Lectures on the Physics of the Nucleus

FZ Dresden-Rossendorf, Feb-April 2008

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0. Introduction: nucleus as a many-body system

1. Reaction theory
1.1. Scattering states, cross sections
1.2 Born approximation
1.3 Partial waves
1.4 Reactions, reaction amplitude, S-matrix
1.5 Optical potential
1.6 Distorted waves, DW Born approximation
1.7 Resonance scattering,
1.8 R-Matrix approach

review briefly

2. Compound nuclear reactions
2.1 Compound nucleus
2.2 Low-energy neutron resonances
2.3 Bohr independence hypothesis
2.4 Hauser-Feshbach theory, Wigner-Ewing
2.5 Fluctuations
2.6 Level densities

3. Pre-equilibrium reactions

Reactions

→ **mainly „direct" reactions**

However, structures in cross section of very different width

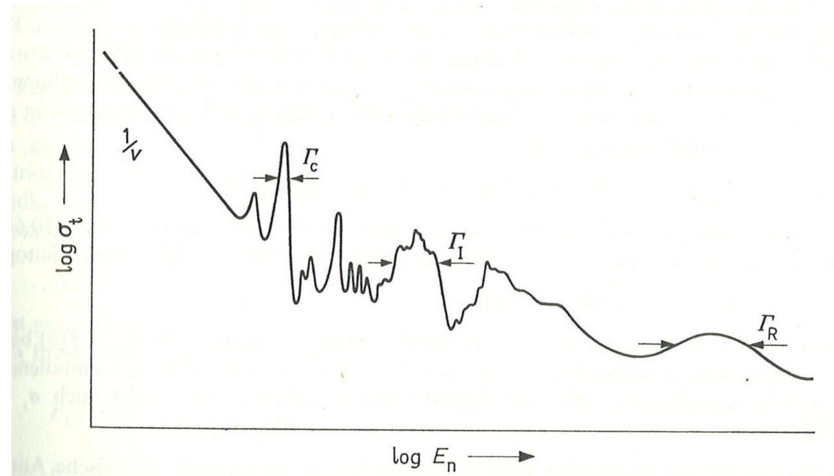
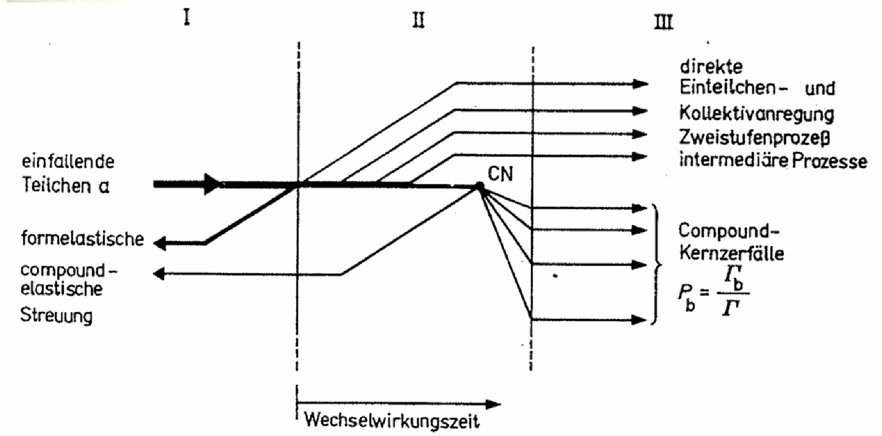


Abb. 10.8
Typischer Verlauf des totalen Wirkungsquerschnitts über einen weiten Energiebereich



Weißkopf picture

Reaction amplitude

(analogous to scattering amplitude in potential scattering)

$$\Psi^{(+)} \xrightarrow[r\to\infty]{} e^{ik_\alpha z_\alpha}\Phi_\alpha + \sum_\beta \boxed{f_{\alpha\beta}(\Omega_\beta)}\,\frac{e^{ik_\beta r_\beta}}{r_\beta}\,\Phi_\beta(\xi_\beta)$$

Eingangskanal Reakt.-ampl. Ausgangskanäle

(nicht ganz unproblem. bei Breakup)

→ Diff. Wirkungsquerschnitt (beachte $v_\beta = \frac{\hbar k_\beta}{\mu_\beta}$)

$$\boxed{\frac{d\sigma}{d\Omega}_{\beta\alpha} = \left(\frac{k_\beta}{k_\alpha}\frac{\mu_\alpha}{\mu_\beta}\right)|f_{\alpha\beta}|^2}$$

from Green function solution of the Coupled-Channels-eqs one obtains (similarly as in potential scattering)

$$\boxed{f_{\alpha\beta} = -\frac{m}{2\pi\hbar^2}\int dr_\alpha'\, d\xi_\alpha'\, \underbrace{e^{-i\vec{k}_\alpha'\vec{z}_\alpha'}\,\Phi_\alpha^*}_{\varphi_\alpha^{(-)*}}\, V_\alpha\, \Psi^{(+)}}$$

$$f_{\alpha\beta}(\Omega) = -\frac{m}{2\pi\hbar^2}T_{\alpha\beta}(\Omega)$$

T-matrix

$$T_{\alpha\beta}(\Omega) = \left\langle \varphi_\alpha^{(-)}\middle|V_\alpha\middle|\Psi_\beta^{(+)}\right\rangle = \left\langle \Psi_\alpha^{(-)}\middle|V_\beta\middle|\varphi_\beta^{(+)}\right\rangle$$

$\varphi_\beta^{(+)}$ $\varphi_\alpha^{(-)}$

(plane wave) Born approximation

post-prior relation

S-Matrix

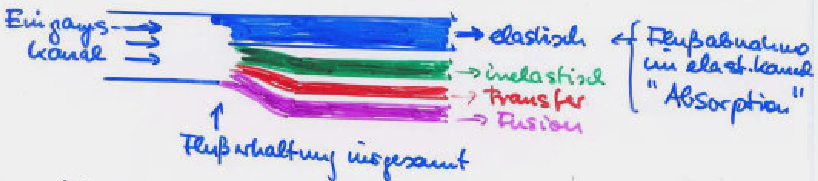
$$S_{\alpha\beta} = \delta_{\alpha\beta} - 2\pi i\, T_{\alpha\beta}$$
$$= \left\langle \varphi_\alpha\middle|\hat{S}\middle|\varphi_\beta\right\rangle$$

better:

Distorted Wave Born Approx. (DWBA) →(later)

Optical Potential:

Effective description of scattering in the presence of open channels



Alternative:

1.) Beschreibung aller Kanäle gleichzeitig; komplex. CC-Problem

2) Beschreibung mit reduzierten Kanälen

→ Flußabnahme, absorptives Potential

insbes.: Beschreibung nur elast. Kanal → "opt. Potential"

Absorptives Pot. ≙ komplexes Potential (nicht-hermitesch, nicht flußerhaltend)

$$\boxed{U^{opt} = V(r) + iW(r)}$$

Realteil   Imag. teil

Zusammenfassend: Eigenschaften des nuklearen Mediums (47)

→ dicht ($\bar{d} \sim r_0$)
Sättigung ($V \sim A$)
geringe Kompressibilität, $a \ll R$ } Flüssigkeit } Fermi-flüssigkeit

→ große mittl. fr. Weglänge: $\lambda \sim R$ → Gas

d.h. Fermioneigenschaften (Pauliprinzip) unterdrücken WW im Medium. (unterdrücken mehr!)

