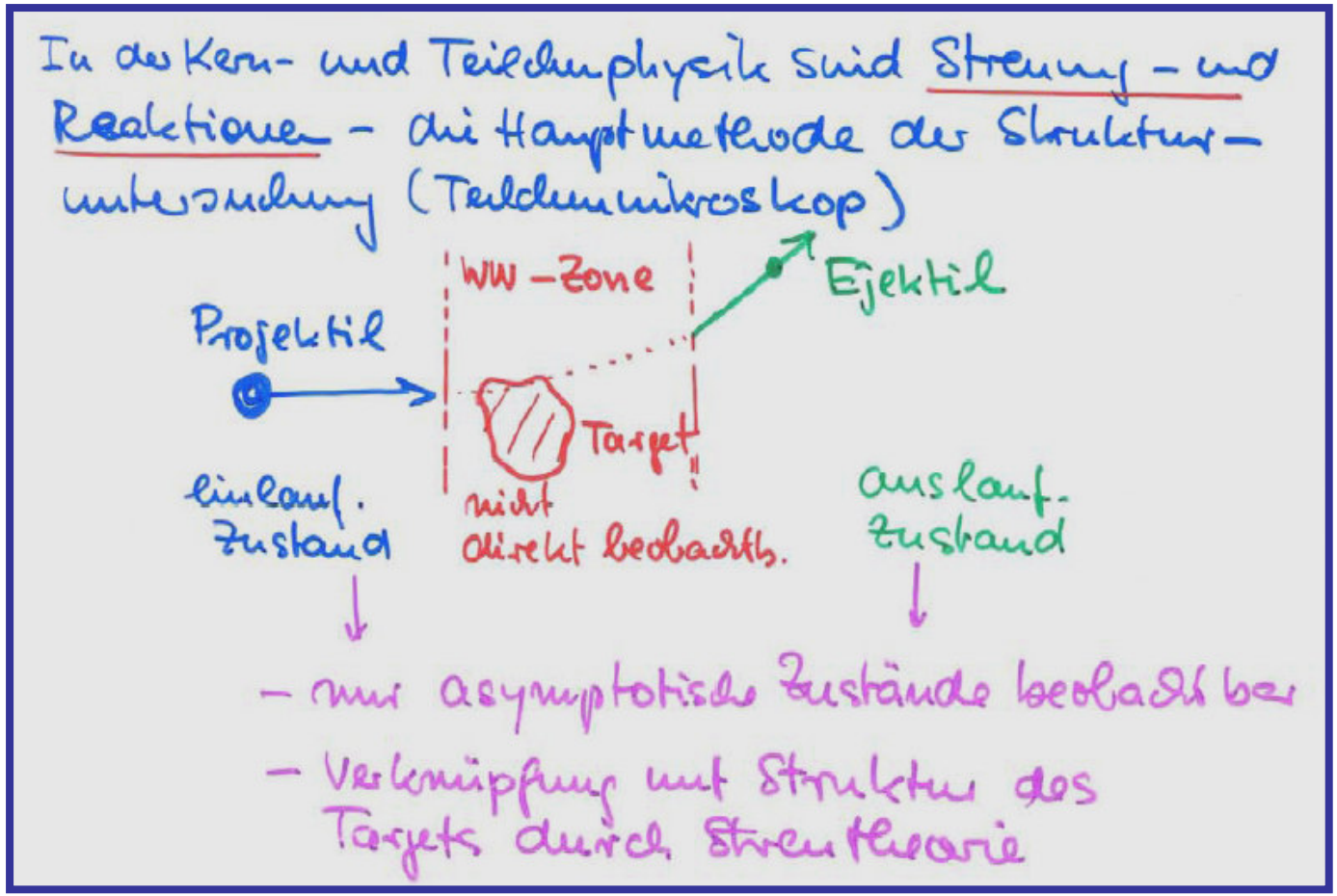


- V^{NN} nicht exakt bekannt
- V^{NN} nicht fundamental!
- kein dominierendes Zentralfeld, V^{NN} stark!
  demnach: Schalenmodell möglich
  (Haxel, Jensen, Suess - Mayer, 1949)
- Wichtigkeit von Modellen (Charakt. Moden des Systems, z. B. Phononen)
  ↕ Beziehung zu Vielteilchentheorie

