Dipole resonances in the gas-droplet model of the nucleus

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Simple relations giving the excitation energy, width, strength function, and transition densities of dipole resonances are derived in the thin diffuse nuclear surface layer approximation. The proton vibrations relative to the neutrons in the interior of the nucleus are dynamically coupled to the proton surface layer oscillations relative to the neutron surface layer by certain boundary conditions at the effective nuclear surface. The proposed model of dipole oscillations unifies the Steinwedel–Jensen (Z. Naturforsch. Teil A **5**, 413 (1950)) and Goldhaber–Teller (Phys. Rev. **74**, 1046 (1948)) models, which can be obtained from it as specific limiting cases. The model in question can easily be generalized to the case of isovector resonances of other multipole orders.

1. INTRODUCTION

The possibility of interpreting the dipole resonances on the basis of both zero[1–3] and first [4–7] sounds has been repeatedly pointed out in the literature. The direct use of the Landau kinetic equation for zero sound, the hydrodynamic equations in the case of first sound, or the microscopic or semimicroscopic equations based on the independent nucleon model is complicated by the occurrence of a relatively sharp change in the density at the nuclear boundary and the relatively small nuclear size. But it is precisely this behavior of the density which allows us to introduce an effective sharp nuclear surface fixed by the locus of the points of maximum density gradient.[8,9] The introduction of an effective surface significantly facilitates the description of the dynamics of the collective motion in the nuclear surface layer.

In the present paper we consider the isovector multipole resonances within the framework of the gas-droplet model of the nucleus[10] in the effective surface approximation. We describe the density-component dynamics in the interior of the nucleus, using either the Landau equation for zero sound[11–14] or the hydrodynamic equations. Because the density in the interior of the nucleus is practically constant, these equations are the same as the equations in an infinite medium. At the effective nuclear surface these oscillations fulfill certain boundary conditions. This method of solving the problem has been used to describe the isoscalar density oscillations[8,15] and the isoscalar current oscillations not involving density variations[16] in nuclei.

In the case of the isovector density oscillations the boundary conditions at the effective nuclear surface are especially simple. The first boundary condition ensures the equality of the mean proton (neutron) velocity in the direction of the normal to the effective surface and the mean velocity for the normal displacement of the effective proton (neutron) surface. The displacement of the effective proton surface relative to the effective neutron surface in the nuclear surface layer gives rise to a restoring force that tends to liquidate this displacement. Connected with these forces is the second boundary condition, which ensures the equality of the normal—to the effective surface—component of the restoring surface force acting on a unit nuclear surface area and the corresponding component of the stress tensor connected with the density oscillation in the interior of the nucleus. These boundary conditions differ from the boundary conditions proposed in Refs. 3–5 and 7, for the latter conditions contain other quantities.

2. RESONANCE ENERGY

Let us consider in greater detail the boundary conditions at the effective surface (ES) that describe the dynamics of the isovector density oscillations in the nuclear surface layer. As the protons oscillate relative to the neutrons, the proton and neutron ES move relative to each other. Let us denote the displacement of the proton (neutron) ES from its equilibrium position by $\zeta_{P(N)}(\mathbf{r}, t)$. The suffix P corresponds to the proton quantities; the suffix N, to the neutron quantities. Then the first boundary condition is that the normal velocity of the proton (neutron) ES should be equal to the mean normal velocity $\mathbf{v}_{P(N)}(\mathbf{r}, t)$ of the particles, i.e.,

$$(\mathbf{v}_{P(N)}(\mathbf{r},\ t)\mathbf{n})|_{\mathrm{ES}}=((d\zeta_{P(N)}(\mathbf{r},\ t)/dt)\mathbf{n})|_{\mathrm{ES}}. \tag{1}$$

The vector $\mathbf{n}$ is oriented along the normal to the ES.

The displacement of the proton ES relative to the neutron ES in the nuclear surface layer gives rise to forces that try to reestablish the equilibrium distance between the proton and neutron ES. The surface energy corresponding to these forces has the form[7,17]

$$E^{(s)}=(B^-/4\pi r_0^4)\int_{\mathrm{ES}} dS(\zeta_N(\mathbf{r},t)-\zeta_P(\mathbf{r},t))^2, \tag{2}$$

where B^- is the isovector stiffness coefficient against displacement of the surface, the nuclear radius is $R=r_0A^{1/3}$, A is the number of nucleons in the nucleus, and dS is an element of nuclear surface area. The coefficient B^- in (2) can be expressed in terms of quantities that are determined by the nuclear energy density functional (see the Appendix). The resultant force acting on a unit surface area in the direction of the normal to the surface is equal to the derivative of the deformation energy (2) with respect to the normal displacement of the proton ES relative to the neutron ES:

$$P^{(s)}=(B^-/2\pi r_0^4)((\zeta_N(\mathbf{r},\ t)-\zeta_P(\mathbf{r},\ t))\mathbf{n})|_{\mathrm{ES}}. \tag{3}$$

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This presure on the ES of the nucleus should be balanced by the corresponding component of the stress tensor $\sigma_{nn}^{-}(\mathbf{r}, t)$ connected with the isovector density oscillations in the interior of the nucleus:

$$P^{(s)}=\sigma_{nn}^{-}(\mathbf{r},\ t)\,|_{\mathrm{ES}}. \qquad (4)$$

1. *Zero sound* + ES. Let us consider the dynamics of the isovector density oscillations in the interior of the nucleus in the zero sound + ES system. In the sharp density-edge approximation the equations in the interior of the nucleus are the same as the equations in an infinite medium. Therefore, the equations for the proton $f_P(\mathbf{r}, \mathbf{p}, t)$ and neutron $f_N(\mathbf{r}, \mathbf{p}, t)$ quasiparticle distribution functions have the form[11-13]

$$\frac{\partial f_i(\mathbf{r},\mathbf{p},t)}{\partial t}+\frac{\mathbf{p}}{M^*}\nabla_r\Big[f_i(\mathbf{r},\mathbf{p},t)+\delta(\varepsilon-\varepsilon_F)$$
$$\times\frac{\pi^2\hbar^3}{p_F M^*}\int d\tau'\sum_{j=P,N}F_{ij}(\mathbf{p},\mathbf{p}')f_j(\mathbf{r},\mathbf{p}',t)\Big]=0. \qquad (5)$$