Consequence of complex potential: damping

1. $\psi(\vec r) \sim e^{i\vec k \vec r}$

$k = \sqrt{\frac{2m}{\hbar^2}(E - V - iW)} = k_r + i k_i \doteq \frac{n}{c}\omega$ Brechungsindex komplex! Optik

(gilt auch lokal, $\rightarrow k(r)$)

$|\psi|^2 = \left|e^{i(\vec k_r + i\vec k_i)\vec r}\right|^2 = e^{-2k_i r}$ "Dämpfung"

Dämpfungslänge ~ Reichw. $\boxed{\lambda \sim \frac{1}{2k_i}}$ Fig. 20

2. Reichweite $\lambda = v\tau = \frac{\hbar v}{2|W|}$, $v = \sqrt{\frac{2E}{m}}$

$= \frac{1}{W}\sqrt{\frac{\hbar^2}{2m}E} \simeq \frac{1}{10\,\text{MeV}}\sqrt{(20\,\text{MeV fm}^2)\,100\,\text{keV}} \sim 4.5\,\text{fm}$

→ Reichweite (mittl. freie Weglänge) $\lambda_{opt} \sim 4\,\text{fm}$

3. Vergleich mit gaskinetischer mittl. fr. Weglänge

$\lambda_{gk} = \frac{1}{\rho\,\sigma_{tot}}$ $\rho = 0.17\,\text{fm}^{-3}$

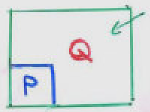
$\sigma_{tot}\big|_{100\,\text{MeV}} \sim 150\,\text{mb} = 15\,\text{fm}^2$ (freie NN-WW)

$\lambda_{gk} \sim 0.4\,\text{fm} \ll \lambda_{opt} \sim 4\,\text{fm}$? Wie kommt das?

$\sigma_{tot}^{frei} \gg \sigma_{tot}^{medium}$ (später!)

Remarks to the microscopic understanding of the Optical Potential:

Absorption, aber Baryonenzahl erhaltung!
→ Fluß in andere Kanäle, d.h. andere Gebiete des Hilbertraums.



$\mathcal{H}$ (Vielteilchen-Hilbertraum)

P elastische Streuung
Q "Rest", un-elastische Prozesse

Projektions methode (Feshbach) bei Beschränkung der Wellenfunktion auf Unterraum

Projektionsop. $\hat P, \hat Q$

$\hat P + \hat Q = 1 \quad \hat P^2 = \hat P, \ \hat Q^2 = \hat Q \quad \hat P \hat Q = \hat Q \hat P = 0$

$\psi = (\hat P + \hat Q)\psi = \psi_P + \psi_Q$

→ Aufstellen einer Gleichung für ψ_P:

Projektionsformalismus: $\hat P + \hat Q = 1$

$$(H - E)\psi = 0$$

$$\hat P (H - E)(\hat P + \hat Q)\psi = 0$$

$$(\underbrace{PHP}_{\substack{H_{PP} \\ \text{(Beschränk. auf P)}}} + \underbrace{PHQ}_{\substack{H_{PQ} \\ \text{(Übergangsop.)}}} - EP)\psi = 0$$

$$(H_{PP} - E)\psi_P = -H_{PQ}\psi_Q$$

entspr. $(H_{QQ} - E)\psi_Q = -H_{QP}\psi_P$ einsetzen

"auflösen" $\psi_Q^{(+)} = \frac{1}{E - H_{QQ} \pm i\varepsilon} H_{QP}\psi_P$ ← Randbedingung; $\varepsilon \to 0$

≙ Green-Funktion

$$\boxed{\Big(H_{PP} + \underbrace{H_{PQ}\frac{1}{E - H_{QQ} \pm i\varepsilon}}_{\text{komplex wegen } (i\varepsilon)\ \hat{=}\ \text{Opt. Potential}} H_{QP} - E\Big)\psi_P = 0}$$

Notes:

1. **This s a formal expression; explicit calculation difficult, since sum over all states in Q (but possible for certain classes of states)**

2. **General principle: when limiting a wavefunction to a subspace of the complete space, one obtains an effective interaction.**

   If the Q-space contains open channels, the effective interaction becomes complex.

3. **Optical potential is analytic as a function of energy. → derive dispersion relations, connecting real and imaginary part**

Dispersive Optical potential:

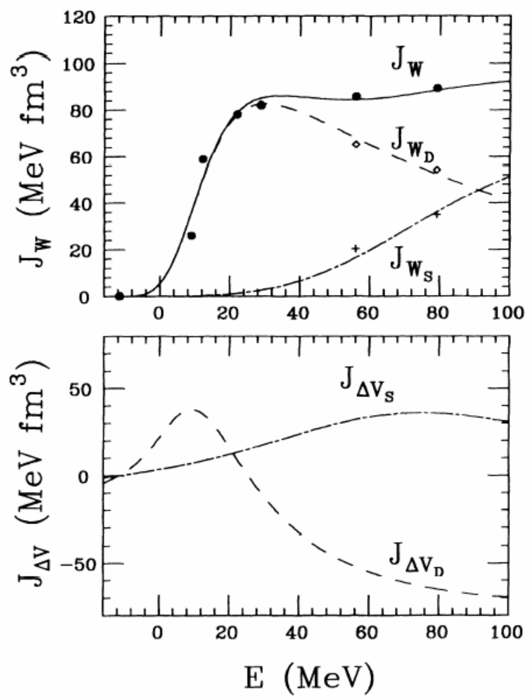
From ist microscopic derivation, the optical potential U(r;E)=V+iW is an analytic function of the energy E (conneted to causality)

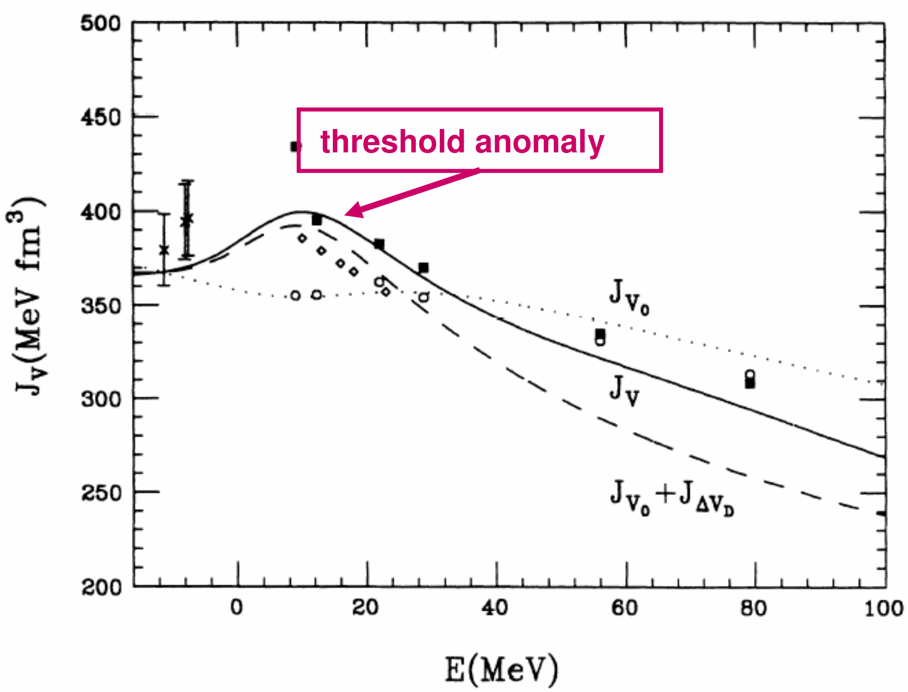
$$\boxed{U(r;E) = \frac{1}{2\pi i}\oint \frac{U(r;E')}{(E'-E)}dE'}$$

