Available online at www.sciencedirect.com
SCIENCE DIRECT®
PHYSICS LETTERS B
Physics Letters B 605 (2005) 101–105
www.elsevier.com/locate/physletb

Effect of a finite-time resolution on Schrödinger cat states in complex collisions

L. Benet [a,b], S.Yu. Kun [a,c,d], Wang Qi [e], V. Denisov [f]

[a] *Centro de Ciencias Físicas, National University of Mexico (UNAM), Cuernavaca, Mexico*
[b] *Centro Internacional de Ciencias, Cuernavaca, Mexico*
[c] *Center for Nonlinear Physics, RSPhysSE, The Australian National University, Canberra ACT 0200, Australia*
[d] *Department of Theoretical Physics, RSPhysSE, The Australian National University, Canberra ACT 0200, Australia*
[e] *Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*
[f] *Institute for Nuclear Research, 03680 Kiev, Ukraine*

Received 9 July 2004; received in revised form 13 October 2004; accepted 3 November 2004

Available online 12 November 2004

Editor: J.-P. Blaizot

Abstract

We study the effect of finite-time resolution on coherent superpositions of clockwise and anticlockwise rotating wave packets in the regime of strongly overlapping resonances of the intermediate complex. Such highly excited deformed complexes may be created in binary collisions of heavy-ions, molecules and atomic clusters. It is shown that time averaging reduces the interference fringes acting effectively as dephasing. We propose a simple estimate of the "critical" time averaging interval. For the time averaging intervals bigger than the critical one the interference fringes wash out appreciably. This is confirmed numerically. We evaluate minimal energy intervals for measurements of the excitation functions needed to observe the Schrödinger cat states. These should be easily observable in heavy-ion scattering. Such an experiment is suggested.

PACS: 03.65.-w; 25.70.Bc; 36.40.-c

Keywords: Schrödinger cat states; Wave packets; Heavy-ion scattering; Strongly overlapping resonances; Atomic clusters

In quantum mechanics a single object can be represented as a superposition of two (or more) spacially separated wave packets. When two wave packets move towards each other and overlap this can produce interference fringes in the position probability distribution. This signifies a coherent superposition of the two spacially distinct states of the same object. Such coherent superpositions are generally called "Schrödinger cat states". Schrödinger cat states have been experimentally created and detected, e.g., for photons in a microwave cavity [1], the Rydberg electron states [2],

E-mail address: benet@fis.unam.mx (L. Benet).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2004.11.005

laser-cooled trapped ions [3], and macroscopically large number of non-interacting Cooper pairs [4]. As was commented in [5], the above examples have one common feature. Namely, Schrödinger cat states [1–4] are quantum superpositions of a small number ($\leqslant 10$) of non-interacting basis states, reflecting either the single-particle nature of the system [2,3] or the absence of interaction between different degrees of freedom [1,4].

A possible realization of Schrödinger cat states for highly-excited strongly-interacting many-body systems has recently been suggested in Ref. [5]. In this proposal the possibility of formation of coherent superpositions of clockwise and anticlockwise rotating wave packets in the regime of strongly overlapping resonances of the intermediate complex has been considered. Such highly excited complexes may be created in binary collisions of heavy-ions, molecules and atomic clusters provided the double-folding potential between the collision partners has a pocket. Calculations support the existence of such pockets for heavy ion [6] and atomic cluster [7,8] systems. When the relative kinetic energy of the colliding partners dissipates into intrinsic excitations they drop into this pocket, forming a highly-excited deformed intermediate complex with strongly overlapping resonances. If the phases of the highly-mixed resonant states, corresponding to different total spin values, are uncorrelated, then the intermediate complex, at any moment of time, is isotropically oriented in angular space. However, in the presence of system specific spin off-diagonal phase correlations, the angle and time dependent probability distribution of the intermediate complex represent wave packets rotating in opposite directions. An example of such a reaction mechanism has been considered [5] for the $^{12}C + {}^{24}Mg$ collision demonstrating the appearance of interference fringes in the time and scattering angle dependent intensity of the decay, when the clockwise and anticlockwise rotating wave packets overlap. It has been evaluated [5] that Schrödinger cat states, which can be created in this collision, involve $\sim 10^4$ strongly-overlapping highly-mixed many-body configurations, each of these being a superposition of $\sim 10^5$ non-interacting basis (Fock) states.

It should be noted that stable wave packets in highly-excited many-body systems have been identified and associated with invariant manifolds in the classical phase space of the system [9,10].

