Isoscalar and isovector giant resonances in the gas-droplet model for deformed nuclei

V. Yu. Denisov

Institute of Nuclear Research, Academy of Sciences of the Ukrainian SSR, Kiev
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Simple expressions are obtained for the magnitudes of the splittings and widths of the isoscalar and isovector giant resonances in deformed nuclei. The computed quantities are in good agreement with the experimental data.

1. INTRODUCTION

The isoscalar and isovector giant resonances in axially symmetric deformed nuclei are split, and have greater widths.[1–4] This splitting has been investigated before in various microscopic and semimicroscopic theories based on the independent-nucleon approximation in a self-consistent field (see Ref. 2 for a review), and also in the simple macroscopic Steinwedel–Jensen[5] and Goldhaber–Teller[6] models by Danos[7] and Okamoto.[8] The independent-nucleon approximation calculations are complicated, and encounter the difficult problem of the detailed description of the nuclear edge. To overcome these difficulties, Strutinskiĭ *et al.*[9] developed the gas-droplet model, in which the nucleon dynamics at the nuclear surface is described with the aid of well known phenomenological constants—the coefficient of surface tension, for example.[4] In this model, the inner-density dynamics in the relatively sharp nuclear edge approximation is, because of the virtual constancy of the density in the nuclear interior, described by the same equations that describe the density dynamics in an infintie medium. The relatively rapid variation of the density at the nuclear edge allows the introduction of an effective sharp nuclear surface, fixed at the locus of the points of maximum density gradient.[10,11] The inner-density oscillations satisfy specific boundary conditions at the effective surface (ES) of the nucleus. This method has been used to investigate various characteristics of the isoscalar giant resonances,[11–14] the isovector giant resonances,[15] and the isoscalar solenoidal current oscillations[16] in spherical nuclei. Let us note that the Steinwedel–Jensen (SJ)[5] and Goldhaber–Teller (GT)[6] models are limiting cases of the model proposed in Ref. 15.

In the case of density oscillations, the nucleon dynamics in the nuclear interior can be described with the aid of either Landau's kinetic equation for zero sound,[17–19] or the hydrodynamic equations.[4,10] At the ES of the nucleus,these oscillations satisfy the simple boundary conditions proposed in Ref. 12 for the isoscalar resonances, and in Ref. 15 for the isovector resonances.

In the case of the isoscalar density excitations the first boundary condition requires the equality of the mean nucleon velocity along the normal to the surface and the corresponding velocity of the ES of the nucleus. The second boundary condition ensures the equality of the normal stress-tensor component connected with the isoscalar density oscillations in the nuclear interior and the surface-tension-related restoring force acting on a unit area of the distorted surface.

For the isovector density oscillations, the first boundary condition ensures the equality of the mean proton (neutron) velocity along the normal to the ES and the normal velocity of the corresponding ES. There arise in the surface layer of the nucleus upon the displacement of the proton ES relative to the neutron ES restoring forces tending to liquidate that displacement.[20,21] Connected with these forces is the second boundary condition, which ensures the equality of the restoring surface force acting on a unit surface area of the nucleus and the corresponding stress-tensor component connected with the isovector density oscillations in the interior of the nucleus.

II. EXCITATION ENERGIES AND MAGNITUDES OF THE RESONANCE SPLITTINGS

1. The zero sound + ES model

Let us consider the dynamics of the density oscillations in the interior of the nucleus is the zero sound + ES approximation. The equations for the distribution functions, $f_p(\mathbf{r},\mathbf{p},t)$ and $f_N(\mathbf{r},\mathbf{p},t)$, for the proton-like and neutron-like quasiparticles in the nuclear interior at zero temperature have the form[17–19]

$$\frac{\partial f_i(\mathbf{r},\mathbf{p},t)}{\partial t}+\frac{\mathbf{p}}{M^*}\nabla_r\Big[f_i(\mathbf{r},\mathbf{p},t)+\delta(\varepsilon-\varepsilon_F)\,(\pi^2\hbar^3/p_FM^*)\times\int d\tau'\sum_{j=P,N}F_{ij}(\mathbf{p},\mathbf{p}')f_j(\mathbf{r},\mathbf{p}',t)\Big]=0. \qquad (1)$$