$$U_D(R,E) = V_0(r,E) + \Delta V(r,E) + iW(r,E)\ .$$

$$\Delta V(r,E) = (E_F - E)\frac{\mathrm{P}}{\pi}\int_{E_F}^{\infty}\frac{W(r,E')}{(E'-E_F)(E-E')}dE'\ .$$ **dispersion relation**

$$J_W(E) = \frac{4\pi}{A_T A_d}\int W(r,E)r^2 dr\ ,$$ **Volume integrals**





Distorted Wave Born Approximation (DWBA)

Born-Approximation für Reaktionsamplitude (S.42) verbessern durch Einführung des Opt.-Pot.

Es war (S.41)

$$(H - E)\Psi^{(+)} = 0$$

$$H = T_\alpha + h_\alpha + U_\alpha^{opt}(r_\alpha) + \underbrace{(V_\alpha - U_\alpha^{opt})}_{V_\alpha^{rest}}$$

"kleines", da Hauptanteil subtrahiert. (besser zu nähern!)

analoge Rechnung (S.41) führt auf

$$\boxed{(T_\alpha + U_\alpha^{opt} - E_\alpha)\psi_\alpha = -\sum_\beta \int ds_\alpha \left(\phi_\alpha^* (V_\alpha - U_\alpha^{opt})\phi_\beta\right)\psi_\beta}$$

Lösen mit GF-Methode wie auf S.42ff

homogene Lsg:

$$(T_\alpha + U_\alpha^{opt} - E_\alpha)\chi_\alpha^{(+)} = 0$$

$\chi_\alpha^{(+)}$: elastische Streuwelle (mit Absorpt.)

$\equiv$ "distorted wave" (relativ zu $e^{i\vec{k}\cdot\vec{r}}$)

$$\boxed{f_{\alpha\beta}^{(DWBA)} = -\frac{\mu}{2\pi\hbar^2}\int dr_\alpha\, ds_\alpha\, \chi_\alpha^{(-)*}(r_\alpha)\left(\phi_\alpha^* V_\alpha^{rest}\phi_\beta\right)\chi_\beta^{(+)}(r_\beta)}$$

Vergl.: $f_{\alpha\beta}^{(BA)} = \;\cdots\cdots\; e^{-i\vec{k}_\alpha'\vec{r}_\alpha}\;\cdots\; V_\alpha \;\cdots\; e^{i\vec{k}_\beta\vec{r}_\beta}$

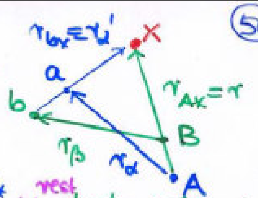
can also be derived from the T-matrix formulation discussed above

Example: Transfer reaction A(a,b)B

Transferreaktion (50)

$A(a,b)B$; $a = b+x$, $B = A+x$

z.B. (d,p)



$$f^{(DWBA)}_{\alpha\beta} = -\frac{\mu}{2\pi\hbar^2}\int d\vec r_\alpha\, d\vec r_\alpha'\, \chi_\alpha^{(-)*}(\vec r_\alpha)\left[\int d\xi_b\, d\xi_A\, d\xi_x\, \phi_a^*\phi_A^*\, V_\alpha^{rest}\, \phi_b\phi_B\right]\chi_\beta^{(+)}(\vec r_\beta)$$

$$=: F_{\alpha\beta}(\vec r_\alpha,\vec r_\beta) = (\phi_a\phi_A | V_\alpha^{rest} | \phi_b\phi_B) \quad \text{"Formfaktor"}$$

$$f^{(DWBA)}_{\alpha\beta} = -\frac{\mu}{2\pi\hbar^2}\, J \iint d\vec r_\alpha\, d\vec r_\beta\, \chi_\alpha^{(-)*}(\vec r_\alpha)\, F_{\alpha\beta}(\vec r_\alpha,\vec r_\beta)\, \chi_\beta^{(+)}(\vec r_\beta)$$

Formfaktor: enthält Kernstruktur.
→ wesentl. Methode zur Untersuchung von Kerneigensch.
(insbes. bei Transfer Einteilchen-eigenschaften)

2) Formfaktor

$$(\phi_a\phi_A | V_{bx} | \phi_b\phi_B) = \int d\xi_x \left(\int d\xi_A\, \phi_A^*\phi_B\right)\left(\int d\xi_b\, \phi_a^*\, V_{bx}\, \phi_b\right)$$

Überlappfunktionen

a) $B = A + x$

Darstellung der Wellenfunktion

$$\Phi_B = \sum_i C_i^B\, \phi_{Ai}(\xi_A)\,\phi_x(\xi_x)\,\varphi_i^B(\vec r_{Ax}) + (\text{kompliz. Konf.})$$

$$(\phi_{A(g.s.)} | \phi_B) = C_1^B\, \varphi_1^B(\vec r_{Ax})\, \phi_x \quad \text{Wellenfkt von } x \text{ in } B$$

(51)

$$\to F_{\alpha\beta}(\vec r_\alpha,\vec r_\beta) = C_1^a C_1^B\, \varphi_1^B(r_{Ax})\, V_{bx}(r_{bx})\, \varphi_1^a(r_{bx})$$

Strukturamplitude — Wellenfkt von x in a und B.