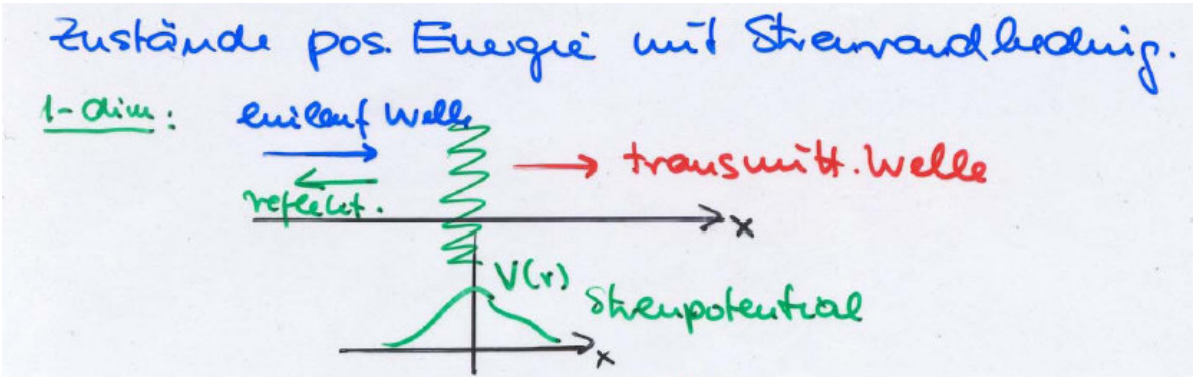
1. Reaction Theory

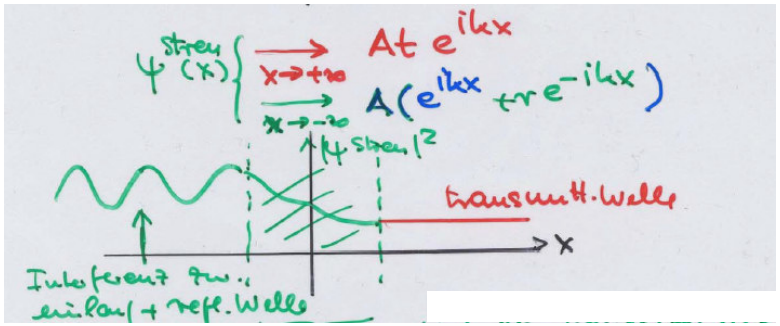
discuss essential aspects of theoretical treatment of scattering and reactions,

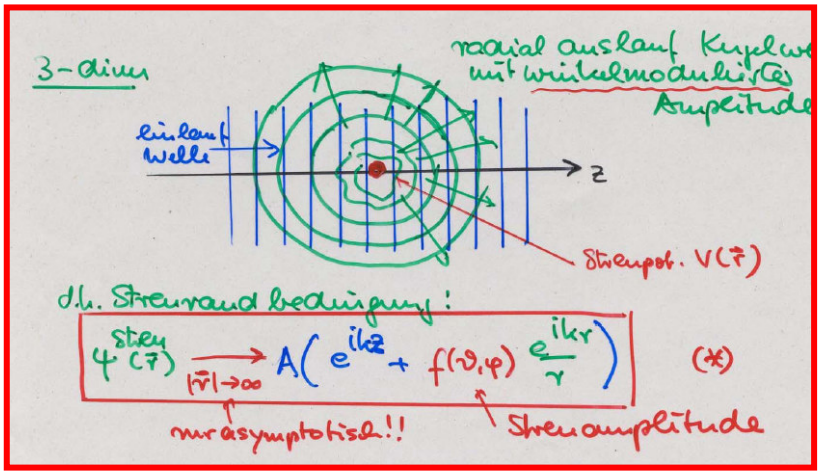
(this is general and not neccessarily confined to nuclear reactions)