It has been unambiguously demonstrated in double-slit like experiments [11] that fullerene beams do indeed represent waves. This supports the conceptual similarity in the quantum mechanical treatment of atomic cluster nanoscale collisions and heavy-ion collisions [12]. Therefore the discussion of Schrödinger cat states created in heavy-ion collisions should be relevant for a study of a possible similar phenomenon in atomic cluster collisions.

It follows from the considerations made in [5] that Schrödinger cat states manifest themselves in the time and scattering angle dependent intensity of a decay of the intermediate complex. Unlike the unimolecular reactions initiated by short laser pulses [13,14], real-time monitoring of the decay of the intermediate complex is not possible for binary collisions. However, it was shown [15,16] that measurements of the cross sections with pure energy resolution enable one to extract the information on the time and scattering angle intensity of the decay of the intermediate complex. This information is equivalent to that obtained in real-time pump/probe laser-pulse experiments, provided the relative contribution of the direct (fast) processes in the energy-averaged cross section is appreciable. Then the energy dependence of the cross section is mainly determined by interference between energy smooth direct reaction amplitude and the fluctuating one, corresponding to time delayed process. In this case, the time dependent intensity of the decay is given by the modulus square of the Fourier transform of the energy fluctuating component of the double differential cross section [16].

It can be seen from Fig. 1 of Ref. [5] that the angular period of the interference fringes is about π/I, where I is the average spin of the intermediate complex. Therefore, in order to observe the effect experimentally, one must measure the cross section with angular resolution better than π/I. Panels (f) and (g) in Fig. 1 of Ref. [5] also provide an indication for a strong time dependence of the interference fringes. Therefore an unanswered question is: what time resolution Δt is necessary to detect the effect? This question is of a crucial importance for planning the experiment, since the required time resolution determines a minimal energy interval $\Delta E = \hbar/\Delta t$ on which the cross section measurements must be carried out in order to resolve the interference fringes.

First we suggest a simple estimate for the time averaging interval Δt_{cr} obtained from the required angular resolution, $\Delta\theta \simeq \pi/I$, needed to resolve angular dependence of the interference fringes. The time it takes the intermediate complex to rotate by the angle π/I is $\pi/I\omega = T/2I$, where $T = 2\pi/\omega$ is the period of rotation and ω is the angular velocity of the intermediate system. This suggests $\Delta t_{\mathrm{cr}} \simeq T/2I$. In what follows this qualitative estimate will be tested and confirmed numerically.

Following Ref. [5], we consider spinless collision partners in the entrance and exit channels. The time and angle dependent intensity of the decay, $P(t,\theta)$, can be expressed as the Fourier component of the energy autocorrelation function of the energy fluctuating collision amplitude. $P(t,\theta)$ has been obtained [5] by summing over very large number of strongly overlapping resonance levels, $\bar{t} \ll \hbar/D$, where $\bar{t}$ is the average life time and D is the average level spacing of the intermediate complex. As a result $P(t,\theta)$ has the form:

$$P(t,\theta) \propto H(t)\exp(-t/\bar{t}) \times \sum_{JJ'}(2J+1)(2J'+1)\big[W(J)W(J')\big]^{1/2} \times \exp\big[i(\Phi-\omega t)(J-J') - \beta|J-J'|t/\hbar\big]P_J(\theta)P_{J'}(\theta). \quad (1)$$

Here $H(t)$ is the Heaviside step function, β is the spin phase-relaxation width, ω is the angular velocity of the coherent rotation, Φ is the deflection given by the total spin (J in $\hbar$ units) derivative of the potential (direct reaction) phase shifts, and the $P_J(\theta)$ are Legendre polynomials. The essential element in the derivation of (1) is the non-vanishing correlation between the energy fluctuating S-matrix elements with different, $J \neq J'$, spin values [15]: $\langle \delta S^J(E)\,\delta S^{J'}(E+\varepsilon)^*\rangle = [\langle|\delta S^J(E)|^2\rangle\langle|\delta S^{J'}(E)|^2\rangle]^{1/2}(\hbar/\bar{t})/[(\hbar/\bar{t}) + \beta|J-J'| + i\hbar\omega(J-J') - i\varepsilon]$, where the brackets stand for averaging over the energy (E). The physical meaning of the inverse spin phase-relaxation width, $\hbar/\beta$, is the characteristic time for the angular spreading of the clockwise and anticlockwise rotating wave packets [15]. This characteristic time scale also determines the time for the decay of the correlations between Fourier components of the fluctuating S-matrix elements (time-dependent collision amplitudes) corresponding to different spin values. The partial average reaction probability is taken in the J-window form, $W(J) = \langle|\delta S^J(E)|^2\rangle \propto \exp[-(J-I)^2/d^2]$, where I is the average spin and $d \ll I$ is the J-window width. This means that we assume a peripheral character of the collision.