→ spektrosk. Faktor $S = |C|^2$

3) Numerisch berechnen:
- Transformation: $(r_{Ax}, r_{bx}) \to (r_\alpha, r_\beta)$; 6-dim. Integral
- Modell für: φ_1^a φ_1^B

$$\to \left.\frac{d\sigma}{d\Omega}\right|_{gemessen} = S_1^a S_1^B\, \frac{d\sigma}{d\Omega}^{(DWBA)} \leftarrow (\text{ohne Ampl. } C_1^a C_1^B)$$

↑ bestimmen

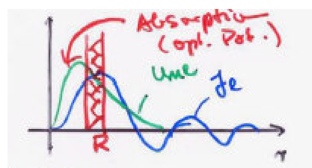
Qualitative discussion:

1. **Plane wave approx.** $\chi_\alpha^{(+)}(r_\alpha) \approx e^{i\vec k_\alpha \vec r_\alpha}$

2. **Zero-range approx** $V_{bx}^{rest}(r_{bx})\varphi_1^a(r_{bx}) \approx D_0\,\delta(r_{bx})$

3. **Let** $\varphi_1^B = u_{nl}(r)Y_{lm}(\Omega)$

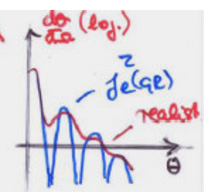
   contributions mainly from $r \approx R$



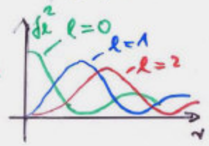
$$f_{lm}^{(ZR)} \approx C_1^A D_0\, j_l(qR)\, u_{nl}(R)\, Y_{lm}(\Omega_q)$$

$$\boxed{\frac{d\sigma}{d\Omega} \sim S_1^A\, j_l^2(qR)}$$

- Stärke: spektrosk. Faktor S_1^A
- Winkelabhängigkeit: $q \sim 2k_\alpha \sin\frac{\theta}{2}$
  → Winkelverteilung oszillierend



- Winkelverteilung sensitiv auf $l \equiv$ Drehimpuls des transf. Teilchens im Kern B
  (erstes Maximum verschiebt sich)
  → Bestimmung der l-Quantenzahl von Zuständen.



Example: Transfer reaction A(a,b)B

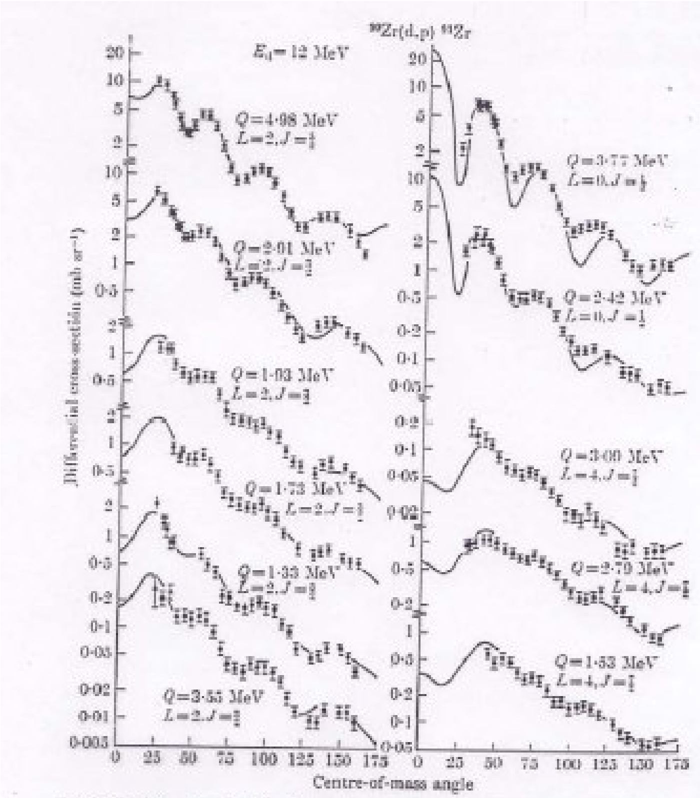
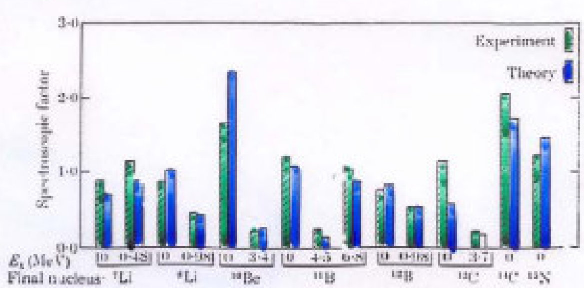


FIG. 15.10. Angular distributions for the reaction ^{90}Zr(d, p)^{91}Zr with 12-MeV deuterons, compared with DWBA calculations including finite-range and non-local corrections. (J. K. Dickens, R. M. Drisko, F. G. Perey, and G. R. Satchler, *Phys. Lett.* 15, 337, 1965.)

Spektroskop. Faktore



Qualitative discussion:

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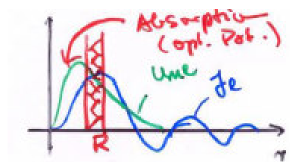
   contributions mainly from $r \approx R$



$f_{lm}^{(ZR)} \approx C_1^A D_0\; j_l(qR)\, u_{nl}(R)\, Y_{lm}^*(\Omega_q)$

$$\boxed{\frac{d\sigma}{d\Omega} \sim S_1^A\, j_l^2(qR)}$$

- Stärke: spektroskop. Faktor S_1^A
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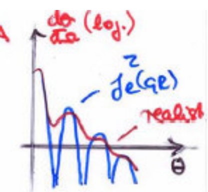

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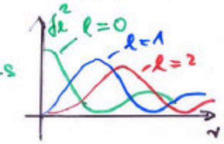


Scattering by a complex potential: Reaction cross section

Partial wave expansion of scatt. amplitude

$$f(\theta) = \frac{1}{2k}\sum^{\infty}(2l+1)i(1-\eta_l)P_l(\cos\theta),$$

Differential cross section

$$\frac{d\sigma_e}{d\Omega} = |f(\theta)|^2 = \frac{1}{4k^2}\left|\sum_{l=0}^{\infty}(2l+1)(1-\eta_l)P_l(\cos\theta)\right|^2$$

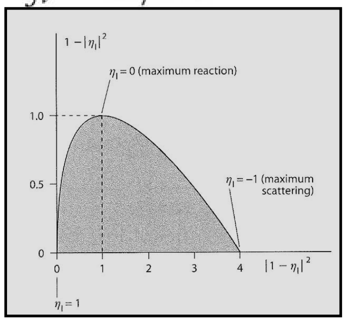
Total (elastic) cross section

$$\sigma_e = \pi\bar{\lambda}^2\sum_{l=0}^{\infty}(2l+1)|1-\eta_l|^2, \qquad \bar{\lambda} = \lambda/2\pi = 1/k.$$

Absorption (reaction) cross sect. : calculated from ingoing flux

$$j_r = -\frac{\hbar}{2im}\int\left(\Psi^*\frac{\partial\Psi}{\partial r} - \Psi\frac{\partial\Psi^*}{\partial r}\right)r^2\,d\Omega. \qquad j_i = \hbar k/m.$$

$$\sigma_r = \pi\bar{\lambda}^2\sum_{l=0}^{\infty}(2l+1)(1-|\eta_l|^2).$$



Total cross section

$$\sigma_{tot} = \sigma_{el} + \sigma_r = \tfrac{2\pi}{k^2}\sum(2\ell+1)(1-\mathrm{Re}\,\eta_\ell)$$

Consequences:

→ $\sigma_{tot} = \frac{4\pi}{k}\mathrm{Im}\,f(0)$ Optical theorem, scattered particles missing in forward scattering ampl.