Here

$$i,\ j=P,\ N,\qquad d\tau'=2d\mathbf{p}'/(2\pi\hbar)^3,$$

M^* is the effective mass of the quasiparticles, and p_F is the limiting Fermi momentum: $F_{PP}(\mathbf{p}, \mathbf{p}')$, $F_{NN}(\mathbf{p}, \mathbf{p}')$ and $F_{PN}(\mathbf{p}, \mathbf{p}')$ are the constants in the proton–proton, neutron–neutron, and proton–neutron quasiparticle interaction amplitudes, respectively. All the interaction constants are expressed in units of $2\pi^2\hbar^3/p_F M^*$. Neglecting the Coulomb interaction, we set, in view of isotopic invariance, $F_{PP}(\mathbf{p}, \mathbf{p}') = F_{NN}(\mathbf{p}, \mathbf{p}')$. It is assumed that the Fermi energies ε_F of the proton and neutron quasiparticles are equal (consequently, the number of protons is equal to the number of neutrons), and that the nuclear temperature is equal to zero. Let us represent the constants in the quasiparticle interaction amplitudes in the form[11-13]

$$F_{ij}(\mathbf{p},\ \mathbf{p}')=F_{0\,ij}+F_{1\,ij}(\mathbf{p}\mathbf{p}')/p_F^2+\dots$$

and limit ourselves to the consideration of only the first term of this expansion. The effective mass M^* is then equal to the nucleon mass M ($M^* = M$). Adding and subtracting the equations (5), we obtain

$$\frac{\partial f^{\pm}(\mathbf{r},\mathbf{p},t)}{\partial t}+\frac{\mathbf{p}}{M}\nabla_r\left[f^{\pm}(\mathbf{r},\mathbf{p},t)+\delta(\varepsilon-\varepsilon_F)\frac{\pi^2\hbar^3}{p_F M}\left(F_0^{\pm}\int d\tau' f^{\pm}(\mathbf{r},\mathbf{p}',t)\right)\right]=0, \qquad (6)$$

where

$$f^{\pm}(\mathbf{r},\ \mathbf{p},\ t)=f_N(\mathbf{r},\ \mathbf{p},\ t)\pm f_P(\mathbf{r},\ \mathbf{p},\ t),\qquad F_i^{\pm}=F_{iPP}\pm F_{iPN}. \quad (7)$$

In the case of plane waves the solutions to Eq. (6) are obtained in Refs. 11–13:

$$f_{\mathbf{k}}^{\pm}(\mathbf{r},\ \mathbf{p},\ t)=(\alpha_N\pm\alpha_P)\delta(\varepsilon-\varepsilon_F)\nu^{\pm}(\mathbf{p},\ \mathbf{k})\exp(i(\mathbf{kr}-\omega t)), \quad (8)$$

where ω is the oscillation frequency, $\mathbf{k}$ is the wave vector, $\alpha_{P,N}$ are the amplitudes, and

$$\nu^{\pm}(\mathbf{p},\ \mathbf{k})=(\cos(\mathbf{p}\wedge\mathbf{k})/(s^{\pm}-\cos(\mathbf{p}\wedge\mathbf{k}))),$$
$$s^{\pm}=\omega/kv_F,\qquad v_F=(2\varepsilon_F/M)^{1/2}. \qquad (9)$$

Here and below $\mathbf{p}\wedge\mathbf{k}$ denotes the angle between the vectors $\mathbf{p}$ and $\mathbf{k}$. The quantities $s^{\pm}$ in (9) are determined from the equations

$$G^{-1}(s^{\pm})\equiv(s^{\pm}/2)\ln((s^{\pm}+1)/(s^{\pm}-1))-1=1/F_0^{\pm}. \qquad (10)$$

For an infinite homogeneous medium the solutions (8) are physical. In a finite medium these solutions can be regarded as a set of formal solutions to the original system of equations (5) or the equations (6). The plane-wave solution (8) constitutes a continuum of solutions with $\mathbf{k}$ vectors differing in magnitude and direction. Since the equations (5) or (6) are a set of linear homogeneous equations, we can construct a more general solution by taking a superposition of the particular solutions (8) in the form

$$f^{\pm}(\mathbf{r},\mathbf{p},t)=\int d\mathbf{k}A(\mathbf{k}\wedge\mathbf{z})f_{\mathbf{k}}^{\pm}(\mathbf{r},\mathbf{p},t). \qquad (11)$$

Here $A(\mathbf{k}\wedge\mathbf{z})$ is a weight function with the aid of which the superposition of plane waves with wave vectors $\mathbf{k}$ is constructed and $\mathbf{z}$ is the preferred direction in space. We are interested in the solution with a fixed frequency; therefore, taking account of the relation between k and ω in (9) and (10), we construct the superposition in only the space of the angles of the vector $\mathbf{k}$.

Let us further consider the isovector oscillations of a density of definite multipole order l, taking

$$\rho_l^{-}(\mathbf{r},t)=\int d\tau f^{-}(\mathbf{r},\mathbf{p},t)=R_l(r)Y_{l0}(\mathbf{r}\wedge\mathbf{z})\exp(-i\omega t), \quad (12)$$

where $R_l(r)$ is some radial function. The dipole resonances correspond to $l = 1$. For convenience of computation, we take the time dependence of the physical quantities in the form $\exp(-i\omega t)$. In this paper we consider the isovector density oscillations in nuclei with a spherical equilibrium shape, for which the generalization to the case of an angular dependence of the form $Y_{lm}(\mathbf{r}, \mathbf{z})$ with $m\neq 0$ is trivial. Substituting (8), (9), and (11) into (12), we find

$$A(\mathbf{k}\wedge\mathbf{z})=Y_{l0}(\mathbf{k}\wedge\mathbf{z}), \qquad (13)$$

$$R_l(r)=(\alpha_N-\alpha_P)\frac{2(4\pi)^2p_F M}{(2\pi\hbar)^3F_0^{-}}j_l(kr), \qquad (14)$$

where $j_l(kr)$ is the spherical Bessel function. The proton (neutron) flux velocity is equal to

$$\mathbf{v}_{P(N)}(\mathbf{r},t)=(1/M\rho_{P(N)})\int d\tau\mathbf{p}f_{P(N)}(\mathbf{r},\mathbf{p},t). \qquad (15)$$