In order to study the effect of time-averaging on the interference fringes we average $P(t,\theta)$ in (1), with a constant weight, on the time interval from $t - \Delta t/2$ to $t + \Delta t/2$. This integration is elementary. Then we calculate the time averaged $P(t,\theta)$ numerically.

Like in Ref. [5], we calculate $P(t,\theta)$ with the set of parameters obtained from the description [15] of the experimental cross section energy autocorrelation functions [17] for $^{12}C + ^{24}Mg$ elastic and inelastic scattering at $\theta = \pi$ [18]. For these collisions the analysis of the oscillations in the cross section energy autocorrelation functions [15] indicates the formation of stable rotational wave packets in spite of the strong overlap of resonance levels in the highly-excited intermediate molecule. The set of parameters is [15]: $\Phi = 0$, $d = 3$, $I = 14$, $\beta = 0.01$ MeV, $\hbar\omega = 1.35$ MeV, and $\bar{t} = 2.2 \times 10^{-21}$ s. It should be noted that the analysis of the angular distributions [18] supports a peripheral reaction mechanism, i.e., $d \ll I$, in the $^{12}C + ^{24}Mg$ elastic scattering.

In Fig. 1 we demonstrate the dependence of the quantity $A\,P(t,\theta)/\langle\sigma(E,\theta)\rangle$ on the time averaging interval at four moments of time. Here $\langle\sigma(E,\theta)\rangle \propto \int_0^\infty dt\,P(t,\theta)$ is the energy average differential cross section for the time-delayed collision. In Fig. 1, $P(t,\theta)$ is scaled with $\langle\sigma(E,\theta)\rangle$ for the reason discussed in Ref. [5]. The constant A is derived from the condition $AP(t=0,\theta=0)/\langle\sigma(E,\theta=0)\rangle = 1$, where $P(t=0,\theta=0)$ is not averaged over the time. Fig. 1(a) shows the two non-overlapping wave packets rotating in backward direction towards each other. In panels (b) and (c) of Fig. 1, the wave packets overlap around $\theta = 180^\circ$ producing interference fringes. Finally, in panel (d) of Fig. 1 the wave packets have passed each other and move apart rotating in the forward direction. Therefore, from Fig. 1 it follows that an increase of the time averaging interval results in a reduction of the amplitude of the interference fringes.

In Fig. 2 we plot the ratio, $r = P^{\Delta t}(t=T/2,\theta)/P^{\Delta t=0}(t=T/2,\theta)$, of the time averaged $P^{\Delta t}(t=T/2,\theta)$ and the $P^{\Delta t=0}(t=T/2,\theta)$ corresponding to ideal time resolution. The time averaging has been