→ max. reaction cross section $\eta_\ell = 0$

$$\sigma_r = \pi\bar{\lambda}^2\sum_{l=0}^{R/\bar{\lambda}}(2l+1) = \pi(R+\bar{\lambda})^2.$$

geometrical + diffraction

$$\sigma_r^{\max} = \sigma_{el}, \quad \sigma_{tot}^{\max} = 2\pi(R+\bar{\lambda})^2$$

twice!

→ Express by log. deriative:

$$f_\ell = R\left.\frac{du_\ell/dr}{u_\ell}\right|_{r=R} = f_R + if_I; \quad f_\ell^{(\mathrm{int})} = f_\ell^{(ext)} \qquad \eta_{\ell=0} = \frac{f_0+ikR}{f_0-ikR}e^{2ikR}$$

→ Cross sections:

$$\sigma_{e,0} = \frac{\pi}{k^2}\left|A_{res} + A_{pot}\right|^2 \qquad A_{res} = \boxed{\frac{-2ikR}{f_0-ikR}} \qquad A_{pot} = \boxed{e^{2ikR}-1}$$

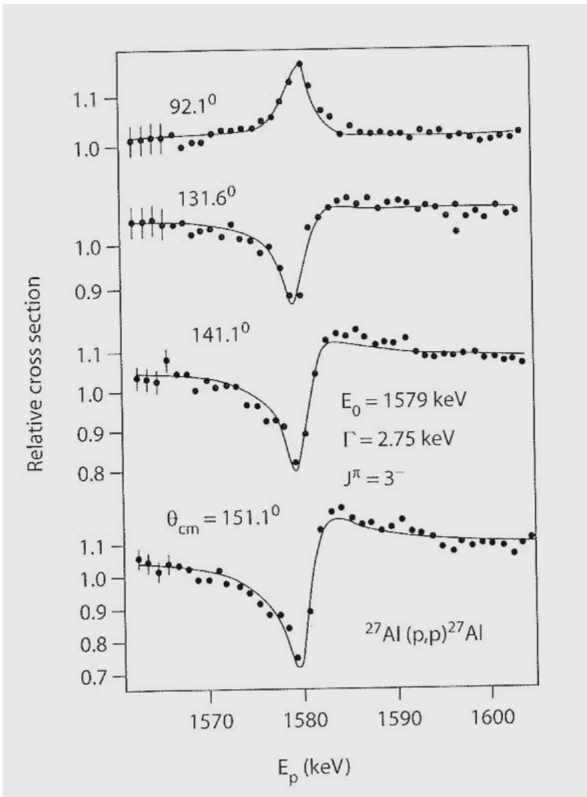
$$\sigma_{r,0} = \frac{\pi}{k^2}\frac{-4kRf_I}{f_R^{\,2} + (f_I - kR)^2}$$

Interference between resonance and potential scattering:

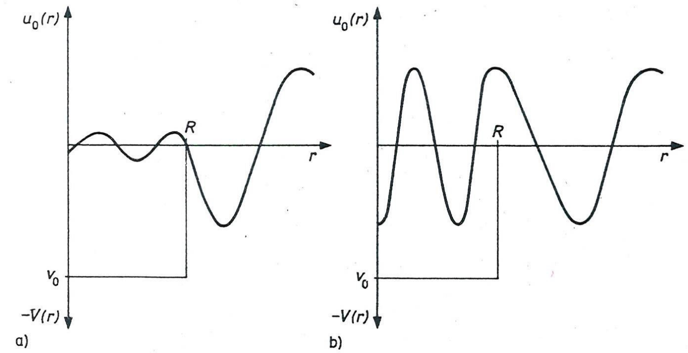
$$\sigma_{e,0} = \tfrac{\pi}{k^2}\left|A_{res} + A_{pot}\right|^2$$

$$A_{res} = \boxed{\frac{-2ikR}{f_0 - ikR}} \qquad A_{pot} = \boxed{e^{2ikR} - 1}$$

$$\sigma_{r,0} = \tfrac{\pi}{k^2}\frac{-4kRf_I}{f_R^{\,2} + (f_I - kR)^2}$$



Single particle resonances (shape resonances)



for simplicity, l=0, square well potential:

Resonance condition:
$$f_R(E=E_r) = R\frac{u_\ell'}{u_\ell}\Big|_{r=R,E=E_R} = 0$$

maximal amplitude in interior:

expansion around E_R
$$f_{0,R}(E) = 0 + (E-E_R)\frac{\partial f_0}{\partial E}\Big|_{E=E_R} + \ldots$$

then, using formulae (2.p back)

with this log. derivative

$$\sigma_{e,0} = \tfrac{\pi}{k^2}\left|e^{2ikR} - 1 + \frac{i\Gamma_\alpha}{(E-E_R)+i\Gamma/2}\right|^2 \qquad \Gamma_\alpha = -\frac{2kR}{(\partial f_R/\partial E)_{E_R}};$$

$$\sigma_{r,0} = \tfrac{\pi}{k^2}\frac{\Gamma_\alpha(\Gamma-\Gamma_\alpha)}{(E-E_R)^2 + (\Gamma/2)^2} \qquad \Gamma = \frac{2kR - 2f_I}{(\partial f_R/\partial E)_{E_R}}$$

Breit-Wigner cross section

Interpretation:

→ relation to life time:
$$\psi(t) = Ae^{(-\frac{i}{\hbar}E_0 t)}e^{-t/\tau} \xrightarrow[FT]{} |\psi(E)|^2 \approx \frac{1}{(E-E_0)^2 + (\hbar/2\tau)^2}$$

→ $\Gamma = \hbar/\tau$ total width, Γ_α entrance channel width, $\Gamma - \Gamma_\alpha = \Gamma_\beta$ exit channel width

→ alternative analysis of resonance condition:
$$\delta_\ell(E)_{E=E_R} = \pi/2; \quad \delta_\ell(E) = \pi/2 + (E-E_R)\left(\partial\delta_\ell/\partial E\right)_{E_R} + \ldots;$$

delay time

$$\Gamma_\alpha/2 = \left(\partial\delta_\ell/\partial E\right)_{E_R}^{-1}$$

→ reduced s.p.width
$$\Gamma_\alpha/2 = -\frac{k_\alpha R}{(\partial f_R/\partial E)_{E_R}} =: (k_\alpha R)\,\gamma_\alpha^2$$