Here

$$d\tau'=2d\mathbf{p}'/(2\pi\hbar)^3;$$

and $i,j=P,N$; M^* is the effective mass of the quasiparticles, p_F is the limiting Fermi momentum, and $F_{PP}(\mathbf{p},\mathbf{p}')$, $F_{NN}(\mathbf{p},\mathbf{p}')$, and $F_{PN}(\mathbf{p},\mathbf{p}')$ are the constants in the proton–proton, neutron–neutron, and proton–neutron quasiparticle interaction amplitudes, respectively. All the interaction constants have been expressed in units of $2\pi^2\hbar^3/p_FM^*$. Neglecting the Coulomb interaction, we assume on the basis of isotopic invariance that $F_{PP}(\mathbf{p},\mathbf{p}')=F_{NN}(\mathbf{p},\mathbf{p}')$. It is assumed in the case of zero-sound excitations that the number of protons is equal to the number of neutrons, and , consequently, that the Fermi energies ε_F of the proton and neutron quasi-

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particles are equal. Let us represent the constants in the quasiparticle interaction amplitudes in the form[17–19]

$$F_{ij}(\mathbf{p}, \mathbf{p}') = F_{0ij} + F_{1ij}(\mathbf{pp}')/p_F^2 + \dots$$

and limit ourselves to only the first term of this expansion. The effective quasiparticle mass M^* then coincides with the nucleon mass M. Adding and subtracting Eq. (1), we obtain

$$\frac{\partial f^\pm(\mathbf{r},\mathbf{p},t)}{\partial t} + \frac{\mathbf{p}}{M}\nabla_r\Big[f^\pm(\mathbf{r},\mathbf{p},t) + \delta(\varepsilon-\varepsilon_F)(\pi^2\hbar^3/p_F M)F_0^\pm \times \int d\tau' f^\pm(\mathbf{r},\mathbf{p}',t)\Big] = 0, \qquad (2)$$

where

$$f^\pm(\mathbf{r},\mathbf{p},t) = f_N(\mathbf{r},\mathbf{p},t) \pm f_P(\mathbf{r},\mathbf{p},t), \quad F_0^\pm = F_{0PP} \pm F_{0PN}, \qquad (3)$$

and the plus and minus indices correspond respectively to isoscalar and isovector quantities. The solutions to Eq. (2) in the case of plane waves are given in Refs. 17–19:

$$f_\mathbf{k}^\pm(\mathbf{r},\mathbf{p},t) = (\alpha_N \pm \alpha_P)\delta(\varepsilon-\varepsilon_F)\nu^\pm(\mathbf{p},\mathbf{k})\exp(i(\mathbf{kr}-\omega t)), \quad (4)$$

where ω is the oscillation frequency, $\mathbf{k}$ is the wave vector,

$$\nu^\pm(\mathbf{p},\mathbf{k}) = \cos(\mathbf{p}\wedge\mathbf{k})/(s^\pm - \cos(\mathbf{p}\wedge\mathbf{k})),$$
$$s^\pm = \omega/kv_F, \quad v_F = (2\varepsilon_F/M)^{1/2}, \quad \alpha_N = \pm\alpha_P, \qquad (5)$$

and α_P and α_N are amplitudes. Here and below $\mathbf{p}\wedge\mathbf{k}$ denotes the angle between the vectors $\mathbf{p}$ and $\mathbf{k}$. The quantities $s^\pm$ in (5) are determined

$$G^{-1}(s^\pm) = (s^\pm/2)\ln((s^\pm+1)/(s^\pm-1)) - 1 = 1/F_0^\pm. \qquad (6)$$

For an infinite homogeneous medium the solutions (4) are physical solutions. In a finite medium these solutions can be regarded as a set of formal solutions with $\mathbf{k}$ vectors differing in their magnitudes and directions. Since (1) and (2) constitute a system of linear homogeneous equations, we can construct a more general solution by taking a superposition of the particular solutions (4) in the form

$$f_l^\pm(\mathbf{r},\mathbf{p},t) = \int d\mathbf{k} A_l^\pm(\mathbf{k},\mathbf{z}) f_\mathbf{k}^\pm(\mathbf{r},\mathbf{p},t). \qquad (7)$$

Here $A_l^\pm(\mathbf{k},\mathbf{z})$ is the weighting function with the aid of which the superposition of the particular solutions with wave vectors $\mathbf{k}$ is constructed, z is the preferred direction in space, and the subscript l will be defined below. We are interested in a solution with a fixed frequency; therefore, taking account of the relation between k and ω in (5) and (6), we carry out the integration in (7) only over the angles of the vector $\mathbf{k}$.