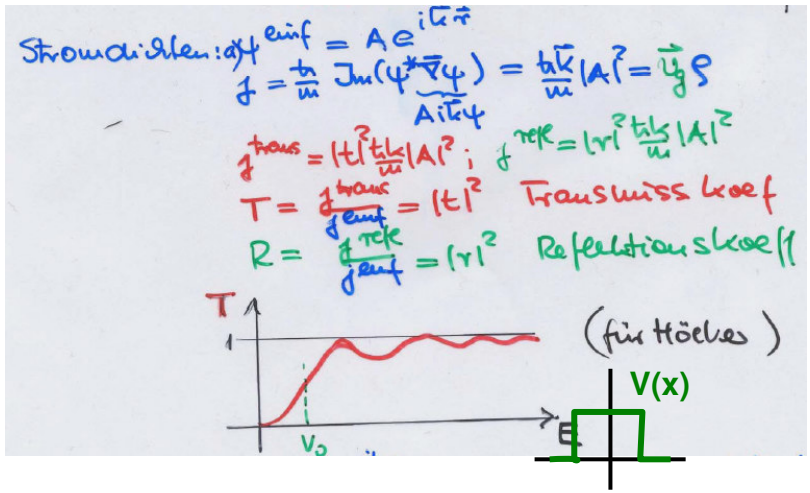


Scattering states, Cross sections

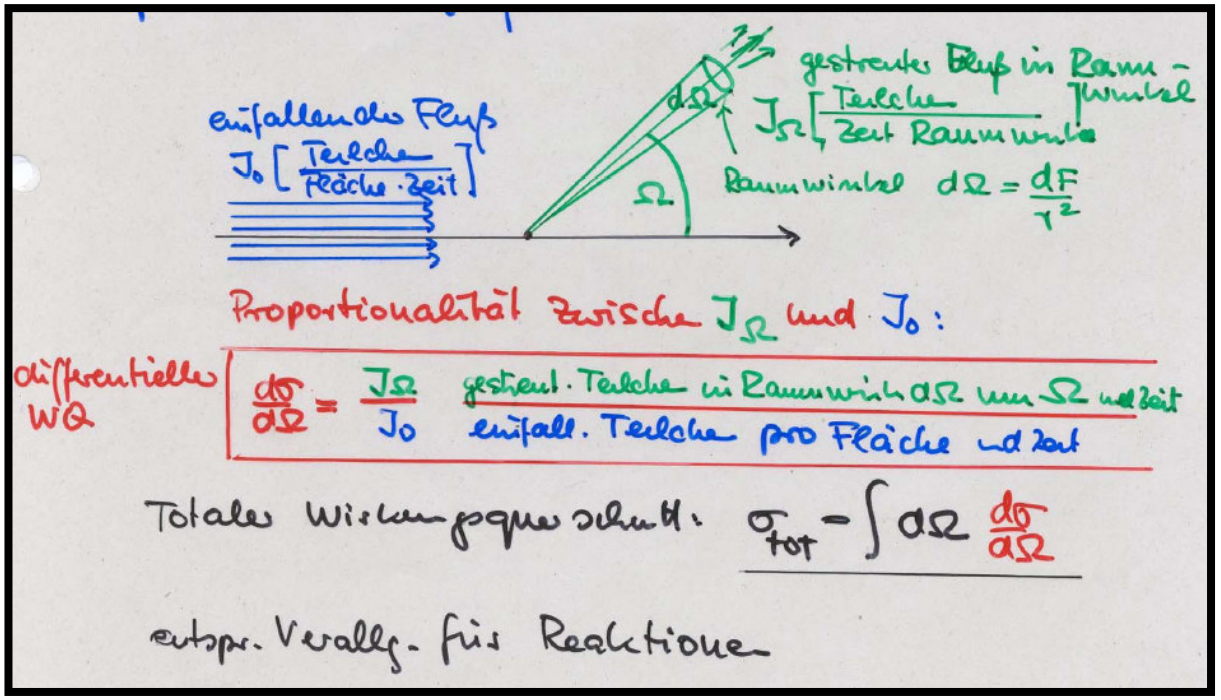


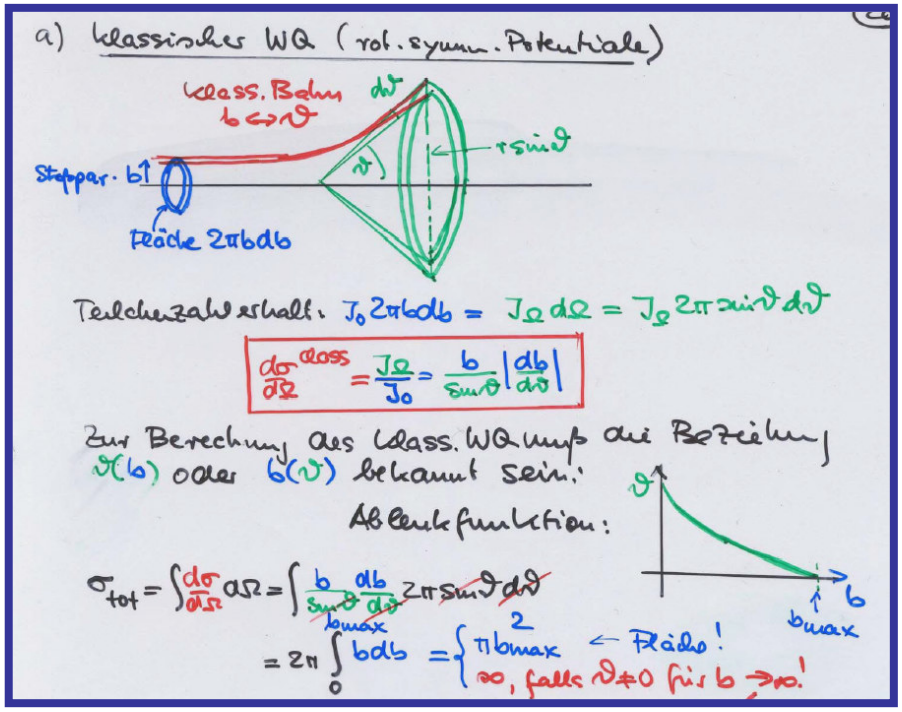


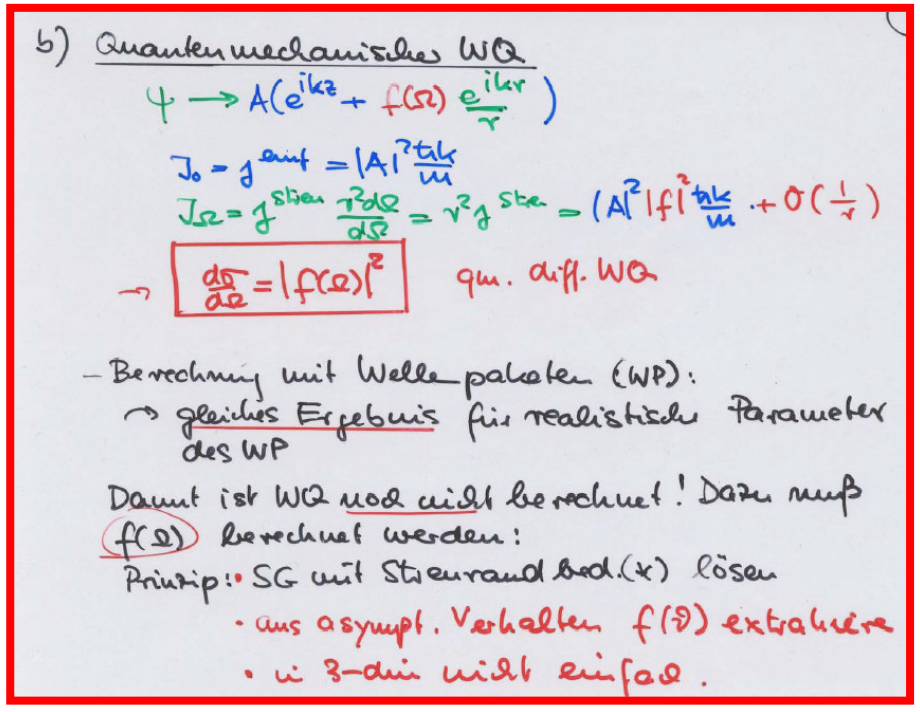




Definition of cross section:







Born Approximation

SG: $\left(-\frac{\hbar^2}{2m}\Delta + V(r) - E\right)\psi(\vec{r}) = 0$

$(\Delta - U + k^2)\psi = 0 \quad ; \; E = \frac{\hbar^2 k^2}{2m}, \; U = \frac{2m}{\hbar^2} V$

$(\Delta + k^2)\psi(\vec{r}) = U(r)\psi(\vec{r}) =: q_{inhom}(\vec{r})$

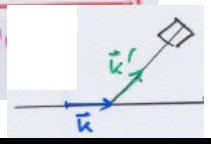
→ formal als inhom. Dgl. schreiben

→ Lösen mit Methode der Greenfunktion:

$$G^{(\pm)}(\vec{r}-\vec{r}') = -\frac{1}{4\pi}\frac{e^{\pm ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$$

$$G^{(\pm)}(\vec{r}-\vec{r}') \longrightarrow -\frac{1}{4\pi}\frac{e^{\pm ikr}}{r} e^{\mp i\vec{k}'\vec{r}'}$$

richtige RB: auslaufende Kugel also $G^{(+)}$



Dann Lsg der inhom SG (S. 21)

$$\psi^{(+)}(\vec{r}) = \psi_0(\vec{r}) + \int d^3r' G^{(+)}(\vec{r}-\vec{r}') U(\vec{r}') \psi^{(+)}(\vec{r}')$$