Here

$$\rho_P=(Z/A)\bar{\rho},\qquad \rho_N=(N/A)\bar{\rho},\qquad \bar{\rho}=3A/4\pi R^3,$$

Z is the number of protons, N is the number of neutrons, and $A = Z + N$. The condition for the preservation of the position of the center of mass gives the relation

$$\int dr r Y_{10}(\mathbf{r}\wedge\mathbf{z})(\rho_{lP}(\mathbf{r},t)+\rho_{lN}(\mathbf{r},t))$$
$$+R\int_{\mathrm{ES}}dS\,Y_{10}(\mathbf{r}\wedge\mathbf{z})(\rho_P(\zeta_{lP}(\mathbf{r},t))_n+\rho_N(\zeta_{lP}(\mathbf{r},t))_n)=0. \quad (16)$$

For the oscillations of multipole order l

$$(\zeta_{lP(N)}(\mathbf{r},t)\mathbf{n})\,|_{\mathrm{ES}}=\alpha_{P(N)}^{(s)}r_0\exp(-i\omega t)Y_{l0}(\mathbf{r}\wedge\mathbf{z}), \qquad (17)$$

where $\alpha^{(s)}_{P(N)}$ is the displacement amplitude for the corresponding effective surface from the equilibrium position. Then we find from (1) with the aid of (8), (11), (13), (15), and (17) that, in the case of dipole oscillations, the amplitudes $\alpha^{(s)}_P$ and $\alpha^{(s)}_N$ are connected by the relation

$$\alpha^{(s)}_P+\alpha^{(s)}_N=0. \tag{18}$$

The protons and neutrons are strongly bound in the nucleus. Consequently, the displacement of the protons from their equilibrium position can be represented in the form of an equivalent displacement of the protons and neutrons. Therefore, let us extend the condition (18) to those other multipole orders of the density oscillations for which the condition (16) is identically fulfilled.

The stress tensor component normal to the surface is

$$\sigma^-_{lnn}(\mathbf{r},t)$$
$$=(1/M)\int d\tau\, p_n p_n f^-(\mathbf{r},\mathbf{p},t)+\bar{\rho}\,(\pi^2\hbar^3/2p_F M)\,F^-\rho_l^-(r,t). \tag{19}$$

Substituting (8), (9), (11), (15), and (17)–(19) into the boundary condition (1) and (4), we obtain a set of two linear homogeneous equations with the unknowns α_P and $\alpha^{(s)}_P$. From the solvability condition for this system we obtain the following characteristic equation for the determination of the magnitude k of the wave vector:

$$g_l(x)\equiv j_l'(x)-(3\varepsilon_F x/4B^-A^{1/3})\,[\,(1-3(s^-)^2+G(s^-))\,j_l''(x)$$
$$+(1-(s^-)^2+(2F_0^-+G(s^-))/3)\,j_l(x)\,]=0. \tag{20}$$

Notice that we are considering the zero-sound vibrations in the case when $N=Z$. For $l=1$ Eq. (20) determines the magnitude of the wave vector of the dipole resonance. In (20) a prime denotes the corresponding derivative of the function with respect to its argument $x=kR$. Let us denote the nth root of Eq. (20) with A nucleons by $x_{ln}(A)$. In a nucleus with A nucleons the isovector-resonance energy $E_{ln}(A)$ corresponding to this root is equal to

$$E_{ln}(A)=D_{ln}(A)\,A^{-1/3}, \tag{21}$$

where

$$D_{ln}(A)=\hbar(s^-v_F/R)\,A^{1/3}x_{ln}(A)=2s^-(\hbar^2\varepsilon_F/2Mr_0^2)^{1/2}x_{ln}(A), \tag{22}$$

s^- being given by Eq. (10). As an example, we show in the upper parts of Figs. 1 and 2 plots of $D_{ln}(A)$ ($0\leqslant l\leqslant 3$) computed with $F_0^-=1.6$, $\varepsilon_F=40$ MeV, $B^-=43.5$ MeV, and $r_0=1.2$ fm. The value of the isovector-stiffness coefficient B^- against surface displacement was chosen by comparing $D_{11}(A)$ with the experimental data. This value differs from the one suggested in Refs. 7 and 17, which was obtained by fitting the mass formula to the nuclear masses. But the mass-formula term connected with B^- is proportional to $(N-Z)^2/A^{4/3}$; therefore, this coefficient cannot be extracted with a good degree of accuracy by fitting the nuclear masses. The quantities $D_{11}(A)$ and $D_{21}(A)$ are in good agreement with the experimental data, which were taken from Ref. (18). In Fig. 2 we show the experimental values only for $l=2$.

2. *First sound* + ES. Let us consider the isovector nuclear excitations in the first sound + ES system. In this case

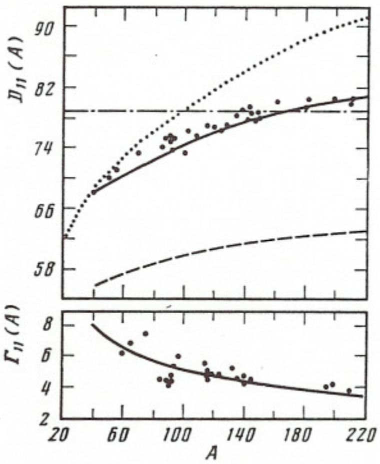

FIG. 1. Dependence of the coefficient $D_{11}(A)$ in the expression (21) for the resonance excitation energy and the resonance width $\Gamma_{11}(A)$ on the number of nucleons in the nucleus. The continuous curves indicate the values for the zero sound + ES system; the dashed curves, those for the first sound + ES system; the dot-dash curves, those for the SJ model; and the dotted curves, those for the GT model.