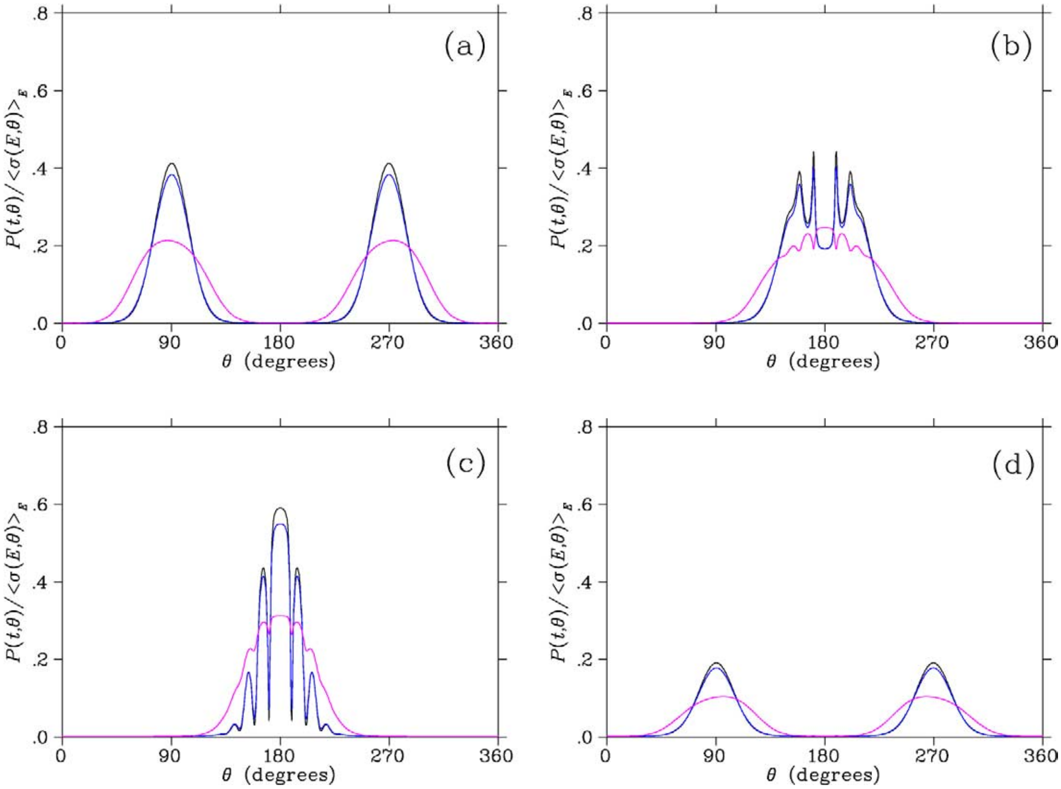

Fig. 1. Time and angular dependence of the decay intensity of the highly-excited intermediate complex for different time averaging intervals: (a) $t = T/4$; (b) $t = 7T/16$; (c) $t = T/2$; (d) $t = 3T/4$. T is the period of one complete revolution of the complex. Black lines correspond to ideal time resolution $\Delta t = 0$; blue $\Delta t = \Delta t_{cr}$; magenta $\Delta t = 5\Delta t_{cr}$. The bigger Δt the more the spread of the wave packets and the stronger reduction of the interference fringes. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

performed around $t = T/2$. At this moment, corresponding to a half of a rotational period, the two wave packets overlap around $\theta = \pi$. The constant unity corresponds to the ideal time resolution. The minima in Fig. 2 reflect the suppression of the peaks of the interference fringes due to the time averaging. One can see that this suppression is not appreciable. The maxima in Fig. 2 signify filling in of the interference minima due to the time averaging. One can see that, for $\Delta t = \Delta t_{cr} = T/2I$, the interference deeps are reduced by a factor $\leqslant 1.5$ only. However, for $\Delta t = 1.5 \times \Delta t_{cr}$, this reduction is given by a factor of about 2.5, and, for $\Delta t = 2 \times \Delta t_{cr}$, by a factor of about 3.5. Therefore, for the time averaging intervals bigger than the critical one, the interference fringes wash out fast and appreciably.

Thus, for the critical time averaging interval, the reduction of the interference maximum/minimum ratio is given by a factor of about 1.5. Since, for the ideal time resolution, the interference maximum/minimum ratio exceeds one order of magnitude (see Fig. 1), this ratio is still about a factor 6 after averaging over Δt_{cr}. In order to have the time resolution Δt_{cr} one has to measure the cross section on the energy interval $\Delta E \simeq \hbar/\Delta t_{cr} = (I/\pi)\hbar\omega$. For the above considered $^{12}C + {}^{24}Mg$ elastic and inelastic scattering [17,18] we have $I = 14$ and $\hbar\omega = 1.35$ MeV [15]. This yields $\Delta E \simeq 6$ MeV. The measurements of the $^{12}C + {}^{24}Mg$ elastic and inelastic scattering at $\theta = \pi$ [18] and the consequent extraction of the cross section energy autocorrelation functions [17] were performed on energy intervals bigger than 10 MeV and 9 MeV, respectively. Clearly, the measurements of the excitation functions on such energy intervals for many backward angles $130^\circ \leqslant \theta \leqslant 180^\circ$ with a fine angular resolution $\Delta\theta \leqslant \pi/3I \simeq 4$–5 degrees and a fine angular step would al-