$\psi_0 = e^{ikz}$; $U\psi^{(+)}$: q_{inhom}

Streuamplitude:

$$f_k(\Omega) = -\frac{1}{4\pi}\int d^3r \, e^{-i\vec{k}'\vec{r}} U(\vec{r}) \psi^{(+)}(\vec{r})$$

formale Lsg!

Näherungslsg. der LS-Gleichung: Iteration (23

$\psi^{(+)(0)} = \psi_0(\vec{r})$ $\quad \mathcal{O}(V^0)$

$\psi^{(+)(1)} = \psi_0(\vec{r}) + \int d^3r' G^{(+)}(\vec{r}-\vec{r}') U(\vec{r}') \psi^{(+)(0)}(\vec{r}')$ $\quad \mathcal{O}(V^1)$

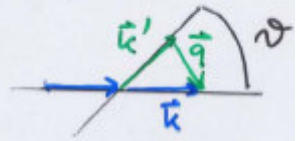
$\psi^{(+)(2)} = \ldots\ldots\ldots\ldots \psi^{(+)(1)}(\vec{r}) \ldots$ $\quad \mathcal{O}(V^2)$

Born'sche Näherung: Verwende $\psi^{(+)(0)}$ in Streuamplitude

$$f_k^{(BA)}(\Omega) = -\frac{1}{4\pi}\int d^3r \, e^{-i\vec{k}'\vec{r}} \left(\frac{2m}{\hbar^2} V(\vec{r})\right) e^{i\vec{k}\vec{r}}$$

$$f_k^{(BA)}(\Omega) = -\frac{m}{2\pi\hbar^2}\int d^3r \, e^{i\vec{q}\vec{r}} V(\vec{r})$$

Born-Approx.

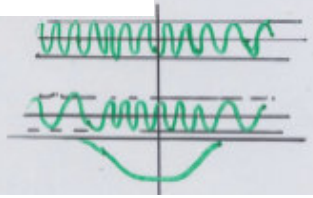


$-\vec{q} = \vec{k} - \vec{k}'$ Impulsübertrag

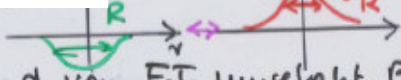
$|\vec{q}| = q = 2k \sin\frac{\vartheta}{2}$

abhängig von Energie (k) und Winkel.

- Güte der Näherung: V klein;
  genauer: V klein gegen E,
  Hochenergienäherung



- Eigenschaft der FT:



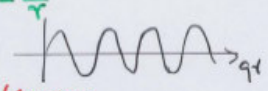
Breite von Fkt und von F.T. umgekehrt prop

→ Streuampl. groß für $q \lesssim \frac{1}{R}$, $2k\sin\frac{\vartheta}{2} \lesssim \frac{1}{R}$ $\Big\rangle$ $kR \ll 1$, $\vartheta \lesssim \frac{1}{kR}$ Vorwärts-Winkel mit E↗

Examples and Applications:

1) Streuung an Punktladung $V_c = \frac{Ze^2}{r}$

$f_c^{(BA)}(q) = -\frac{2m}{\hbar^2 q} \int r\,dr \sin qr \cdot \frac{1}{r}$



Oszill., nicht konverg.

→ konvergenzerz. Faktor $V \to V' = Ve^{-r/a}$; $a \to \infty$

$\sin qr = \frac{1}{2i}(e^{iqr} - e^{-iqr})$

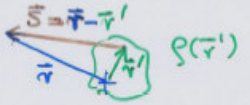
$\to f_c^{(BA)}(q) = -\frac{2m}{\hbar^2}\frac{Ze^2}{q^2 + \left(\frac{1}{a}\right)^2} \to 0 = f_{Rutherford}(q)$ Rutherf.-Amplitude

$\frac{d\sigma^{(BA)}}{d\Omega_c} = |f_c^{(BA)}|^2 = \frac{4m^2(Ze^2)^2}{(\hbar^2 q^2)^2} = \left(\frac{Ze^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}} = \frac{d\sigma}{d\Omega}\Big|_{Rutherf.}$

→ für Coulomb: $\frac{d\sigma}{d\Omega}^{class} \equiv \frac{d\sigma}{d\Omega}^{(BA)} \equiv \frac{d\sigma}{d\Omega}^{q.m.}$! (Zufall)

2) endliche Ladungsverteil.

