# Lectures on the Physics of the Nucleus

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0. Introduction: nucleus as a many-body system

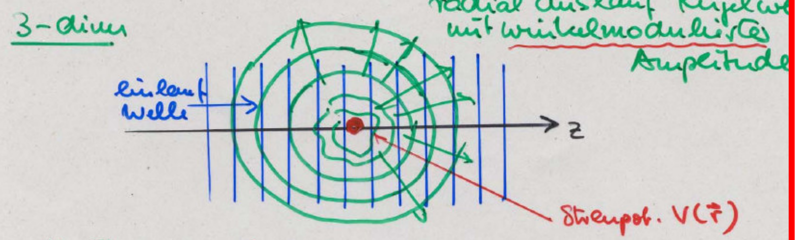
1. Reaction theory
1.1 Scattering states, cross sections
1.2 Born approximation
1.3 Partial waves
1.4 Reactions, reaction amplitude, S-matrix
1.5 Optical potential
1.6 Distorted waves, DW Born Approximation
1.7 Resonace scattering, R-Matrix

2. Compound nucleus
   and compound nuclear reactions
3. Single particle models
4. Pre-equilibrium reactions
5. Many body methods (brief)
6. Collectice motion (mainly vibrations)

Summary from last lecture:

## 3-dimensional scattering problem:

3-dim




d.h. Streurandbedingung:

$$\boxed{\psi^{\text{Streu}}(\vec r) \xrightarrow[|\vec r|\to\infty]{} A\left(e^{ikz} + \underbrace{f(\vartheta,\varphi)}_{\text{Streuamplitude}}\frac{e^{ikr}}{r}\right)} \quad (*)$$

nur asymptotisch!!

differential cross section

$$\boxed{\frac{d\sigma}{d\Omega} = |f(\Omega)|^2}$$

Lippmann-Schwinger Eq. (formal solution)

$$\psi^{(+)}(\vec r) = \underbrace{\psi_0(\vec r)}_{e^{ikz}} + \int d^3r'\, G^{(+)}(\vec r - \vec r\,')\underbrace{U(\vec r\,')\psi^{(+)}(\vec r\,')}_{\varrho_{\text{inhom}}}$$

**Born approximation**

Streuamplitude:

$$\boxed{f_k(\Omega) = -\frac{1}{4\pi}\int d^3r\, e^{-i\vec k'\vec r}\,U(\vec r)\,\psi^{(+)}(\vec r)}$$

formale Lsg!

## Born Approximation

$$f_k^{(BA)}(\Omega) = -\frac{1}{4\pi}\int d^3r\, e^{-i\vec k'\vec r}\left(\frac{2m}{\hbar^2}V(\vec r)\right)e^{i\vec k\vec r}$$

$$\boxed{f_k^{(BA)}(\Omega) = -\frac{m}{2\pi\hbar^2}\int d^3r\, e^{i\vec q\vec r}\, V(\vec r)} \quad \text{Born-Approx.}$$

$-\ \vec q = \vec k - \vec k'$ Impulsübertrag

$|\vec q| = q = 2k\sin\frac{\vartheta}{2}$

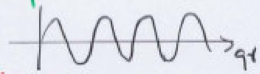
abhängig von Energie ($k$) und Winkel.


**high energy approximation**

## Examples and Applications:

1) Streuung an Punktladung $V_c = \frac{zZe^2}{r}$

$$f_c^{(BA)}(q) = -\frac{2mZe^2}{\hbar^2 q}\int \cancel{r}\,dr\,\sin qr\,\frac{1}{\cancel{r}}$$




Oszill., nicht konverg.

→ konvergenzerzeugender Faktor $V \to V' = V e^{-r/a};\ a \to \infty$

$\sin qr = \frac{1}{2i}\left(e^{iqr} - e^{-iqr}\right)$

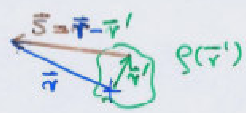
$$\to f_c^{(BA)}(q) = -\frac{2mZe^2}{\hbar^2}\,\frac{1}{q^2 + \underbrace{\left(\frac{1}{a}\right)^2}_{\to 0}} = f_{Ruth}(q)$$ Rutherford-Amplitude

$$\frac{d\sigma^{(BA)}}{d\Omega_c} = \left|f_c^{(BA)}\right|^2 = \frac{4m^2(Ze^2)^2}{(\hbar^2 q^2)^2} = \left(\frac{Ze^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}} = \frac{d\sigma}{d\Omega}\Big|_{Ruth}$$

→ für Coulomb: $\frac{d\sigma}{d\Omega}^{class} \equiv \frac{d\sigma}{d\Omega}^{(BA)} \equiv \frac{d\sigma}{d\Omega}^{q.m.}$ ! (Zufall)

2) endliche Ladungsverteil.

$V(\vec r) = \int d^3r'\,\rho(\vec r')\,V_c(\vec r - \vec r')$




$$f_q^{(BA)}(\Omega) = -\frac{m}{2\pi\hbar^2}\int d^3r\, e^{i\vec q\vec r}\, V(\vec r)$$

$\vec r = \vec s + \vec r'$, $V(\vec r) = \int d^3r'\,\rho(\vec r')\,V_c(\vec s)$

$$= \underbrace{\int d^3r'\, e^{i\vec q\vec r'}\rho(\vec r')}_{=:\,F(\vec q)}\ \underbrace{\int d^3r\, e^{i\vec q\vec s}\, V_c(\vec s)\left(-\frac{m}{2\pi\hbar^2}\right)}_{f_{Ruth}\ !}$$

($d^3r \to d^3s$)

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\Big|_{Ruth}\,|F(\vec q)|^2}$$

mit $F(\vec q) = \int d^3r\, e^{i\vec q\vec r}\rho(\vec r)$ Formfaktor ≙ Fouriertransf. der Ladungsverteilung

→ Messung der Ladungsverteil.!