$\sigma^-_{l\mu\nu}(r,t)$ is diagonal, and is related to the isovector volume compressibility K^- of the nucleus and the dynamical component $\rho_l^-(\mathbf{r},t)$ of the density in the interior of the nucleus, i.e., $\sigma^-_{l\mu\nu}(\mathbf{r},t)=\delta_{\mu\nu}(K^-/9)\times\rho_l^-(\mathbf{r},t)$.

From the hydrodynamical equations for a two-component medium we obtain with the aid of the boundary conditions (1) and (4) a characteristic equation for the determination of the magnitude k of the wave vector in the first sound + ES system for a nucleus with any N and Z:

$$g_P(x)\equiv g_l'(x)-(K^-x/3B^-A^{1/3})\,(NZ/A^2)\,j_l(x)=0. \tag{23}$$

Notice that, for $N=Z$, Eq. (20) can be transformed into

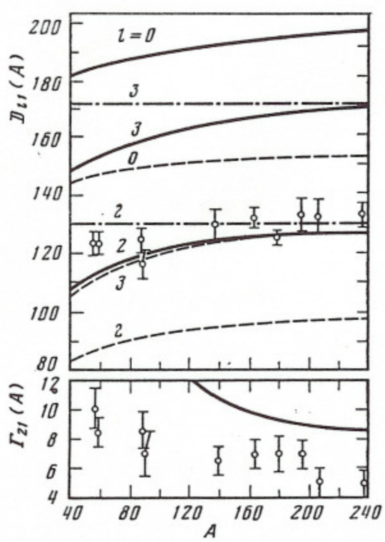

FIG. 2. Dependence of the coefficient $D_{l1}(A)$ in the expression (21) for the resonance excitation energy for $l=0, 2, 3$ (these numbers are indicated in the figure) and of the resonance width $\Gamma_{21}(A)$ on the number of nucleons in the nucleus. The designations are the same as in Fig. 1.

(23) by formally setting $\varepsilon_F = K^-/6$, $F_0^- = 0$, and $s^- = 1/\sqrt{3}$. These substitutions effect a formal transition from zero to first sound. By substituting the root of Eq. (23) into (21) and (22) [after making the indicated substitutions in (22)], we can find the resonance energies in the first sound + ES system with arbitrary N and Z.

The first sound + ES type oscillations unify the Steinwedel–Jensen (SJ) and Goldhaber–Teller (GT) models. In the SJ model the effective proton surface does not undergo displacement relative to the effective neutron surface, which corresponds to the situation in which $B^- \to \infty$ (see Ref. 5). The characteristic equation $j_l'(x) = 0$ in this model[4,5] is automatically obtained from (23) by going over to the limit $B^- \to \infty$. On the other hand, in the GT model the isovector volume compressibility is $K^- = \infty$ and B^- is finite. In this limit we obtain from (21)–(23) the dependence of the resonance energy on the number of nucleons in the nucleus. This dependence coincides up to a numerical factor $\sqrt{NZ/A^2} \simeq 1/2$ with the dependence found in Ref. 6.

For certain K^- and B^- values we can find the isovector-resonance excitation energies in the first sound + ES system in the SJ and GT models. In Figs. 1 and 2 the dependence $D_{l1}(A)$, (21), $l = 0$, 1, 2, 3, computed for the first sound + ES system with $B^- = 43.5$ MeV and $K^- = 450$ MeV, is compared with the $D_{\rm SJ}(A)$ obtained in the SJ model for $l = 1, 2$ and with the $D_{\rm GT}(A)$ obtained in the GT model. Usually in the GT model the coefficient $d_{\rm GT}$ in the dependence $D_{\rm GT}(A) = d_{\rm GT} A^{1/6}$ is chosen so as to obtain good agreement with experiment in the region of light nuclei (see Fig. 1). Then the coefficient $d_{\rm GT} \simeq 37$ MeV and differs insignificantly from the value $d_{\rm GT} \simeq 41$ MeV obtained with the aid of (21)–(23) in the appropriate limiting case.

Comparison with the experimental data shows that the zero sound + ES model agrees well with experiment in a broad range of nuclei, whereas the SJ model provides a good description of the region of heavy nuclei and the GT model describes only the region of light nuclei. Let us also note that a good description of the experimental resonance energies is also possible in the first sound + ES system. Thus, for example, for $700 \lesssim K^- \lesssim 800$ MeV and $60 \leqslant B^- \leqslant 70$ MeV the dipole-resonance excitation energies computed in the first sound + ES system differ from the experimental values by not more then 5%. These isovector volume compressibility modulus (K^-) values are significantly higher than the values $K^- \approx 450$–500 MeV obtained[5,14] from the mass formulas. But the mass formula term containing K^- is proportional to $(N-Z)^2/A$; therefore, the value of K^-, like that of B^-, cannot be determined with a high degree of accuracy by fitting the nuclear masses.

The energies of the resonances of multipole order $l \geqslant 1$ in both the zero and + ES and first sound + ES systems depend weakly on the parameters F_0^-, K^-, and B^-. Thus, a 20% change in these parameters leads to not more than a 10% change in the resonance energies. In the case of zero sound the resonance energy depends very weakly on the constant F_0^- in the quasiparticle interaction amplitude. Thus, the position of a resonance does not change by more than 10% when F_0^- is varied in the range from 0.5 to 4. The monopole resonance energies depend more strongly on these parameters.