$V(\vec{r}) = \int d^3r' \rho(\vec{r}')\, V_c(\vec{r} - \vec{r}')$



$f_q^{(BA)}(\Omega) = -\frac{m}{2\pi\hbar^2}\int d^3r\, e^{i\vec{q}\cdot\vec{r}} V(\vec{r})$

$\vec{s} + \vec{r}'$ $= \int d^3r' \rho(\vec{r}') V_c(\vec{s})$

$= \underbrace{\int d^3r' e^{i\vec{q}\cdot\vec{r}'} \rho(\vec{r}')}_{=: F(\vec{q})} \underbrace{\int d^3r\, e^{i\vec{q}\cdot\vec{s}}\, V_c(\vec{s})\left(-\frac{m}{2\pi\hbar^2}\right)}_{f_{Rutherford}!}$

$\boxed{\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\Big|_{Rutherford} |F(\vec{q})|^2}$

mit $F(\vec{q}) = \int d^3r\, e^{i\vec{q}\cdot\vec{r}} \rho(\vec{r})$ Form faktor $\hat{=}$ Fouriertransf. der Ladungsverteilung

→ Messung der Ladungsvert.!

4) Bestimmung der Ladungsverteilung

α) Inverse Fouriertransformation

$\rho(\vec{r}) = \frac{1}{(2\pi)^3}\int_{\mathbb{R}^3_q} e^{-i\vec{q}\cdot\vec{r}} F(\vec{q})\, d^3q$

Betrag + $\vec{q}$? (d.h. Winkel, Energie)

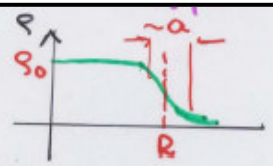
→ Fehlerband (Fourier-Bessel Analyse)

β) Ansatz für $\rho(\vec{r})$ und Bestimmung des Parameters

i) Nukleon $\rho_c = \frac{a^3}{8\pi} e^{-ar} \to F(q^2) = \frac{1}{\left(1 + q^2/a^2\hbar^2\right)^2}$

experiment $a \sim 4.2$ fm $\to \sqrt{\langle r^2 \rangle} \simeq 0.86$ fm

ii) Kern $\rho_c(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{a}\right)}$



exp.: $R = r_0 A^{1/3}$, r_0 const $\simeq 1.15$ fm

$a \simeq$ const $\simeq 0.6$ fm

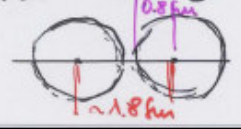
$\rho_0 \simeq$ const $\simeq 0.14\,\text{fm}^{-3} \simeq \frac{1}{6}\frac{\text{Nukl}}{\text{fm}^3}$

Flüssigkeitseigenschaften, Sättigung

$V \propto R^3 \sim A$, $a \ll R$ (außer für kleine Kerne)

$\frac{\text{Vol}}{\text{Nukl}} = \frac{V}{A} = \frac{4\pi}{3} r_0^3$ → mittl. Abstand der Nukl.

$\bar{d} \simeq \sqrt[3]{\frac{V}{A}} \sim 1.6 \sim 1.8$ fm



→ dichte Packung!

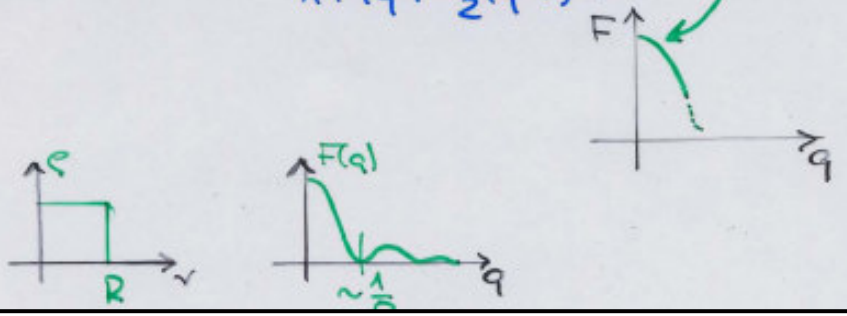
Applications of BA (contd.)