Let us consider the isoscalar and isovector density oscillations of specific multipole order l in an axially symmetric, slightly deformed nucleus. (The $l = 1$ case corresponds to dipole oscillations.) Nuclei with $A \gtrsim 40$ are only slightly deformed[22,23]; therefore the problem can be solved by the method of perturbation theory.

In the case of deformed nuclei the radius of the ES has the form

$$R(\theta) = R_0(1 + \beta_L Y_{L0}(\theta)), \quad R_0 = r_0 A^{1/3}, \qquad (8)$$

where L is the multipolarity, β_L is the deformation parameter, and $\theta = \mathbf{r}\wedge\mathbf{z}$.

In an axially symmetric deformed nucleus isoscalar or isovector density oscillations of multipolarity l and angular-momentum component m are accompanied by oscillations in the corresponding density that have the same frequency and angular-momentum component, but are of different multipolarity l', with $|L-l| \leqslant l'(\neq l) < |L+l|$ (in first-order perturbation theory in terms of the parameter β_L). Consequently the density oscillations have the form

$$\rho_l^\pm(\mathbf{r},t) \equiv \int d\tau f_l^\pm(\mathbf{r},\mathbf{p},t) = \Big[\alpha_{lm}\Phi_{lm}^\pm(r)Y_{lm}(\mathbf{r},\mathbf{z}) + \beta_L\sum_{l'}\alpha_{l'm}\Phi_{l'm}^\pm(r)Y_{l'm}(\mathbf{r},\mathbf{z})\Big]\exp(-i\omega_{lm}t), \qquad (9)$$

where the $\Phi_{lm}^\pm(r)$ are certain radial functions and the α_{lm} and $\alpha_{l'm}$ are amplitudes. For computational convenience, the time dependence of the physical quantities has been taken in the form $\exp(-i\omega_{lm}t)$. Substituting (4), (5), and (7) into (9), we obtain

$$A_l^\pm(\mathbf{k},\mathbf{z}) = i^{-l}\alpha_{lm}Y_{lm}(\mathbf{k},\mathbf{z}) + \beta_L\sum_{l'} i^{-l'}\alpha_{l'm}Y_{l'm}(\mathbf{k},\mathbf{z}), \quad (10)$$

$$\Phi_{lm}^\pm(r) = c_V^\pm j_l(k_{lm}r), \quad \Phi_{l'm}^\pm(r) = c_V^\pm j_{l'}(k_{lm}r),$$
$$c_V^\pm = (\alpha_N \pm \alpha_P)\cdot 2(4\pi)^2 p_F M/(2\pi\hbar)^3 F_0^\pm, \qquad (11)$$

where $j_l(x)$ is the spherical Bessel function,

$$k_{lm} = k_l + \Delta k_{lm}, \quad \omega_{lm} = \omega_l + \Delta\omega_{lm}, \quad \Delta\omega_{lm} = s^\pm v_F \Delta k_{lm}, \qquad (12)$$

ω_l and k_l are the frequency and wave vector corresponding to density oscillations of multipolarity l in a spherically symmetric nucleus, and $\Delta\omega_{lm}$ and Δk_{lm} are the corrections to the frequency ω_l and wave vector k_l that arise as a result of the deformation of the nucleus.

Knowing the distribution function $f_{P(N)}(\mathbf{r},\mathbf{p},t)$, we can compute the velocity of the proton (neutron) flux:

$$\mathbf{v}_{lP(N)}(\mathbf{r},t) = (1/M\rho_{P(N)})\int d\tau\, \mathbf{p} f_{lP(N)}(\mathbf{r},\mathbf{p},t) \qquad (13)$$

and the stress-tensor components

$$\sigma_{l\mu\nu}^\pm(\mathbf{r},t) = (1/M)\int d\tau\, p_\mu p_\nu f_l^\pm(\mathbf{r},\mathbf{p},t) + \delta_{\mu\nu}F_0^\pm\bar\rho(\pi^2\hbar^3/2p_F M)\rho_l^\pm(\mathbf{r},t). \qquad (14)$$