3. THE TRANSITION DENSITIES

For small isovector-density oscillations in a nucleus with a spherical equilibrium shape the transition density can be represented in the form of a sum of volume and surface components:

$$\delta\rho_{lP(N)}(\mathbf{r}, t) = \rho_{lP(N)}(\mathbf{r}, t)\, y(r) - \rho_{P(N)}(\zeta_{lP(N)}(\mathbf{r}, t))_{\mathbf{n}}(dy(r)/dr). \tag{24}$$

Here $y(r)$ is the shape function of the density distribution about the nucleus. This function can be taken in, for example, the form[1)]

$$y(r) = (1 + \exp((r-R)/d))^{-1}, \tag{25}$$

where R and d are the nuclear radius and diffuseness, respectively. Using the boundary conditions (1) and (4), we find from (24) the radial dependence of the transition density to be the time-independent quantity

$$\begin{aligned}\delta\rho_l^-(r) &= (\delta\rho_N(\mathbf{r}, t) - \delta\rho_P(\mathbf{r}, t))/((\alpha_N - \alpha_P)\, Y_{l0}(\mathbf{r}\wedge\mathbf{z}) \\ &\times \exp(-i\omega t)) = j_l(x_{ln}(A)\, r/R)\, y(r) \\ &- j_l'(x_{ln}(A))/x_{ln}(A)\,(R\, dy(r)/dr).\end{aligned} \tag{26}$$

Here $x_{ln}(A)$ is given by (20) or (23). Figure 3 shows the $l = 0$, 1, $n = 1$ transition densities computed for a nucleus with $A = 2Z = 2N = 208$ in the zero sound + ES system from the formula (26) with the aid of the shape function (25). The shape-function parameters, $R = 6.6$ fm and $d = 0.55$ fm, for a nucleus with $A = 208$ were taken from Ref. 19. The transition density (26) for $l = 1$ is compared in Fig. 3 with the transition densities obtained in the SJ and GT models and in the random phase approximation.[20] For convenience of comparison, all the transition densities are normalized at the peaks to unity. Notice that in the SJ model the transition density is given by only the first term in (26), while in the GT model it is given by only the second term. When both terms are taken into account, the transition density contains a greater surface contribution than obtains in the SJ model and a greater volume contribution than obtains

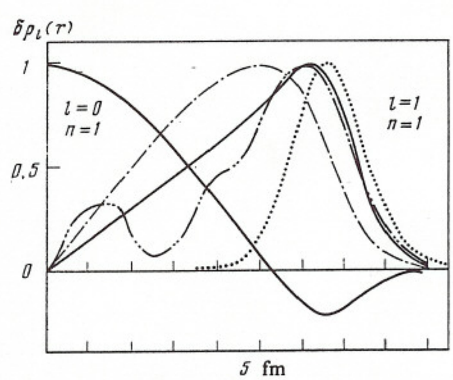

FIG. 3. Radial dependence of the transition density for $l = 0$, 1. The dot-dash curve with two dots depicts the dependence computed in the random phase approximation. The remaining designations are the same as in Fig. 1

in the GT model. The transition densities computed in the random phase approximation[20] and in the theory of finite Fermi systems[21] exhibit similar behavior.

The root of the characteristic equation (20) or (23) depends on the number A of nucleons in the nucleus; therefore, the relative contribution to the transition density from the first and second terms in (26) varies with A. Thus, as the number of nucleons in the nucleus decreases, the role of the second term in (26) becomes greater, i.e., the surface contribution to the transition density increases.

Qualitatively, the transition densities have the same radial dependence in the first sound + ES system as in the zero sound + ES system.

4. THE STRENGTH FUNCTION

To find the strength function, let us consider the response of a nucleus to an external field

$$V_{ext}^{\omega}(t)=v_{ext}^{\omega}(t)+[v_{ext}^{\omega}(t)]^{+},$$
$$v_{ext}^{\omega}(t)=\lambda(t)\hat{q}(\mathbf{r}), \quad \lambda(t)=\lambda\exp(-i(\omega+i\eta)t). \tag{27}$$

Here $\eta\to+0$, λ and ω are the amplitude and frequency, respectively, of the external field and $\hat{q}(\mathbf{r})$ is a single-particle operator, which we choose in the form of a multipole operator:

$$\hat{q}(\mathbf{r})=\sum_{i=1}^{A}q_l(\mathbf{r}_i), \quad q_l(\mathbf{r})=r^lY_{l0}(\mathbf{r}\wedge\mathbf{z})t_z, \quad l\geqslant1,$$
$$\hat{q}(\mathbf{r})=\sum_{i=1}^{A}q_0(\mathbf{r}_i), \quad q_0(\mathbf{r})=r^2t_z, \quad l=0; \tag{28}$$
$$t_z=\begin{cases}-{}^1/_2 & \text{for protons}\\ {}^1/_2 & \text{for neutrons}\end{cases}$$

The strength function $S(\omega)$ can be expressed in terms of the imaginary part of the response function[13,14]:

$$S(\omega)=(-1/\pi)\,\mathrm{Im}\,\mathfrak{R}(\omega). \tag{29}$$

The response function is connected by the simple relations[13]

$$\mathfrak{R}(\omega)=\int d\mathbf{r}\hat{q}_l(\mathbf{r})\,(\delta\rho_{lP}^{\omega}(\mathbf{r},t)+\delta\rho_{lN}^{\omega}(\mathbf{r},t))/\lambda(\omega) \tag{30}$$

with the transition densities $\delta\rho_{lP(N)}^{\omega}(\mathbf{r},t)$ in the presence of the external field (27), (28). Here and below, for simplicity of computation, we choose the external field in the form of the first term in (27).

Let us investigate in detail the response to the external field for $l\geqslant1$ in the zero sound + ES system, and give only the final expressions in the $l=0$ case. From the expressions obtained for the zero sound + ES system we can, by making the appropriate formal substitutions, go over to the expressions for the first sound + ES system.