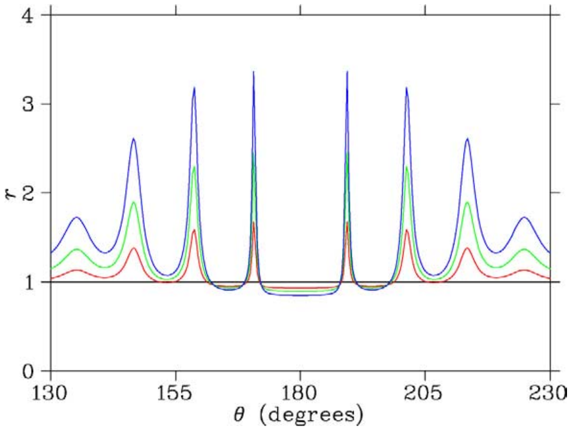

Fig. 2. Ratio $r = P^{\Delta t}(t = T/2, \theta)/P^{\Delta t=0}(t = T/2, \theta)$ of the time averaged $P^{\Delta t}(t = T/2, \theta)$ and the $P^{\Delta t=0}(t = T/2, \theta)$ corresponding to ideal time resolution. The time averaging has been performed around $t = T/2$ with T being the period of one complete revolution of the complex. Black color is for ideal time resolution $\Delta t = 0$; red $\Delta t = t_{cr}$; green $\Delta t = 3t_{cr}/2$; blue $\Delta t = 2t_{cr}$. The minima of the curves reflect suppression of the peaks of the interference fringes (see Fig. 1) due to the time averaging. The maxima signify filling in of the interference minima (see Fig. 1) due to the time averaging. The bigger the time averaging interval Δt the greater deviation from the constant unity. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

low to search for Schrödinger cat states in $^{12}\mathrm{C} + {}^{24}\mathrm{Mg}$ elastic scattering. Such an experiment would be desirable since the Schrödinger cat states predicted [5] for $^{12}\mathrm{C} + {}^{24}\mathrm{Mg}$ scattering involve $\sim 10^4$ many-body configurations of the highly excited intermediate complex. To the best of our knowledge, the internal interactive complexity of these quantum macroscopic superpositions dramatically exceeds [5] all those previously experimentally realized.

Acknowledgements

We are grateful to Francois Leyvraz and Thomas H. Seligman for useful discussions and suggestions. One of us (L.B.) acknowledges financial support from the projects IN-101603 (DGAPA-UNAM) and 43375-E (CONACyT).

References

[1] M. Brune, et al., Phys. Rev. Lett. 77 (1996) 4887.
[2] M.W. Noel, C.R. Stroud Jr., Phys. Rev. Lett. 77 (1996) 1913.
[3] C.J. Myatt, et al., Nature (London) 403 (2000) 269.
[4] J.R. Friedman, V. Patel, W. Chen, S.K. Tolpygo, J.E. Lukens, Nature (London) 406 (2000) 43.
[5] S.Yu. Kun, L. Benet, L.T. Chadderton, W. Greiner, F. Haas, Phys. Rev. C 67 (2003) 011604(R).
[6] W. Greiner, J.Y. Park, W. Scheid, Nuclear Molecules, World Scientific, Singapore, 1995.
[7] E. Engel, et al., Phys. Rev. B 48 (1993) 1862.
[8] U.R. Schmitt, E. Engel, R.M. Dreizler, Phys. Rev. B 50 (1994) 14674.
[9] T. Papenbrock, T.H. Seligman, H.A. Weidenmüller, Phys. Rev. Lett. 80 (1998) 3057.
[10] T. Papenbrock, T.H. Seligman, J. Phys. A: Math. Gen. 34 (2001) 7423.
[11] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw, A. Zeilinger, Nature 401 (1999) 680.
[12] R. Schmidt, H.O. Lutz, R. Dreizler (Eds.), Nuclear Physics Concepts in the Study of Atomic Cluster Physics, Lecture Notes in Physics, vol. 404, Springer, Heidelberg, 1992.
[13] A.H. Zewail, Femtochemistry, vols. 1–2, World Scientific, Singapore, 1995.
[14] A.H. Zewail, Nobel Lectures 1999, in: I. Grenthe (Ed.), Nobel Lectures, Chemistry 1996–2000, World Scientific, Singapore, 2003.
[15] S.Yu. Kun, A.V. Vagov, W. Greiner, Phys. Rev. C 63 (2001) 014608.
[16] S.Yu. Kun, A.V. Vagov, L.T. Chadderton, W. Greiner, Int. J. Mod. Phys. E 11 (2002) 273.
[17] B. Ghosh, R. Singh, Pramana J. Phys. 29 (1987) 1555.
[18] M.C. Mermaz, et al., Phys. Rev. C 24 (1981) 1512.