4) Bestimmung der Ladungsverteilung

α) Inverse Fouriertransformation

$$\rho(\vec r) = \frac{1}{(2\pi)^3}\int_{\mathbb{R}^3_q} e^{-i\vec q\vec r}\, F(\vec q)\, d^3q$$

$F(\vec q)$ bekannt $\forall\,\vec q$? (d.h. Winkel, Energien)

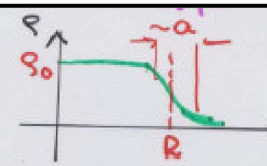
→ Fehlerband (Fourier-Bessel Analyse)

β) Ansatz für $\rho(\vec r)$ und Bestimmung der Parameter

i) Nukleon $\rho_c = \frac{a^3}{8\pi}e^{-ar} \to F(q^2) = \frac{1}{\left(1 + q^2/a^2\hbar^2\right)^2}$

experiment $a \sim 4.2$ fm → $\sqrt{\langle r^2\rangle} \simeq 0.86$ fm

ii) Kern $\rho_c(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{a}\right)}$



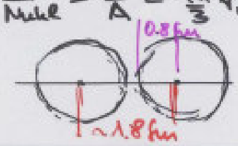

exp.:
- $R = r_0 A^{1/3}$, $r_0 \approx$ const $\simeq 1.15$ fm
- $a \approx$ const $\simeq 0.6$ fm
- $\rho_0 \approx$ const $\simeq 0.16\ \text{fm}^{-3} \simeq \frac{1}{6}\ \frac{\text{Nukl}}{\text{fm}^3}$

Flüssigkeitseigenschaften, Sättigung

$V \propto R^3 \sim A$, $a \ll R$ (außer für kleine Kerne)

$\frac{\text{Vol}}{\text{Nukl}} = \frac{V}{A} = \frac{4\pi}{3}r_0^3$ → mittl. Abstand der Nukl.

$\bar d \simeq \sqrt[3]{\frac{V}{A}} \sim 1.6 \sim 1.8$ fm

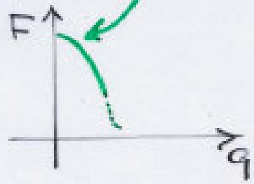



→ dichte Packung!

# Applications of BA (contd.)

5) Qualitatives Verhalten:

a) $F(\vec q) = \int d^3r\, \rho(\vec r)\, \underbrace{e^{i\vec q\cdot\vec r}}_{1 + i\vec q\vec r - \frac{1}{2}(\vec q\vec r)^2 + \dots} = 1 - \frac{q^2}{6}\langle r^2\rangle + \dots$




b)

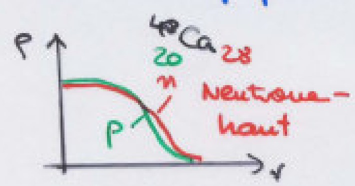

6) Elektronenstreuung mißt Ladungsverteilung $\rho_{Charge}$

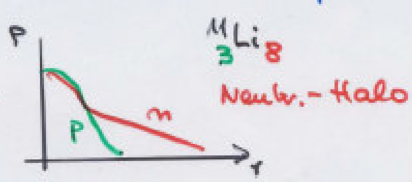
Massenverteilung?

Bei stabilen Kernen approx: $\rho_{mass} = \frac{A}{Z}\rho_{Charge}$

aber interessante Effekte insbes. bei

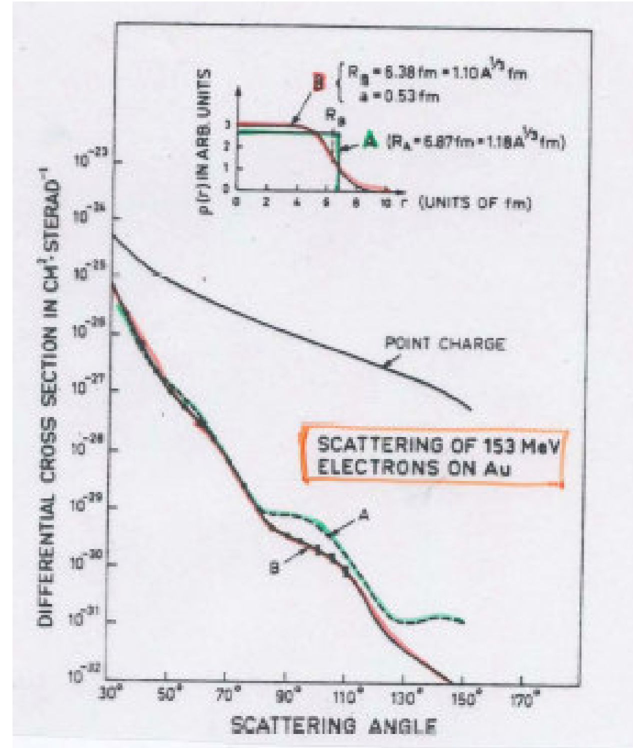
Kernen mit großem Neutronenüberschuß (exot. Kerne)




Messung $\rho_m$: Hadronstreuung: p, π, → "optisches" Pot.

schwache WW: ν ← (Vorschlag)



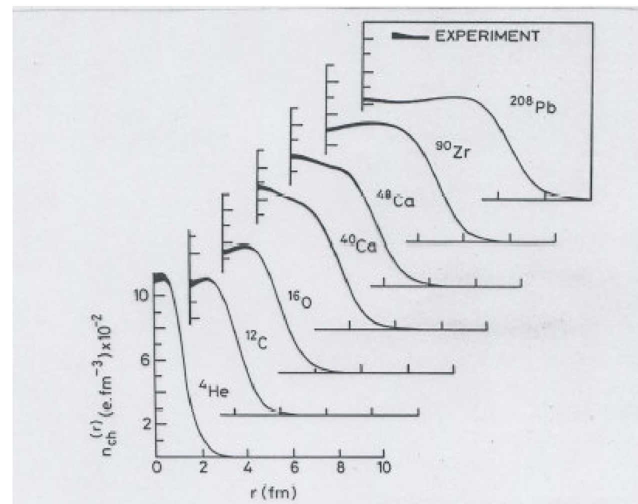




Figure 3.4 Charge densities $n_{ch}(r)$ of several nuclei measured by electron scattering. The width of the line indicates the experimental uncertainty. [*from B. Frois, Proceedings of the Niels Bohr Centennial Symposium, North-Holland, Amsterdam, 1986, p. 125.*]