The functions $\rho_{lP(N)}(\mathbf{r},t)$ and $\zeta_{lP(N)}(\mathbf{r},t)$ in the expression for the transition density $\delta\rho_{lP(N)}^{\omega}(\mathbf{r},t)$ contain a contribution due to the external field. In order to compute this contribution, we must find the quasiparticle distribution function in the presence of the external field (27), (28). The potential (27) produces a force field that acts on the quasiparticles. Therefore, the system's kinetic equations for the quasiparticle distribution functions contain terms connected with (27).[13] Subtracting one kinetic equation from the other, we obtain an equation for the distribution function $f_{\omega}^{-}(\mathbf{r},\mathbf{p},t)$ in the presence of an external field:

$$\frac{\partial f_{\omega}^{-}(\mathbf{r},\mathbf{p},t)}{\partial t}+\frac{\mathbf{p}}{M}\nabla_r\left[f_{\omega}^{-}(\mathbf{r},\mathbf{p},t)-\delta(\varepsilon-\varepsilon_F)\frac{\pi^2\hbar^3}{p_FM}\left(F_0^{-}\int d\tau' f_{\omega}^{-}(\mathbf{r},\mathbf{p}',t)\right)\right]=\lambda(t)\delta(\varepsilon-\varepsilon_F)\frac{\mathbf{p}}{M}\nabla_rW(\mathbf{r}), \tag{31}$$

where

$$W(\mathbf{r})=r^lY_{l0}(\mathbf{r}\wedge\mathbf{z}). \tag{32}$$

Let us, for convenience of computation, replace the multipole external field in this equation by

$$W(\mathbf{r})=\frac{(2l+1)!!}{4\pi(i\varkappa)^l}\int d\Omega_{\varkappa}\exp(i\boldsymbol{\varkappa}\mathbf{r})\,Y_{l0}(\boldsymbol{\varkappa}\wedge\mathbf{z}), \tag{33}$$

which goes over into (32) in the limit as $\varkappa r\to0$. The distribution function satisfying Eq. (31) with the external field (33) can easily be found, since the external field (33) is a superposition of plane waves with wave vectors $\boldsymbol{\varkappa}$ having different directions. Equation (31) is linear; therefore, finding its general solution for plane waves, and then constructing a superposition of the plane-wave solutions in the space of the angles of the vector $\boldsymbol{\varkappa}$ with weight $Y_{l0}(\boldsymbol{\varkappa}\wedge\mathbf{z})$, we obtain the general solution to the equation with the external field (33). The obtained solution gives in the 0 limit $\varkappa r\to$ the distribution function $f_{\omega}^{-}(\mathbf{r},\mathbf{p},t)$ with the multipole external field (32).

The homogeneous plane-wave solution to Eq. (31) was obtained above [see (8)]. The particular solution to the equation with the external field (33) has the form

$$\bar{\varphi}_{\varkappa,\omega}(\mathbf{r},\mathbf{p},t)=\frac{(2l+1)!!}{4\pi(i\varkappa)^l}\delta(\varepsilon-\varepsilon_F)\frac{\lambda(t)\exp(i\boldsymbol{\varkappa}\mathbf{r})\nu^{-}(\mathbf{p}\wedge\boldsymbol{\varkappa})}{1-F_0^{-}G^{-1}(\omega/\varkappa v_F)}. \tag{34}$$

The functions $\nu^{-}(\mathbf{p},\boldsymbol{\varkappa})$ and $G(s)$ in (34) are defined in (9) and (10). The general solution to Eq. (31) with the external field (33) has the form

$$f_{\omega}^{-}(\mathbf{r},\mathbf{p},t)=\int d\Omega_{\mathbf{k}}Y_{l0}(\mathbf{k}\wedge\mathbf{z})\,(f_{\mathbf{k}}^{-}(\mathbf{r},\mathbf{p},t)+\bar{\varphi}_{\mathbf{k},\omega}(\mathbf{r},\mathbf{p},t)). \tag{35}$$

Here the arbitrariness in the direction of the $\boldsymbol{\varkappa}$ vector allowed us to set $\mathbf{k}/k=\boldsymbol{\varkappa}/\varkappa$. Let us, using the boundary conditions (1) and (4), express the amplitudes $\alpha_{P(N)}^{(s)}$ and $\alpha_{P(N)}$ in terms of the external field amplitude λ. Then, substituting the shape function $y(r)=\theta(R-r)$ into (23), we find with the aid of (30) the response function

$$\mathfrak{R}_{lA}(\omega)=-l(\bar{\rho}R^{2l+1}/M\omega^2)[\xi_lj_l(kR)(kRg_l(kR+i\eta))^{-1}-1], \tag{36}$$

where

$$\xi_l=l[1+{}^3/_5\Delta_0(l-1)(\varepsilon_F/B^{-})A^{-1/3}],$$

and $\Delta_0=1$. Substituting (36) into (28), we obtain the strength function

$$S_{lA}(\omega)=(^3/_8\pi)(lAR^{2l-2}/M)\xi_l\sum_n \omega^{-1}(j_l(x_{ln}(A))$$
$$/(x_{ln}^2(A)|g_l'(x_{ln}(A))|))\delta(\omega-E_{ln}(A)/\hbar), \quad (37)$$

where $x_{ln}(A)$ is the nth root of Eq. (20).

Knowing the strength function, we can compute its mth moment

$$S_{lA}^{(m)}=\hbar^{m+1}\int d\omega\,\omega^m S_{lA}(\omega)=\sum_n S_{lAn}^{(m)}. \quad (38)$$

The first strength-function moment, which is related to the resonance intensity, is the most important moment in the analysis of collective excitations in nuclei. In the case of the multipole external field (28) the first moment of the strength function has a model-independent value,[5,14,20] $\overline{S}_{lA}^{(1)}$. By measuring the $S_{lAn}^{(1)}$ in units of $\overline{S}_{lA}^{(1)}$, we can find the degree of exhaustion of the model-independent energy-weighted sum rule (EWSR) by the nth resonance of multipole order $l\geqslant 1$:

$$S_{lAn}^{(1)}/\overline{S}_{lA}^{(1)}=2\xi_l j_l(x_{ln}(A))/(x_{ln}^2(A)|g_l'(x_{ln}(A))|). \quad (39)$$