R-Matrix Theory

- alternative to partial wave representation

- useful for parametrizatin of compound nucleus reactions

$$\psi = \frac{u_l(kr)}{r} Y_{\ell m}(\vartheta,\varphi);$$

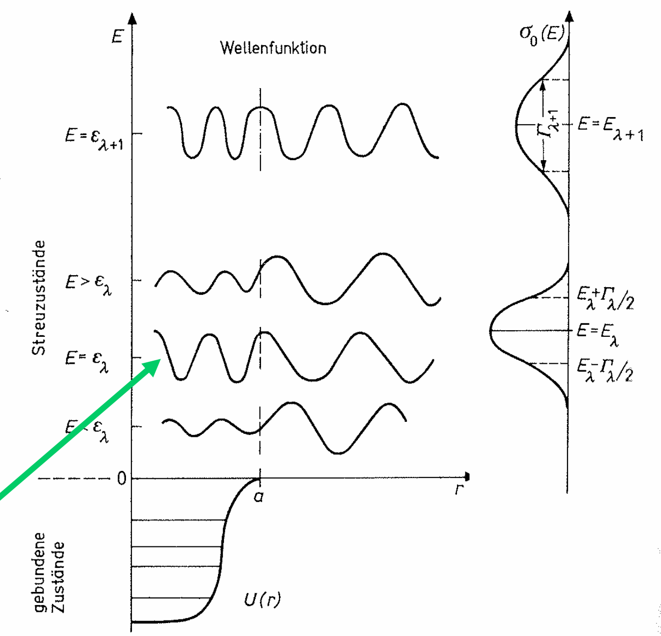
$$R_\ell := \frac{u_\ell}{r u_\ell{}'}\bigg|_{r=a}$$ R-“Matrix“

$$\xrightarrow[r>a]{} \sum_\hbar i^\ell (2\ell+1)\tfrac{1}{r}(h_\ell^{(-)} - S_\ell h_\ell^{(+)})$$

$$S_\ell = e^{2i\delta_\ell}$$

$$R_\ell \leftrightarrow S_\ell \leftrightarrow \delta_\ell = \Phi_\ell + \arctan\frac{P_\ell R_\ell}{1+Q_\ell R_\ell}$$

with $R_\ell^{-1}[h_\ell^{(\pm)}] = (Q_\ell + iP_\ell)^{(*)}$



Complete set of basis fcts. in interior space (r<a) . $\{\varepsilon_\ell, \omega_\ell(r)\}$

with $\omega_\ell(r=a)=0$

(single particle potential resonances) :

Expand any fct. u_E in set and calculate R-Matrix

$$R_\ell(E) = \sum_\lambda \frac{\gamma_\lambda^2}{\varepsilon_\ell - E}; \quad \gamma_\ell = \tfrac{\hbar}{2ma}\omega_\ell^2(a)$$

reduced s.p. width

cross section

(for single resonace):

Breit-Wigner shape

$$\sigma_{\ell=0}^{\lambda_0} = \frac{\pi}{k^2}\frac{\Gamma_{\lambda_0}{}^2}{(E-E_{\lambda_0})^2 + \frac{\Gamma_{\lambda_0}{}^2}{4}};$$

$$E_{\lambda_0} = \varepsilon_{\lambda_0} - \gamma_{\lambda_0}^2 Q_{\lambda_0}; \Gamma_{\lambda_0} = 2\gamma_{\lambda_0} P_{\lambda_0}$$

Generalization to Reactions:

Interior CC problem: $H\Psi_\lambda = E_\lambda \Psi_\lambda$

Exterior channel fct. $\varphi_c(r)$

Reduced widths

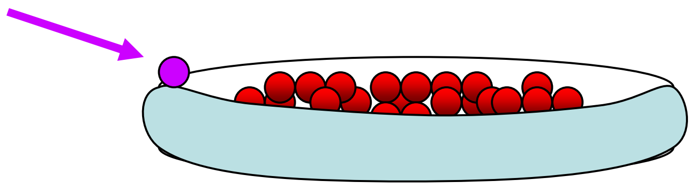
$$\gamma_{\lambda c} = \left(\frac{\hbar^2}{2\mu_\lambda a_c}\right)^{1/2} \int dF\, \varphi_c^* \Psi_\lambda$$

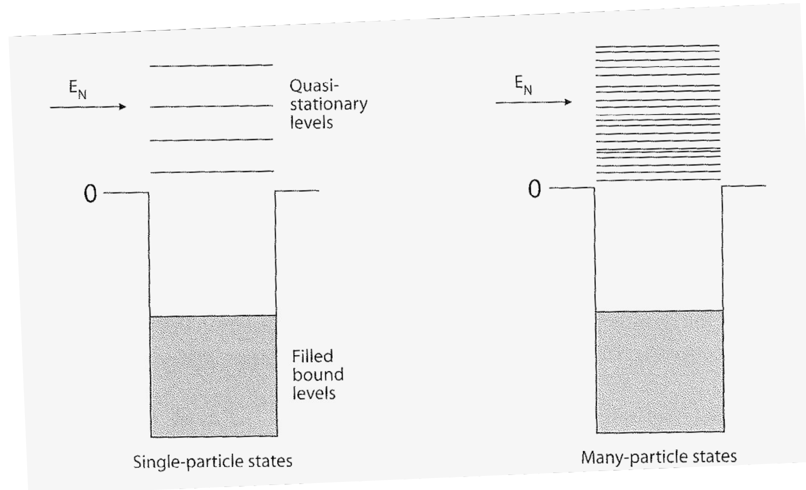
Multi-channel R-Matrix

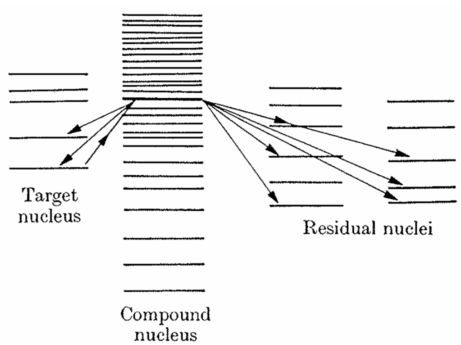
$$R_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

The Compound Nucleus

Bohr picture







.4. Energy diagram illustrating the excitation of intermediate compound-nucleus states in a nuclear reaction.

Loss of memory of incident channel (except for conserved quantities (angular mom.,parity, etc.)

Bohr independence hypothesis:

Formation and decay of CN independent

Strongly fluctuating cross sections

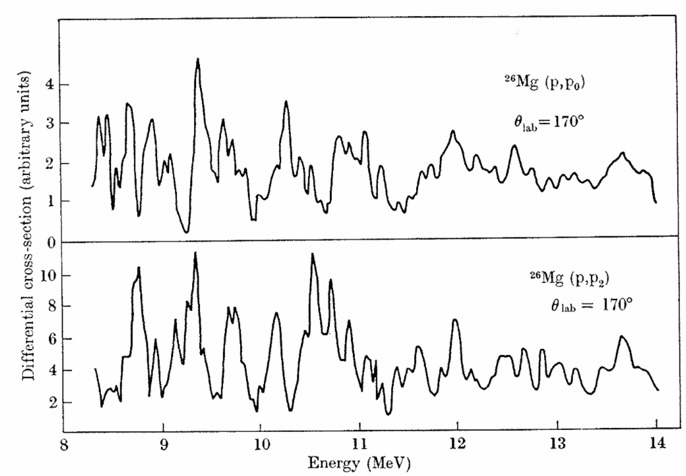
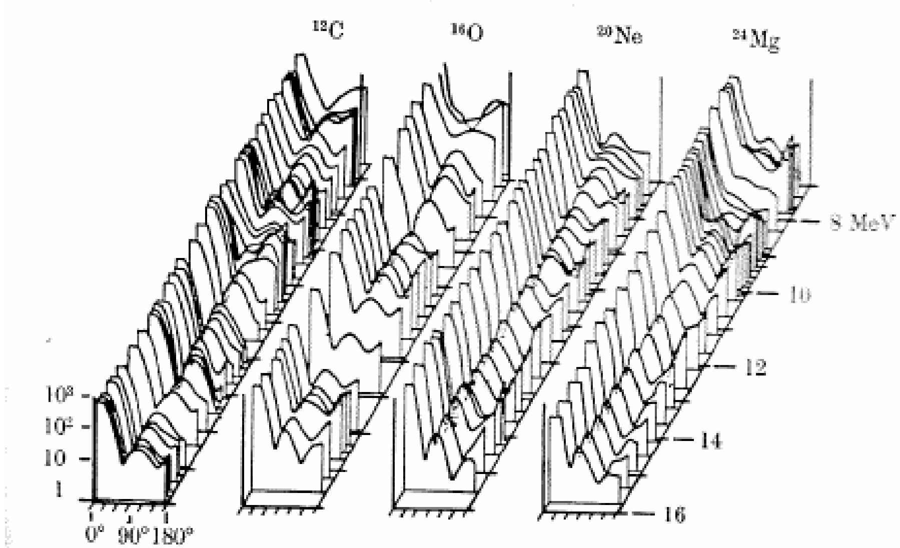


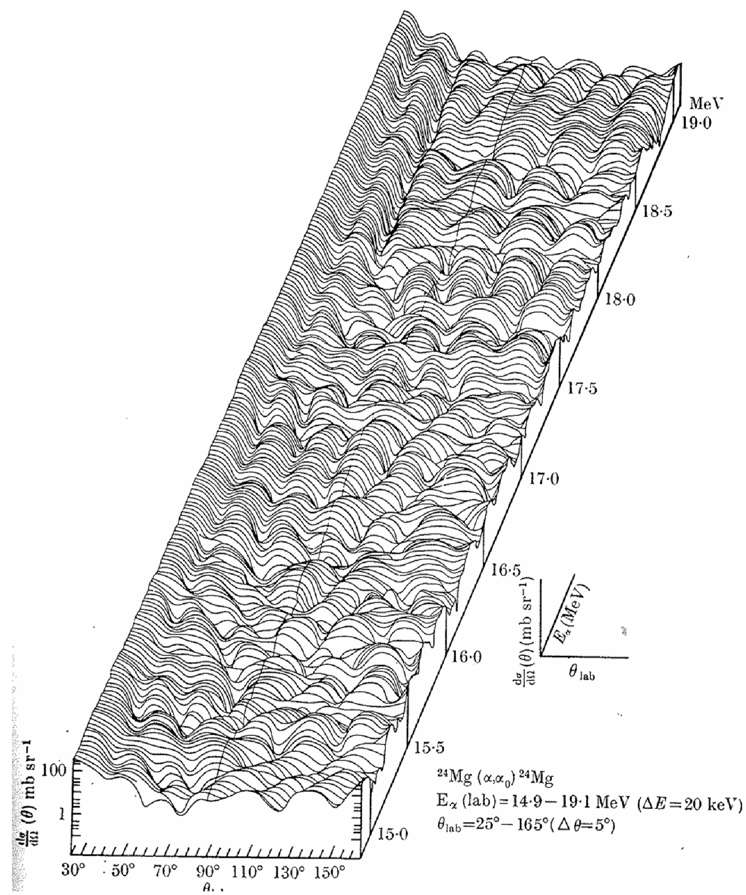
FIG. 11.8. Excitation functions for the reactions ^{26}Mg(p, p$_0$) and ^{26}Mg(p, p$_2$), showing the decrease in the amplitude of the fluctuations as the energy increases. This is due to the increasing contribution of direct processes to the reactions. (O. Häusser, P. von Brentano, and T. Mayer-Kuckuk, *Phys. Lett.* 12, 226, 1964.)

Systematic behavior of cross sections

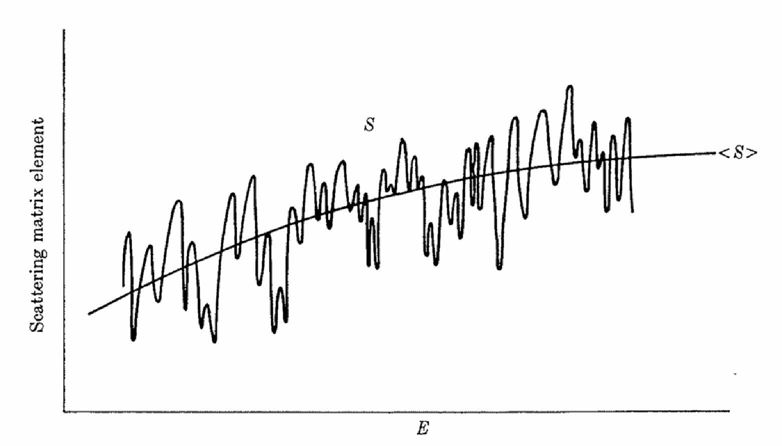
Angular differential cross sections for in a broad energy range protons on different nuclei



Angular differential cross sections alpha scattering on Mg in fine energy steps



Averaging of cross sections



general relations

$$\left.\begin{aligned} \sigma_E &= \frac{\pi}{k^2}\,|1 - S|^2, \qquad \sigma_R = \frac{\pi}{k^2}\,(1 - |S|^2) \\ \sigma_T &= \sigma_E + \sigma_R = \frac{2\pi}{k^2}\,(1 - \mathrm{Re}\,S) \end{aligned}\right\}.$$

devide into average and fluctuating part

$$S = \langle S \rangle + S_{fl}; \quad \langle S_{fl} \rangle = 0$$

the average cross section is typically given by the optical model

Average elastic, reaction and total cross sections

$$\langle \sigma_E \rangle = \frac{\pi}{k^2}\langle |1 - S|^2 \rangle = \frac{\pi}{k^2}\{|1 - \langle S \rangle|^2 - |\langle S \rangle|^2 + \langle |S|$$

$$\langle \sigma_R \rangle = \frac{\pi}{k^2}\langle (1 - |S|^2) \rangle = \frac{\pi}{k^2}(1 - \langle |S|^2 \rangle),$$

$$\langle \sigma_T \rangle = \frac{2\pi}{k^2}\langle (1 - \mathrm{Re}\,S) \rangle = \frac{2\pi}{k^2}(1 - \mathrm{Re}\,\langle S \rangle).$$