# Partial waves:

- Methode zur exakten Berechn. der Streuamplitude im Sinne einer Entwicklung
- Klassifizierung der exp. Information

**Bahndrehimpuls:**

klassisch $\vec{\ell} = \vec{r} \times \vec{p}$

$\rightarrow$ q. mech. $\hat{\vec{L}} = \vec{r} \times \hat{\vec{p}}$

$[L_z, L^2] = 0$

Zentralsymm. Potential: $V(r)$

daher: $[H, L^2] = [H, L_z] = [L^2, L_z] = 0$

EF von H und EF von $L^2, L_z$, also Ansatz

$$\boxed{\psi(\vec{r}) = R_{\ell \cancel{m}}(r)\, Y_{\ell m}(\vartheta, \varphi)}$$

$Y_{lm}(\theta,\varphi)$ **spherical harmonics**

(H hängt nicht von $L_z$ ab!)

→„radial" Schödinger equation

$$\boxed{\left(\frac{1}{r}\partial_r^2 r - \frac{\ell(\ell+1)}{r^2} - U(r) + k^2\right) R_\ell(r) = 0 \; ; \; k^2 = \frac{2mE}{\hbar^2}}$$

1) Lsg für $V \equiv 0$ (freies Teilchen, kont. Spektrum)

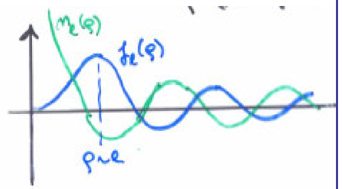
$$\left(\partial_r^2 - \frac{\ell(\ell+1)}{r^2} - \cancel{U(r)} + k^2\right) u_{\ell k}(\vec{r}) = 0$$

**Normailized solutions: Bessel, Neumann and Hankelfct.**

$$j_\ell(\rho) \xrightarrow[\rho\to\infty]{} \tfrac{1}{\rho}\sin(\rho - \tfrac{\ell\pi}{2})$$

$$n_\ell(\rho) \xrightarrow[\rho\to\infty]{} \tfrac{1}{\rho}\cos(\rho - \tfrac{\ell\pi}{2})$$

$$h_\ell^{(\pm)}(\rho) = n_\ell \pm i\, j_\ell \xrightarrow[\rho\to\infty]{} i^{\pm\ell}\frac{e^{\pm i\rho}}{\rho}$$

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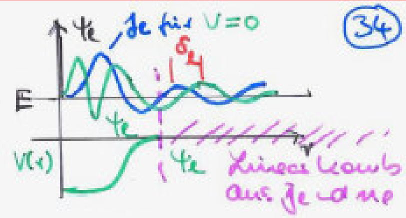
2) Lsg. für $V \neq 0$ (für kurzreichw. Pot.)

$$\psi_{k\ell m}(\vec{r}) = \varphi_\ell(\varrho)\, Y_{\ell m}(\Omega)$$




$$\varphi_\ell(\varrho) \xrightarrow[r \gg R]{} \alpha\, j_\ell(\varrho) + \beta\, n_\ell(\varrho)$$

$\rightarrow \frac{1}{\varrho}\sin(\varrho - \frac{\ell\pi}{2})$ $\quad \frac{1}{\varrho}\cos(\varrho - \frac{\ell\pi}{2})$

$\alpha^2 + \beta^2 = 1$, Parametris

$\alpha = \cos\delta_\ell \quad \beta = \sin\delta_\ell$

$$\boxed{\varphi_\ell(\varrho) \xrightarrow[r\to\infty]{} \frac{1}{\varrho}\sin\left(\varrho - \frac{\ell\pi}{2} + \delta_\ell\right)}$$

asymptotische Phasenverschiebung gegenüber Lsg mit $V=0$

$$= \frac{e^{-i\delta_\ell}}{2i}\left(h_\ell^{(-)}(\varrho) + \underbrace{e^{2i\delta_\ell}}_{=:S_\ell} h_\ell^{(+)}(\varrho)\right)$$

"Streumatrix"

Phasenverschiebung $\delta_\ell(k)$, $\ell = 0, 1, 2, \ldots \infty$

enthält asympt. Information über Wirkung des Streupotentials.

## 3) Streuamplitude $\Leftrightarrow$ Phasenverschiebung

Streuampl. entwickelbar nach Kugelfkt

$$f_k(\vartheta, \cancel{\varphi}) = \sum (2\ell+1)\, f_\ell(k)\, P_\ell(\cos\vartheta)$$

$$\boxed{f_\ell(k) = \frac{1}{k} \sin\delta_\ell\, e^{i\delta_\ell} = \frac{1}{2ik}(S_\ell - 1)}$$

## 4) Wirkungsquerschnitt

$$\underline{\frac{d\sigma}{d\Omega} = |f_k(\vartheta)|^2 = \frac{1}{k^2} \left| \sum_{\ell=0}^{\infty} (2\ell+1) \sin\delta_\ell\, e^{i\delta_\ell} P_\ell(\cos\vartheta) \right|^2}$$

$$= \frac{1}{k^2} \sum_\ell \sum_{\ell'} \cdots\cdots\cdots P_\ell(\cos\vartheta) P_{\ell'}(\cos\vartheta)$$

→ Interferenz der verschiedenen Partialwellen

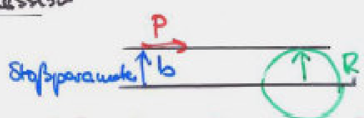
$$\sigma_{tot} \quad \int \frac{d\sigma}{d\Omega} d\Omega = (\text{Orthog. der Legendre Polynome})$$

$$\underline{= \frac{4\pi}{k^2} \sum_\ell (2\ell+1) \sin^2\delta_\ell}$$ inkohärente Überlagerung der PW

## 5) Konvergenz der PW-Entwicklung

Wieviele Drehimpulse $\ell$ tragen zur Summe bei?

klassisch




R Größe des Potentials

$$\hbar\ell = pb \leq pR \quad \rightarrow \quad \ell \leq \frac{p}{\hbar} R = kR = \sqrt{\frac{2mE}{\hbar^2}}\, R$$

q.m.