Replacing the field (32) by $W(\mathbf{r})\sim\int d\Omega_\varkappa(\exp(i\varkappa\mathbf{r})-1)$, and repeating the operations that were carried out when (32) was replaced by (33) in the $l\geqslant 1$ case, we obtain for $l=0$ the response function

$$\mathfrak{R}_{0A}(\omega)=(2\bar{\rho}R^5/M\omega^2)[\xi_0 j_2(kr)/(kRg_0(kR+i\eta))+1], \quad (40)$$

the strength function

$$S_{0A}(\omega)=(^3/_2\pi)(AR^2/M)\xi_0$$
$$\times\sum_n \omega^{-1}j_2(x_{0n}(A))/(x_{0n}^2(A)|g_0'(x_{0n}(A))|)$$
$$\times\delta(\omega-E_{0n}(A)/\hbar) \quad (41)$$

and the degree of exhaustion of the model-independent EWSR by the nth resonance

$$S_{0An}^{(1)}/\overline{S}_{0A}^{(1)}=10\xi_0 j_2(x_{0n}(A))/(x_{0n}^2(A)|g_0'(x_{0n}(A))|). \quad (42)$$

For $l=0$ we have the quantity

$$\xi_0=(1+{}^3/_2(\varepsilon_F/B^-)A^{-1/3}(1+F_0^-)).$$

Setting formally $\varepsilon_F=K^-/6$, $F_0^-=0$, $S^-=1/\sqrt{3}$, and $\Delta_0=0$ in (35), (37), (39)–(42), we obtain expressions for the strength function, the response function, and the degree of exhaustion of the model-independent EWSR by the nth resonance in the first sound + ES system. Notice also that (39) yields in the limit $B^-\to\infty$ the same expression obtained in the SJ model for $S_{lAn}^{(1)}/\overline{S}_{lA}^{(1)}$ [cf. (39), after the formal substitutions have been made in it in the limit $B^-\to\infty$, with the expression (6.688) in Ref. 5].

In both the case of zero sound + ES and the case of first sound + ES oscillations, the degree of exhaustion of the model-independent EWSR by the first root of Eq. (20) or (23) for the resonances of multipole order $l\geqslant 1$ is close to unity, and depends weakly on the parameters F_0^-, B^-, and K^-. Thus, the quantity $S_{lAn}^{(1)}/\overline{S}_{lA}^{(1)}$ changes by not more than 10% when the parameters are changed by 20%. In the case of the zero sound + ES system the degree of exhaustion of the model-independent EWSR depends very weakly on the constant F_0^- in the quasiparticle interaction amplitude. Thus, $S_{lAn}^{(1)}/\overline{S}_{lA}^{(1)}$ changes by not more than 10% when F_0^- is varied from 0.5 to 4. The degree of exhaustion of the EWSR depends more strongly on these parameters in the case of the resonances of multipole order $l=0$.

Notice that the degree of exhaustion of the model-independent EWSR in the zero sound + ES system can be affected by the terms connected with the constants $F_1^\pm$ in the quasiparticle interaction amplitude.[5] But since $|F_1^\pm|\ll 1$ (Ref. 1), the contribution of these terms to $S_{lAn}^{(1)}/\overline{S}_{lA}^{(1)}$ is small compared to the leading terms (39) and (42). The effect of $F_1^\pm$ on the resonance excitation energies in the zero sound + ES system can also be neglected (see also Ref. 15, where the weak effect of F_1^+ on the excitation energies of the isoscalar resonances is demonstrated).

In Table I we present the degrees of exhaustion, computed with the same parameter values used in the computation of the excitation energies, of the model-independent EWSR for the first ($n=1$) resonances of multipole orders $l=0$, 1, 2, and 3 in the zero sound + ES, first sound + ES, and SJ models. In the region of medium-weight and heavy nuclei the computed degrees of exhaustion are in good agreement with Bertrand's experimental data.[18] The degrees of exhaustion of the model-independent EWSR depend weakly on the number A of nucleons in the nucleus. Thus, when A is varied in the range from 40 to 250, the degree of exhaustion of the model-independent EWSR deviates from the values given in Table I by not more than 4%.

The contribution from the next ($n\geqslant 2$) roots of Eqs. (20) and (23) to the degree of exhaustion of the model-independent EWSR is negligibly small in comparison with the contribution from the first roots of these equations.

5. RESONANCE WIDTHS

It follows from the experimental investigations of the decay of the giant dipole resonances that, in the region of heavy nuclei, the contribution of the exit width to the total width is small.[18,20,22] Therefore, we can estimate the width of the giant resonances in heavy nuclei by considering only the quasiparticle collisions. As a result of the quasiparticle-qua-

TABLE I. Degrees of exhaustion of the model-independent EWSR.

l	Zero sound + ES %	First sound + ES, %	Steinwedel-Jensen model, %	Experiment, %
0	~85	~73	–	–
1	~98	~95	86	80–120
2	~94	~94	78	70–100
3	~90	~93	72	–

siparticle collisions, the wave number k of the zero sound becomes complex.[11-13] The width Γ is proportional to the imaginary part of k:

$$\Gamma=\hbar s^{-} v_F \operatorname{Im} k=\hbar s v_F \gamma^0. \tag{43}$$

In the case of zero-sound excitations in a Fermi liquid, when the excitation energy $E \ll \varepsilon_F$, γ^0 is connected with the classical attenuation factor $\tilde{\gamma}$ by the relation[11-13]

$$\gamma^0=\tilde{\gamma}[1+(E/2\pi T)^2]. \tag{44}$$

Here T is the temperature of the system. Knowing $\tilde{\gamma}$, we can find with the aid of (43) and (44) the width of the giant resonance of multipole order l in a nucleus with A nucleons if we substitute into (44) the expression for $E=E_{ln}(A)$ from (21). To find $\tilde{\gamma}$, we substitute for the collision integral in the system of equations (5) the approximate expression given in terms of the mean free time τ^0:

$$I(f_{P(N)}(\mathbf{r},\mathbf{p},t))=-1/(i\tau^0)\int d\Omega_{\mathbf{k}} A(\mathbf{k}\wedge\mathbf{z})\Big[f_{\mathbf{k}P(N)}(\mathbf{r},\mathbf{p},t)-1/p_F^3$$
$$\times\Big(\int d\tau' f_{\mathbf{k}P(N)}(\mathbf{r},\mathbf{p}',t)+3\cos(\mathbf{p}\wedge\mathbf{k})$$
$$\times\int d\tau' \cos(\mathbf{p}'\wedge\mathbf{k}) f_{\mathbf{k}P(N)}(\mathbf{r},\mathbf{p},t)\Big)\Big]. \tag{45}$$