Here

$$\bar\rho = 3/4\pi r_0^3, \quad \rho_P = (Z/A)\bar\rho, \quad \rho_N = (N/A)\bar\rho,$$

Z is the number of protons in the nucleus, N is the number of neutrons, and $A = N + Z$. The last term in (14) arises as a result of the interaction between the quasiparticles.[12,15]

The boundary conditions at the ES of the nucleus for the isoscalar density excitations have the form[12]

$$(1/A)((N\mathbf{v}_N(\mathbf{r},t) + P\mathbf{v}_P(\mathbf{r},t))\mathbf{n})|_{\mathrm{ES}} = (\mathbf{v}^+(\mathbf{r},t)\mathbf{n})|_{\mathrm{ES}} = (dR(\mathbf{r},z;t)/dt), \qquad (15)$$

$$\sigma_{nn}^+(\mathbf{r},t)|_{\mathrm{ES}} = \sum_{\mu\nu}\sigma_{\mu\nu}^+(\mathbf{r},t)(\mathbf{e}_\mu\mathbf{n})(\mathbf{e}_\nu\mathbf{n})|_{\mathrm{ES}} = 2\varkappa(H(t) - H_0). \qquad (16)$$

(The explicit expressions obtained for $v_\mu^+(\mathbf{r},t)$ and $\sigma_{\mu\nu}^+(\mathbf{r},t)$ with the aid of (4), (5), (7), and (13)–(16) are given in Appendix 1.) Here

TABLE I. Values of the coefficients $E^+_{l1}(A)A^{1/3}$, $\Delta E^+_{l1}(A)A^{1/3}/\beta_2$, and $\Delta E^+_{lmn}(A)A^{1/3}/\beta_2$ for different l, as computed in the zero sound + ES approximation.

l	$E^+_{l1}(A)A^{1/3}$	$\Delta E^+_{l1}(A)A^{1/3}/\beta_2$	m	$\Delta E^+_{lm1}(A)A^{1/3}/\beta_2$
0	130	–	0	–
2	63	52	2	26
			1	−13
			0	−26
3	107	73	3	−41
			2	0
			1	−25
			0	−32
4	152	105	4	55
			3	14
			2	−16
			1	−34
			0	−40

$$R(\mathbf{r},\mathbf{z};t)=R(\theta)\left[1+\left(\alpha^{(s)}_{lm}Y_{lm}(\mathbf{r},\mathbf{z})+\beta_L\sum_{l'}\alpha^{(s)}_{l'm}Y_{l'm}(\mathbf{r},\mathbf{z})\right)\times\exp(-i\omega_{lm}t)\right] \tag{17}$$

is the radius of the dynamical ES of the nucleus, $\alpha^{(s)}_{lm}$ is the ES oscillation amplitude,

$$\mathbf{n}=\mathbf{e}_r-\beta_L\frac{dY_{L0}(\theta)}{d\theta}\mathbf{e}_\theta \tag{18}$$

is the normal to the ES of the nucleus, $\sigma^+_{nn}(\mathbf{r},t)$ is the stress-tensor component normal to the ES, the $\mathbf{e}_\mu$ are unit vectors of the spherical coordinate system, $\varkappa$ is the surface-tension coefficient,[4] and $H(t)$ and H_0 are the mean curvatures of the dynamical and static ES, respectively.

Retaining, in the case of small isoscalar density oscillations of multipolarity l in an axially symmetric deformed nucleus, retaining the terms of first order in β_L, we find that the difference between the dynamical and static mean curvatures is equal to

$$\begin{aligned}H(t)-H_0=\Big[\alpha^{(s)}_{lm}\Big(&(l+2)(l-1)Y_{lm}(\mathbf{r},\mathbf{z})\\&-\beta_L((L(L+1)+(l+2)(l-1))\\\times Y_{lm}(\mathbf{r},\mathbf{z})Y_{L0}(\theta)&+2(dY_{L0}(\theta)/d\theta)(dY_{lm}(\mathbf{r},\mathbf{z})/d\theta))\Big)\\+\beta_L\sum_{l'}\alpha^{(s)}_{l'm}(l'+2)(l'-1)&Y_{l'm}(\mathbf{r},\mathbf{z})\Big]\exp(-i\omega_{lm}t)/2R_0.\end{aligned} \tag{19}$$