$\ell < \rho_{max} = kR$ → PW-Entwicklung konvergiert schnell für nicht zu hohe Energie

→ Bsp. NN-Streuung

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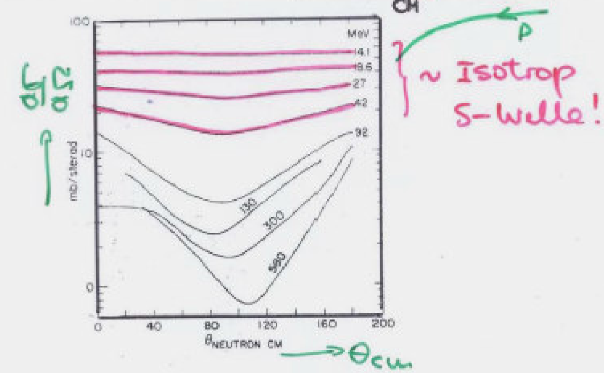
Bsp: pn-Wirkungsquerschnitt


~ Isotrop S-Welle!

Figure 2.1 Neutron-proton scattering cross sections $d\sigma/d\Omega$, as a function of the center-of-mass angle of the deflected neutron, for various laboratory energies of the incident neutron (MeV).

$\ell < \ell_{max} = 1$ für $E_{cm} \lesssim 40$ MeV (S-Welle)

$$\ell_{max} \approx R\frac{p}{\hbar} = R\sqrt{\frac{2\mu E_{cm}}{\hbar^2}} \qquad \left[E_{cm} = \frac{p^2}{2\mu},\ \mu = \frac{m}{2}\right]$$

Reichweite

$$= R\sqrt{\frac{E_{cm}}{2} \Big/ \frac{\hbar^2}{2m}} \qquad \left[\frac{\hbar^2}{2m} = \frac{\hbar^2 c^2}{2mc^2} = \frac{(200\ \text{MeV fm})^2}{2 \cdot 1000\ \text{MeV}} \simeq 20\ \text{MeV fm}^2\right]$$

$$\simeq R[\text{fm}] \sqrt{\frac{E_{cm}[\text{MeV}]}{40}}$$

→ $R \approx 1$ fm, kurzreichweitig!

## Scattering with Coulomb interaction (infinite range)

$$\left(\frac{1}{r}\partial_r^2 r - \frac{\ell(\ell+1)}{r^2} + \frac{2\eta k}{r} - U_{nuc}(r) + k^2\right)\varphi_\ell(k) = 0$$

langreichweitig, auch asympt. wirksam $\qquad \eta = \frac{mZe^2}{\hbar k}$ Coul. parameter

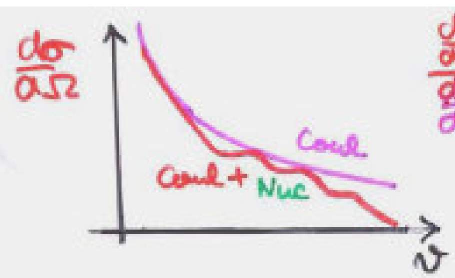
$$\varphi_\ell^{(+)} \to \frac{1}{\rho}\sin\left(\rho - \eta\ln 2\rho - \frac{\ell\pi}{2} + \sigma_\ell + \delta_\ell'\right)$$

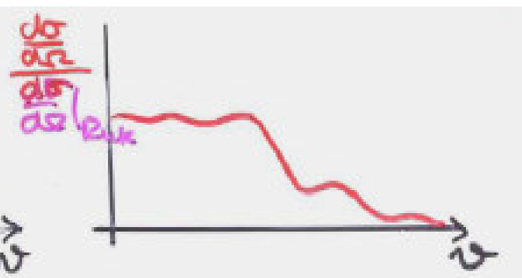
Coulomb phase shift

nuclear phase shift

Resultat:

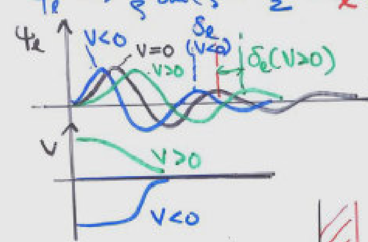
$$f_k(\Omega) = \underbrace{f_{Coul}(\Omega)}_{e^{i\sigma} f_{Ruth}(\Omega)} + \underbrace{f_{nuc}(\Omega)}_{\sum(2\ell+1)\sin\delta_\ell' e^{i\delta_\ell'} P_\ell(\cos\vartheta)}$$




## Sign of phase shift

$$\varphi_\ell \to \frac{1}{\rho}\sin\left(\rho - \frac{\ell\pi}{2} + \delta_\ell\right)$$



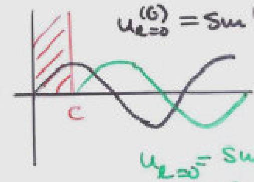

$\delta_\ell > 0$, $V<0$, attraktiv

$\delta_\ell < 0$, $V>0$, repulsiv

(modulo $\pi$!)

Bsp.: Harte Kugel ($\ell = 0$)

$\delta_0 = -kc$




$u^{(0)}_{\ell=0} = \sin kr$

$u_{\ell=0} = \sin k(r-c) = \sin(kr + \delta_0)$

## Low energy scattering: „scattering length“, E -> 0

$\delta_\ell(k) \xrightarrow[kR\ll\ell]{} (kR)^{2\ell+1}$ (s.z.B. harte Kugel)

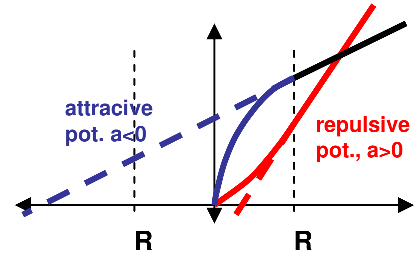
→ für $k\to 0$ trägt nur $\ell=0$ bei (s.o. Konvergenz)