Cross sections defined by the average S-Matrix

$$\tilde{\sigma}_E = \frac{\pi}{k^2}\,|1 - \langle S \rangle|^2,$$

$$\tilde{\sigma}_R = \frac{\pi}{k^2}\,(1 - |\langle S \rangle|^2),$$

and

$$\tilde{\sigma}_T = \frac{2\pi}{k^2}\,(1 - \mathrm{Re}\,\langle S \rangle).$$

Thus

$$\langle \sigma_T \rangle = \tilde{\sigma}_T,$$

However,

sections, since

$$\langle |S|^2 \rangle \neq |\langle S \rangle|^2.$$

It is thus convenient to introduce the fluctuation cross-section

$$\sigma_{FL} = \frac{\pi}{k^2}\,(\langle |S|^2 \rangle - |\langle S \rangle|^2),$$

so that the observed energy-averaged cross-sections are related calculated from $\langle S \rangle$ by the expressions

$$\langle \sigma_E \rangle = \tilde{\sigma}_E + \sigma_{Fl},$$

$$\langle \sigma_R \rangle = \tilde{\sigma}_R - \sigma_{Fl},$$

Evaluation with CN resonances

To evaluate σ_{Fl} it is necessary to use the explicit expression for the scattering amplitude in a resonance reaction (see Chapter 14):

$$S = \mathrm{e}^{2\mathrm{i}\delta}\left(1 - \sum_S \frac{\mathrm{i}\Gamma_{\mathrm{n}}^S}{E - E_S + \frac{1}{2}\mathrm{i}\Gamma^S}\right), \tag{5.39}$$

Let the averaging over the resonances be defined by

$$\langle f(E)\rangle = \frac{1}{I}\int_{E-\frac{1}{2}I}^{E+\frac{1}{2}I} f(E)\,\mathrm{d}E \tag{5.40}$$

so that

$$\langle S\rangle = \frac{1}{I}\int_{E-\frac{1}{2}I}^{E+\frac{1}{2}I} \mathrm{e}^{2\mathrm{i}\delta}\left(1 - \sum_S \frac{\mathrm{i}\Gamma_{\mathrm{n}}^S}{E - E_S + \frac{1}{2}\mathrm{i}\Gamma^S}\right)\mathrm{d}E$$

$$= \mathrm{e}^{2\mathrm{i}\delta}\left(1 - \sum_S \frac{\pi\Gamma_{\mathrm{n}}^S}{I}\right). \tag{5.41}$$

If we define the strength function $\frac{\overline{\Gamma}_n}{D} = \frac{1}{I}\sum_S \Gamma_n^S$

$$\langle S\rangle = \mathrm{e}^{2\mathrm{i}\delta}\left(1 - \frac{\pi\overline{\Gamma}_{\mathrm{n}}}{D}\right).$$

Thus

$$\langle\sigma_{\mathrm{R}}\rangle = \frac{\pi}{k^2}\left\{1 - \left(1 - \frac{\pi\overline{\Gamma}_{\mathrm{n}}}{D}\right)^2\right\} - \sigma_{\mathrm{Fl}} = \frac{\pi}{k^2}\frac{2\pi\overline{\Gamma}_{\mathrm{n}}}{D} - \sigma_{\mathrm{Fl}},$$

since $\overline{\Gamma}_{\mathrm{n}} \ll D$.

The cross-section for the formation of a compound nucleus

$$\langle\sigma_{\mathrm{CN}}\rangle = \frac{1}{I}\sum_S \int_{E-\frac{1}{2}I}^{E+\frac{1}{2}I} \frac{\pi}{k^2}\frac{\Gamma_{\mathrm{n}}^S\Gamma^S}{(E - E_S)^2 + \frac{1}{4}\Gamma_S^2}\,\mathrm{d}E$$

$$= \frac{\pi}{k^2}\frac{2\pi}{I}\sum_S \Gamma_{\mathrm{n}}^S = \frac{\pi}{k^2}\frac{2\pi\overline{\Gamma}_{\mathrm{n}}}{D}.$$

Therefore

$$\langle\sigma_{\mathrm{R}}\rangle = \langle\sigma_{\mathrm{CN}}\rangle - \sigma_{\mathrm{Fl}}.$$

But

$$\langle\sigma_{\mathrm{R}}\rangle = \langle\sigma_{\mathrm{CN}}\rangle - \langle\sigma_{\mathrm{CE}}\rangle,$$

and therefore

$$\sigma_{\mathrm{Fl}} \equiv \langle\sigma_{\mathrm{CE}}\rangle.$$

The total energy-averaged elastic cross-section is thus the s total shape elastic and the total energy-averaged compou cross-sections,

$$\langle\sigma_{\mathrm{E}}\rangle = \tilde{\sigma}_{\mathrm{E}} + \langle\sigma_{\mathrm{CE}}\rangle.$$

$$\left\langle\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right\rangle = \frac{1}{4k^2}\langle|(1 - \langle S\rangle - \tilde{S})P_0(\cos\theta)|^2\rangle$$

$$= \frac{1}{4k^2}\{|1 - \langle S\rangle|^2 + \langle|\tilde{S}|^2\rangle\},$$

the cross-terms vanishing because $\langle\tilde{S}\rangle = 0$. Thus

$$\left\langle\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right\rangle = \frac{\mathrm{d}\sigma_{\mathrm{E}}}{\mathrm{d}\Omega} + \left\langle\frac{\mathrm{d}\sigma_{\mathrm{CE}}}{\mathrm{d}\Omega}\right\rangle.$$

… thus relation between different cross sections:

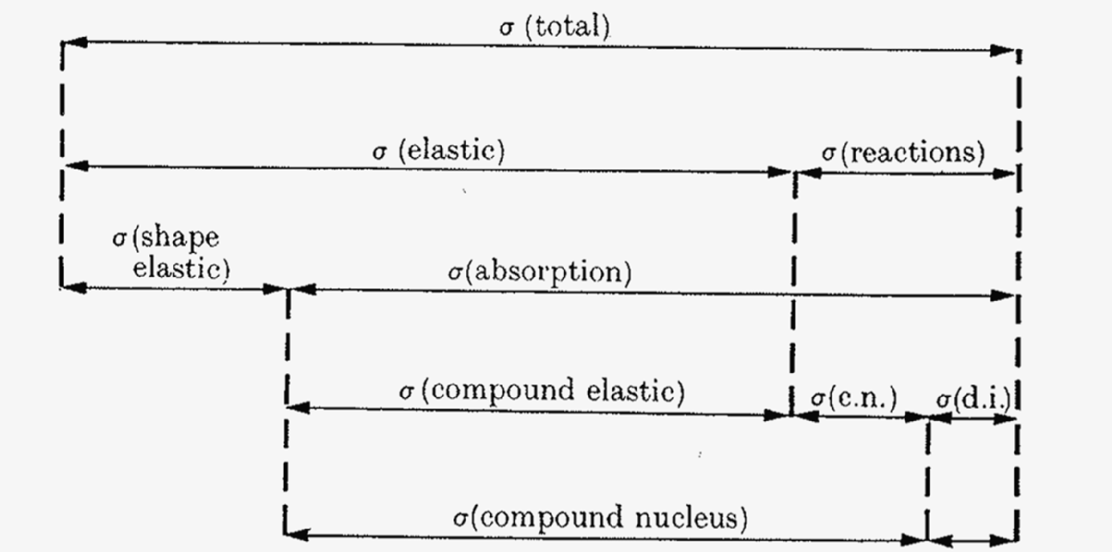


FIG. 5.7. Diagram illustrating the various components of the total cross-section.