5) Qualitatives Verhalten:

a) $F(\vec{q}) = \int d^3r\, \rho(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}$

$e^{i\vec{q}\cdot\vec{r}} \approx 1 + i\vec{q}\vec{r} - \frac{1}{2}(\vec{q}\vec{r})^2 + \ldots$

$= 1 - \frac{q^2}{6}\langle r^2\rangle + \ldots$

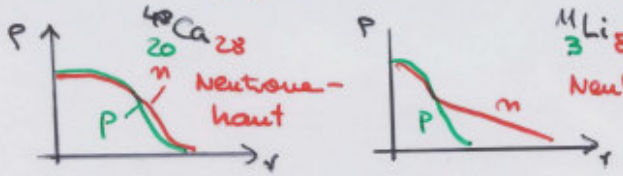


6) Elektronenstreuung misst Ladungsverteilung (Ladung) Masse Verteilung?

Bei stabilen Kernen approx: $\rho_{mass} = \frac{A}{Z}\rho_{Ladung}$

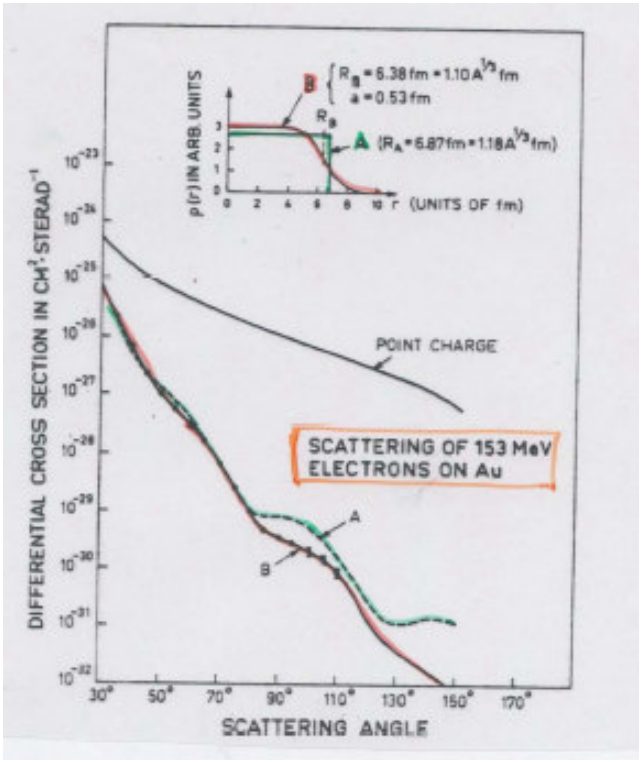
aber interessante Effekte insbes bei:

Kernen mit großem Neutronenüberschuß (exot. Kerne)



Messung für: Hadronstreuung: $p, \pi, \ldots$ → "optisches" Pot.

schwache WW: ν ← (Umkehrung)



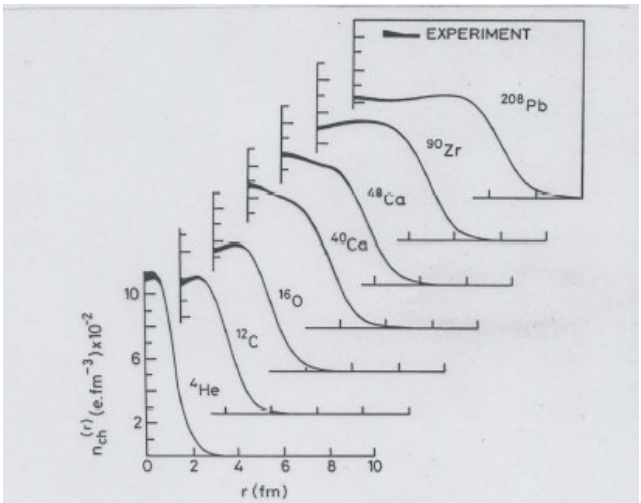


Figure 3.4 Charge densities $n_{ch}(r)$ of several nuclei measured by electron scattering. The width of the line indicates the experimental uncertainty. [*from B. Frois, Proceedings of the Niels Bohr Centennial Symposium, North-Holland, Amsterdam, 1986, p. 185.*]