→ Entwicklung $\tan\delta_0 = -ak + bk^3 + \dots \simeq \sin\delta_\ell$

→ $\sigma_{tot} \xrightarrow[kR\ll 1]{} \frac{4\pi}{k^2}\sin^2\delta_0 \approx \frac{4\pi}{k^2}a^2k^2 = 4\pi a^2$

$a \equiv$ „Streulänge“ ≙ Streu-Radius bei kleinen Energien

wave function $\quad u_{\ell=0}(r) \xrightarrow[E\to 0, r>R]{} k(r-a)$



## Reactions

Bisher Streuung ohne Änderung des inneren Zustands des Kerns: "elastisch"

Kern hat jedoch innere Freiheitsgrade. Dann

→ Streuung und innere Anregung: inelastisch } nicht-elastische

→ Streuung mit Teilchenaustausch: Transfer } Reaktion

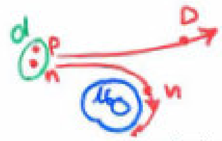
Allg. Notation:

(proj) (Target)

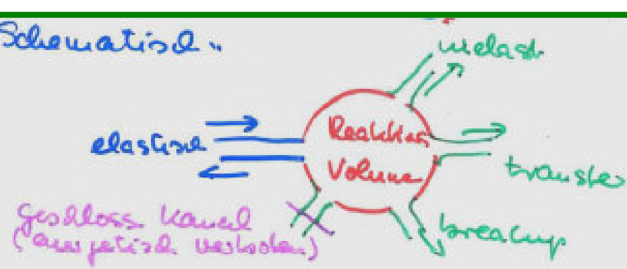
(Bsp. $d + {}^{16}O_{g.s.}$)

| | Bsp: | | | |
|---|---|---|---|---|
| $a + A \to a + A$ | $d + {}^{16}O_{g.s}$ | elastisch | | "Kanäle" |
| $\to a + A^*$ | $d + {}^{16}O(3^-)$ | inelastisch | | |
| $\to b + B$ | $p + {}^{17}O$ | n-stripping | transfer | |
| | $t + {}^{15}O$ | n-pickup | | |
| $\to b + x + A$ | $p + n + {}^{16}O$ | breakup | | |
| $\to C^*$ | ${}^{18}O^*$ | fusion | | |

zB. n-stripping




Schematisch:




Nomenklatur bei Kernreaktionen: $A(a,b)B$

Ejektil, Restkern, Target, projektil

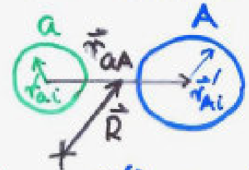
Bsp. ${}^{16}O(d,p){}^{17}O$

## Nuclear reaction theory –

## sketch in analogy to scattering theory

Partition: Aufteilung der Nukleonen auf zwei (oder mehrere) Kerne: A+a oder B+b

Aufteilung des Hamiltonop.:




$$H = \sum T_i + \sum_{i<j} V_{ij}$$

$$= \underbrace{\frac{P^2}{2M}}_{\to 0 \text{ (CM-System)}} + \underbrace{\frac{P_{aA}^2}{2\mu}}_{T_{aA} \text{ rel. kin. Energie}} + \underbrace{\left(\sum_{i\in a} \frac{P_{ai}'^2}{2m} + \sum_{i<j\in a} V_{ij}\right)}_{=: h_a \text{ (Struktur a)}} + \underbrace{\left(\sum_{i\in A} \frac{P_{Ai}'^2}{2m} + \sum_{i<j\in A} V_{ij}\right)}_{=: h_A \text{ (Struktur von A)}} + \underbrace{\sum\sum_{i\in a, j\in A} V_{ij}}_{=: V_{aA} \text{ WW}}$$

$$\to \boxed{H = T_\alpha + V_\alpha + h_\alpha}$$

$\alpha = \{a, A\}$ Partitionsindex

$h_\alpha = h_a + h_A$

entspr. $H = T_\beta + V_\beta + h_\beta$ $\quad \beta = \{b, B\}$ usw.

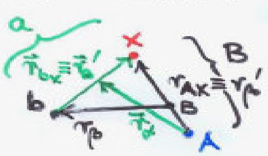
Annahme in Reaktionstheorie: Kernstruktur bekannt

$$\left.\begin{array}{l} h_a \phi_{an} = \varepsilon_{an} \phi_{an} \\ h_A \phi_{Am} = \varepsilon_{Am} \phi_{Am} \end{array}\right\} \quad h_\alpha \phi_{\alpha i} = \varepsilon_{\alpha i} \phi_{\alpha i} \qquad \begin{array}{l} \phi_{\alpha i} = \phi_{an} \phi_{Am} \\ \varepsilon_{\alpha i} = \varepsilon_{an} + \varepsilon_{Am} \end{array}$$

Ham.op. der Relativbeweg.: $H_\alpha = T_\alpha + V_\alpha(r_\alpha, \xi_\alpha)$

Koordinaten: abhängig von Partition. $\underline{\vec r_\alpha = r_{aA}}$ Part. koord.

z.B. Transfer: $\underset{(=b+x)}{a} + A \to b + \underset{(=A+x)}{B}$




innere Koord der Part. α:

$$\underbrace{\underbrace{\vec r_{bx}}_{\equiv \vec r_\alpha'} , \underbrace{\vec r_{bi}', \vec r_{Ai}', \vec r_{xi}'}_{\xi}}_{S_\alpha}$$

entspr. β.

Streufunktion:

$(H - E)\, \Psi^{(+)} = 0 \quad \leftarrow$ Streurandbeding (s.u.)

Ansatz: $\boxed{\Psi^{(+)} = \sum_\alpha \sum_i \psi_{\alpha i}(\vec r_\alpha)\, \phi_{\alpha i}(S_\alpha)}$

$\psi_{\alpha i}$: Relativfunktion; $\phi_{\alpha i}$: Kanalfkt.