After substituting (45) into (5), we find in the limit $\omega\tau^0 \gg 1$ that

$$\tilde{\gamma}=1/(s^{-}\tau^0 v_F), \tag{46}$$

where s^{-} is defined in (9), (10). The mean free time of the quasiparticles is connected with the viscosity of the Fermi liquid by the relation[12]

$$\tau^0=5\eta/(\bar{\rho} v_F^2 M). \tag{47}$$

The viscosity of the Fermi liquid is inversely proportional to the square of the temperature[12,13]:

$$\eta=\eta_0/T^2, \tag{48}$$

where η_0 is the coefficient of proportionality. Substituting (44) and (46)–(48) into (43), we obtain

$$\Gamma_{ln}(A)=a[(2\pi T)^2+E_{ln}^2(A)], \tag{49}$$

where

$$a=\hbar\bar{\rho}\varepsilon_F/(10\pi^2\eta_0),$$

and $E_{ln}(A)$ is defined in (21). It follows from (49) that in the limit $2\pi T \ll E_{ln}(A)$

$$\Gamma_{ln}(A)=aE_{ln}^2(A). \tag{50}$$

The widths are proportional to the square of the resonance excitation energy.

In Ref. 23 a semiphenomenological description of the giant dipole resonance width is given. It is proposed in that paper that the widths of the resonances in spherical nuclei are described by the formula (50). The parameter a is determined there by fitting (50) to the experimental data, and is found to be equal to $a=0.019\pm0.005\ \text{MeV}^{-1}$. Choosing $a=0.02\ \text{MeV}^{-1}$, we find that the quasiparticle-viscosity parameter for the Fermi liquid is equal to $\eta_0=1.84\times10^{-21}$ MeV-sec/fm. The giant dipole resonance widths in spherical nuclei, as computed with the aid of (20)–(22) and (50), then agree well with the experimental values (see Fig. 1). The quadrupole-resonance widths computed with the aid of (20)–(22) and (50) agree less well with the experimental data (see Fig. 2). But let us note that the isovector quadrupole resonance is observed in inelastic electron scattering reactions.[18,20] It is weakly excited in these reactions, and has a large width. Therefore, it is difficult to separate it out from the observed inelastic scattering spectrum, and it is difficult to determine sufficiently reliably its position, width, and model independent EWSR strength.[20]

6. CONCLUSION

In the paper we have investigated those giant resonances in the gas-droplet nuclear model in the effective surface approximation which can be considered on the basis of zero and first sounds. In the case of zero sound the energies, widths, and model-independent EWSR strengths of the resonances are in good agreement with the experimental values. But good agreement with the experimental data can also be achieved in the first sound + ES system when the isovector volume compressibility modulus K^{-} is sufficiently large, specifically, when K^{-} lies in the range from 700 to 800 MeV. Therefore, the problem of interpreting the resonances is a complicated one.

In the case of nuclei with nonspherical equilibrium shapes the resonances split.[20,24]

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APPENDIX

Let us write the nuclear energy density functional in the form

$$\mathscr{E}(\rho_P(r),\ \rho_N(r))=b_V(\rho_P(r)+\rho_N(r))+\varepsilon(\rho_P(r),\ \rho_N(r))$$
$$+(\beta_P+\gamma_P/\rho_P(r))(\nabla\rho_P(r))^2+(\beta_N+\gamma_N/\rho_N(r))(\nabla\rho_N(r))^2$$
$$+\beta_{PN}\nabla\rho_P(r)\nabla\rho_N(r), \tag{A1}$$

where $\varepsilon(\rho_P(r),\rho_N(r))$ is some polynomial function of the proton and neutron densities $\rho_P(r)$ and $\rho_N(r)$ respectively, $\beta_P, \beta_N, \beta_{PN}, \gamma_P$ and γ_N are constants, and b_V is the energy per nucleon in infinite nuclear matter. The equilibrium proton and neutron density distributions can be found from the system of variational equations

$$\frac{\delta\mathscr{E}(\rho_P(r),\rho_N(r))}{\delta\rho_{P,(N)}(r)}=\mu_{P,(N)}, \tag{A2}$$

where $\mu_{P,(N)}$ is the chemical potential of the protons (neutrons).

A displacement of the proton ES by ζ relative to the neutron ES leads to a general shift of the proton and neutron density distributions:

$$\rho_P(r)\to\rho_P(r-\zeta/2),\qquad \rho_N(r)\to\rho_N(r+\zeta/2). \tag{A3}$$

The surface energy then depends on ζ, and, in the case of semi-infinite nuclear matter, has the form

$$B^s(\zeta)=4\pi r_0^2 \int_{-\infty}^{+\infty} dr[\mathscr{E}(\rho_P(r-\zeta/2),\rho_N(r+\zeta/2))-\mu_P\rho_P(r-\zeta/2) -\mu_N\rho_N(r+\zeta/2)]. \quad (A4)$$

Then the isovector-stiffness coefficient against surface displacement is defined as

$$B^-=r_0^2\frac{d^2B^s(\zeta)}{d\zeta^2}\bigg|_{\zeta=0}. \quad (A5)$$

Substituting (A4) into (A5), and taking (A2) into consideration, we obtain

$$B^-=2\pi r_0^4\int_{-\infty}^{+\infty} dr\left[\beta_{PN}\left(\frac{\partial}{\partial r}\rho_P(r)\frac{\partial^3}{\partial r^3}\rho_N(r)+\frac{\partial}{\partial r}\rho_N(r)\frac{\partial^3}{\partial r^3}\rho_P(r)\right) -2\frac{\partial^2\epsilon(\rho_P(r),\rho_N(r))}{\partial\rho_P(r)\,\partial\rho_N(r)}\frac{\partial\rho_P(r)}{\partial r}\frac{\partial\rho_N(r)}{\partial r}\right]. \quad (A6)$$

[1)]For a more exact approximation to the shape function, obtained with the aid of the nuclear energy density functional, see Ref. 9.

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