$\alpha$: Partition Zustände $=: \alpha$ (Kanäle)

$$[T_\alpha - \underbrace{(E - \varepsilon_\alpha)}_{=: E_\alpha}]\, \psi_\alpha = -\sum_\beta \int ds_\alpha \underbrace{(\phi_\alpha^* V_\alpha \phi_\beta)}_{=: V_{\alpha\beta}}\, \psi_\beta(\vec r_\beta)$$

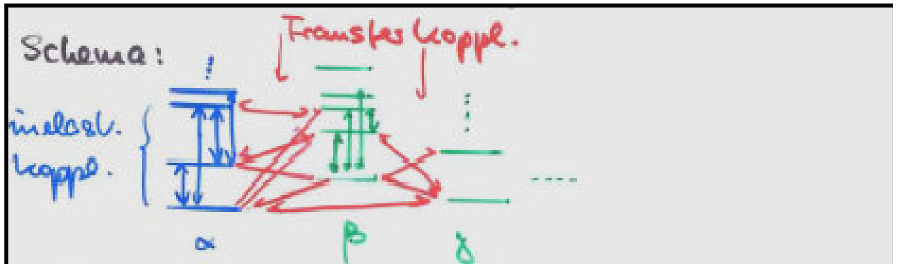
$=: V_{\alpha\beta}$ Kopplungspot. "Formfaktor"

$$\boxed{[T_\alpha - E_\alpha]\, \psi_\alpha = -\sum_\beta \int ds_\alpha V_{\alpha\beta} \psi_\beta}$$

Gekoppelte Gleichungen für Relativfunktion (Coupled Channels, CC)

(gekopp. Dgl., falls nur inelast. Kanäle
gekopp. Integro-Dgl. wenn Transferkanäle)

Schema:




Näherung: Nur bestimmte (wichtige) Kopplungen berücksichtigen!

## Reaction amplitude

(analogous to scattering amplitude in the potential scatterin)

$$\Psi^{(+)} \xrightarrow[r\to\infty]{} \underbrace{e^{ik_\alpha z_\alpha}\Phi_\alpha}_{\text{Eingangskanal}} + \sum_\beta \underbrace{f_{\alpha\beta}(\Omega_\beta)}_{\text{Reakt.-ampl.}}\frac{e^{ik_\beta r_\beta}}{r_\beta}\underbrace{\Phi_\beta(\xi_\beta)}_{\text{Ausgangskanäle}}$$

(nicht ganz unproblem. bei Breakup)

→ Diff. Wirkungsquerschnitt (beachte $v_\beta = \frac{\hbar k_\beta}{\mu_\beta}$)

$$\boxed{\frac{d\sigma}{d\Omega}_{\beta\alpha} = \left(\frac{k_\beta}{k_\alpha}\frac{\mu_\alpha}{\mu_\beta}\right)|f_{\alpha\beta}|^2}$$

**from Green function solution of the CC-equations one obtains (simlarly to the scattering)**

$$\boxed{f_{\alpha\beta} = -\frac{m}{2\pi\hbar^2}\int dz'_\alpha d\xi'_\alpha \underbrace{e^{-ik'_\alpha z'_\alpha}\Phi^*_\alpha}_{\varphi_\alpha^{(-)*}} V_\alpha \Psi^{(+)}}$$

$$f_{\alpha\beta}(\Omega) = -\frac{m}{2\pi\hbar^2}T_{\alpha\beta}(\Omega)$$

T-matrix

$$T_{\alpha\beta}(\Omega) = \left\langle \varphi_\alpha^{(-)} \middle| V_\alpha \middle| \Psi_\beta^{(+)} \right\rangle = \left\langle \Psi_\alpha^{(-)} \middle| V_\beta \middle| \varphi_\beta^{(+)} \right\rangle$$

(plane wave) Born approximation

$\varphi_\beta^{(+)}$ $\varphi_\alpha^{(-)}$

post-prior relation

better: distorted wave BA (later)

S-Matrix

$$S_{\alpha\beta} = \delta_{\alpha\beta} - 2\pi i\, T_{\alpha\beta} = \left\langle \varphi_\alpha \middle| \hat{S} \middle| \varphi_\beta \right\rangle$$

## Optical Potential:

Effective description of scattering in the presence of open channels


Alternative:

1.) Beschreibung aller Kanäle gleichzeitig; komplex. CC-Problem

2) Beschreibung mit reduzierten Kanälen

→ Flußabnahme, absorptives Potential

insbes.: Beschreibung nur elast. Kanal → "opt. Potential"

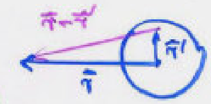
Absorptives Pot. ≡ komplexes Potential (nicht-hermitesch, nicht flußerhalt.)

$$\boxed{U^{opt} = V(r) + iW(r)}$$

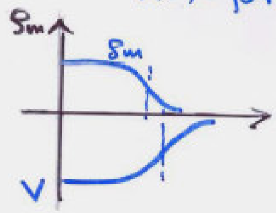
Realteil  Imag.teil

Form:

Realteil: Faltung




$$V(\vec{r}) = \int d^3r' \rho_m(\vec{r}')\, \underbrace{V^{NN}(\vec{r}-\vec{r}')}_{\text{kurzreichweitig } (\sim 1\,fm)}$$




Parametrisierung (Woods-Saxon-Pot.)

$$V(r) = V_0(E) \frac{1}{1+\exp\left(\frac{r-R_r}{a_r}\right)}$$

$R_r = r_{0r} A^{1/3}$; $r_{0r} \sim 1.2\,fm$ (größer als bei $\rho_m$)

$a_r \sim 0.65\,fm$ (etwas größer) "



Imaginärteil: nicht so klar, da abhängig vom Reaktionsmechanismus, der zu Absorption führt.

Parametrisierung:

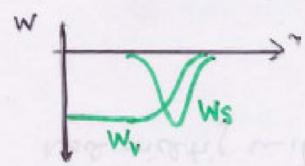
"Volumen"absorption: $W_v = \frac{W_0(E)}{1+\exp\left(\frac{r-R_i}{a_i}\right)}$

"Oberflächen"absorpt.: $W_s \sim \frac{dW_v}{dr}$

$R_i \lesssim R_r$

$a_i \simeq a_v$

$\Rightarrow W_0 < 0$ (sm)




- Mechanismus der Anregung anderer Kanäle
- abhängig von Probe (p, π, A ..)
- oft beides (z.B. p.)

Bestimmung der Parameter durch Anpassung an Daten: s. S. 45

→ Stark energieabhängig!

(→ Energieabh. von $V^{NN}$

→ Effektivität der Kopplung an andere Kanäle)

## Optical Potential:

complex potenial -> complex index of refraction

Komplexes Pot. ≡ "Optisches" Potential

≙ komplexer Brechungsindex

$$\left(-\frac{\hbar^2}{2m}\Delta + V + iW\right)\psi = i\hbar\partial_t\psi = E\psi$$

Sei V, W konstant (z.B. im Innern des Kern)

$$\psi(\vec r) \sim e^{i\vec k\vec r}$$

$$k = \sqrt{\frac{2m}{\hbar^2}(E - V - iW)} = k_r + ik_i \stackrel{!}{=} \frac{n}{c}\omega$$

Brechungsindex komplex!

Optik

(gilt auch lokal, → $k(r)$)

Dann: $|\psi|^2 = |e^{i(\vec k_r + i\vec k_i)\vec r}|^2 = e^{-2k_i r}$ Dämpfung

Dämpfungslänge ~ Reichw. $\boxed{\lambda \sim \frac{1}{2k_i}}$ Fig. 20

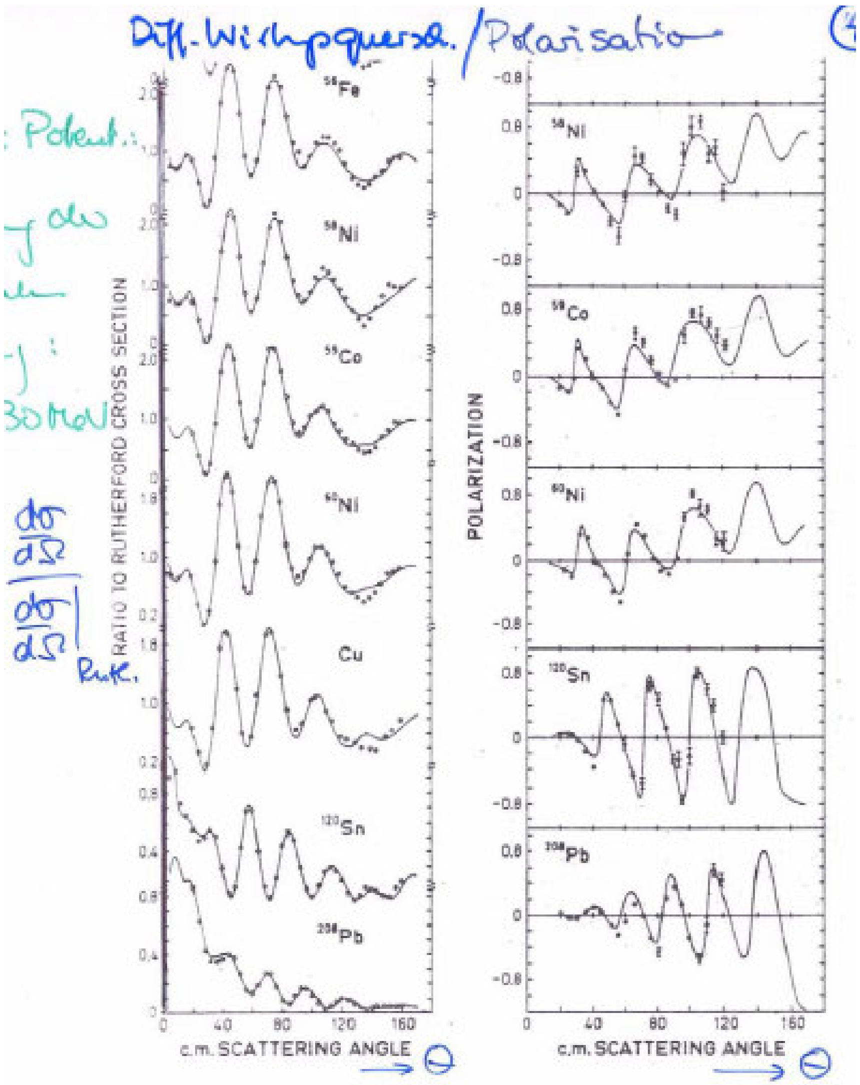
(4)

Zahlenbsp.: $E = 100$ MeV, $V \sim 20$ MeV, $W \simeq 10$ MeV

$$k = \sqrt{\frac{1}{20}(80 - 10i)}\ \mathrm{fm}^{-1} = \sqrt{4 - \tfrac{1}{2}i}\ \mathrm{fm}^{-1} = 2\left(1 - \tfrac{i}{16}\right)\mathrm{fm}^{-1}$$

$$k_i \sim \tfrac{1}{8}\ \mathrm{fm}^{-1}; \quad \lambda \sim \frac{1}{2k_i} \sim 4\ \mathrm{fm}$$

**Optical pot. fit of elastic scattering; p + A, 30 MeV**




**Energy dependence of strength of opt. Potential**

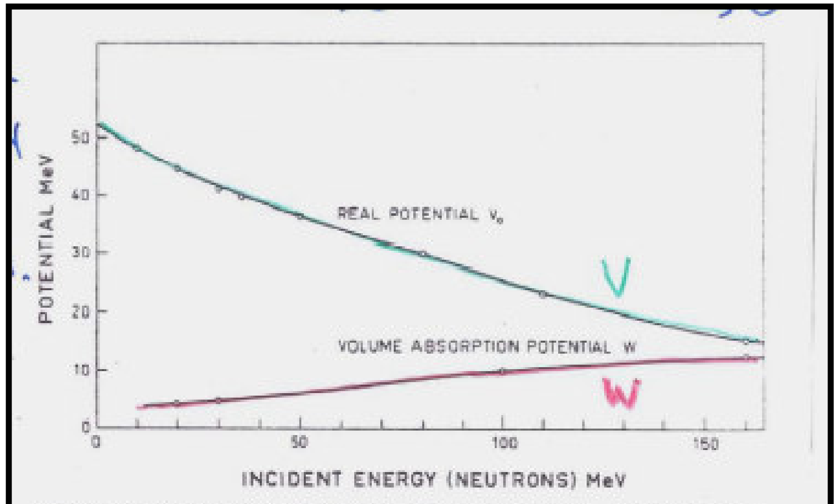



**Figure 2-29** The figure is based on the optical model analyses by L. Rosen, J. G. Beery,

Alternative view on absorption by OM:

→ **Derive continuity eq. from Schrödinger eq.**

$$\frac{d\rho}{dt} + \text{div}\,\vec{j} = \boxed{\frac{2}{\hbar} W \rho} \quad (= 0 \text{ für konst. Potential, s.S. 14})$$

$$\rho = \psi^*\psi\,, \quad \vec{j} = \frac{\hbar}{m_i}\,\text{Im}(\psi^*\nabla\psi)$$

Absorptionsrate: Integration über große Kugel

$$\underbrace{\frac{d}{dt}\int\rho\,dV}_{N} + \underbrace{\int \text{div}\,\vec{j}\,dV}_{=0 \text{ (Gauß'scher Satz)}} = \frac{2}{\hbar} W N \quad (W = \text{const})$$

$$\frac{dN}{dt} = \frac{2}{\hbar} W N$$

$$N(t) = N_0\, e^{-t/\tau}\,; \quad \tau = -\frac{\hbar}{2W} \;\rightarrow\; W<0 \rightarrow \text{Absorption!}$$

$\tau$ Absorptionszeit

Reichweite $\lambda = v\tau = \frac{\hbar v}{2|W|}$, $v = \sqrt{\frac{2E}{m}}$

$$= \frac{1}{W}\sqrt{\frac{\hbar^2}{2m}E} \approx \frac{1}{10\,\text{MeV}}\sqrt{(20\,\text{MeV fm}^2)\,100\,\text{MeV}} \sim 4.5\,\text{fm}$$

→ Reichweite (mittl. freie Weglänge) $\lambda_{opt} \sim 4$ fm

→ groß gegen $\bar{d} \sim 1.8$ fm, mittl. Abstand

→ vergleichbar mit Kerngröße $R = r_0 A^{1/3}$

Vergleich mit gaskinetischer mittl. fr. Weglänge

$\lambda_{gk} = \frac{1}{\rho\,\sigma_{tot}}$ $\quad \rho = 0.17\,\text{fm}^{-3}$

$\sigma_{tot}\big|_{100\,\text{MeV}} \sim 150\,\text{mb} = 15\,\text{fm}^2$ (freie NN-WW)

$\lambda_{gk} \sim 0.4\,\text{fm} \ll \lambda_{opt} \sim 4\,\text{fm}$ $\quad$ ? Wie kommt das?

$\sigma_{tot}^{frei} \gg \sigma_{tot}^{medium}$ (später!)

(47)

Zusammenfassend: Eigenschaften des nuklearen Mediums

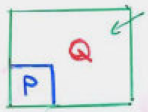
→ dicht ($\bar{d} \sim r_0$)
Sättigung ($V \sim A$)
geringe Kompressibilität, $a \ll R$ } Flüssigkeit } Fermi-Flüssigkeit

→ große mittl. fr. Weglänge: $\lambda \sim R \rightarrow$ Gas

d.h. Fermioneneigenschaften (Pauliprinzip) unterdrücken WW mit Medium.
(unten darüber mehr!)

## Remarks to the microscopic understanding of the Optical Potential:

Absorption, aber Baryonenzahl erhaltung!
→ Fluß in andere Kanäle, d.h. andere Gebiete des Hilbertraums.




$\mathcal{H}$ (Vielteilchen-Hilbertraum)

P elastische Streuung
Q "Rest", un-elastische Prozesse

Projektions methode (Feshbach) bei
Beschränkung der Wellenfunktion auf Unterraum
Projektionsop. $\hat{P}, \hat{Q}$

$$\hat{P}+\hat{Q}=1 \quad \hat{P}^2=\hat{P},\ \hat{Q}^2=\hat{Q} \quad \hat{P}\hat{Q}=\hat{Q}\hat{P}=0$$

$$\psi=(\hat{P}+\hat{Q})\psi=\psi_P+\psi_Q$$

→ Aufstellen einer Gleichung für $\psi_P$:

Projektionsformalismus: $\hat{P}+\hat{Q}=1$

$$(H-E)\psi=0$$

$$\hat{P}(H-E)(\hat{P}+\hat{Q})\psi=0$$

$$(\underbrace{PHP}_{H_{PP}}+\underbrace{PHQ}_{H_{PQ}}-EP)\psi=0$$

$H_{PP}$ (Beschränk. auf P) $H_{PQ}$ (Übergangsop.)

$$(H_{PP}-E)\psi_P=-H_{PQ}\psi_Q$$

entspr. $(H_{QQ}-E)\psi_Q=-H_{QP}\psi_P$

einsetzen

"auflösen" $\psi_Q^{(+)}=\frac{1}{E-H_{QQ}+i\varepsilon}H_{QP}\psi_P$ ← Rand Bedingung; $\varepsilon\to 0$

≙ Green-Funktion

$$\boxed{\Big(\underbrace{H_{PP}+H_{PQ}\frac{1}{E-H_{QQ}+i\varepsilon}H_{QP}}_{\text{komplex wegen } (i\varepsilon)\ \hat{=}\ \text{Opt. Potential}}-E\Big)\psi_P=0}$$

**Notes:**

1. **This s a formal expression; explicit calculation difficult, since sum over all states in Q (but possible for certain claases of states)**
2. **Opt. Potential usually determined empirically by fir to elastic scattering (as above)**
3. **Derivation of an optical potential also possible from a different perspective (Feshbach, Porter, Weißkopf, 1954): Overlapping compound nuclear resonances can be represented by the cross section of a complex potential (later)**
4. **General principle:**

   **when limiting a wavefunction to a subspace of the complete spce, one obtains an effective interaction.**

   **If the Q-space contains open channels, the effective interaction becomes complex.**

**Thank you for your interest!**

**……until the next time!**