Substituting (A1)–(A4) (see Appendix 1) and (17)–(19) into the boundary conditions (15) and (16), we obtain for each m a system of linear equations for the relative amplitudes $\alpha_{lm}(\alpha_P+\alpha_N)$, $\alpha_{l'm}(\alpha_P+\alpha_N)$, $\alpha^{(s)}_{lm}$, and $\alpha^{(s)}_{l'm}$. From the solvability condition for this system of equations in the zeroth order approximation in β_L we find for the determination of the wave vector k_l for a spherically symmetric nucleus an equation having the same form as the one found in Ref. 12:

$$g_l^{+(0)}(x)\equiv j_l'(x)-(3\varepsilon_F xA^{1/3}/B^{(s)}(l-1)(l+2))\times(c_2^+j_l''(x)+c_1^+j_l(x))=0. \tag{20}$$

Here and below the primes denote differentiation of the spherical Bessel functions with respect to their argument $x=k_lR_0$,

$$c_1^\pm=1-(s^\pm)^2+(2F_0^\pm+G(s^\pm))/3,\quad c_2^\pm=1-3(s^\pm)^2+G(s^\pm), \tag{21}$$

and $B^{(s)}=4\pi r_0^2\varkappa$ is the coefficient of $A^{2/3}$ in the mass formula.[4] The root $x^+_{ln}(A)=k_{ln}R_0$ of Eq. (20) practically does not depend on A (n is the number of the root of the equation). Thus, as A is varied from 40 to 250, the quantity $x^+_{ln}(A)$ for $l=0$, 2, 3, and 4 changes by not more than 5%. Retaining in the solvability condition the terms linear in β_L, we obtain an expression for the determination of the quantity $\Delta x^+_{lmn}(A)$:

$$\begin{aligned}\Delta x^+_{lmn}(A)=\Delta k_{lmn}R_0=&-\beta_L x_{ln}^+(A)((2L+1)/4\pi)^{1/2}\langle l0L0|l0\rangle\\&\times\langle lmL0|lm\rangle[1+(L(L+1)c_2^+(3\varepsilon_F A^{1/3}x/B^{(s)}(l+2)(l-1))\\&\times(j_l'(x)-j_l(x)/x)/x^2-(1/2)L(L+1)j_l(x)/x^2\\&-(1+2L(L+1)/((l+2)(l-1)))\\&\times j_l'(x)/x)/(g_l^{+(0)}(x))']|_{x=x^+_{ln}(A)}.\end{aligned} \tag{22}$$

The excitation energy $E^+_{lmn}(A)$ of the resonance of multipolarity l, with component m, is connected with $x^+_{ln}(A)$ and $\Delta x^+_{lmn}(A)$ by the relations

$$E^\pm_{lmn}(A)=E^\pm_{ln}(A)+\Delta E^\pm_{lmn}(A), \tag{23}$$

where

$$E_{ln}^\pm(A)=\hbar(v_F/r_0)s^\pm x_{ln}^\pm(A)A^{-1/3}, \tag{24}$$

$$\Delta E^\pm_{lmn}(A)=E^\pm_{ln}(A)(\Delta x^\pm_{lmn}(A)/x_{ln}(A)). \tag{25}$$

It follows from (22)–(25) that the resonance of multipolarity l splits up in an axially symmetric deformed nucleus into $l+1$ components. In an axially symmetric deformed nucleus the components whose m's are equal in magnitude but opposite in sign have the same excitation energy. For $B_L>0$ the component with $m=0$ has the minimum energy $E^\pm_{ln\min}(A)=E^\pm_{l0n}(A)$, while the component with $m=l$ has the maximum energy $E^\pm_{ln\max}(A)=E^\pm_{lln}(A)$. The magnitude of the splitting of the nth resonance of multipolarity l is equal to

$$\Delta E_{ln}^\pm(A)=E^\pm_{ln\max}(A)-E^\pm_{ln\min}(A). \tag{26}$$

The lowest ($n=1$) resonances of multipolarities $l=2$ and 3 in spherical nuclei correspond to experimentally observed resonances, and exhaust the major portion of the model-independent energy-weighted sum rule.[12,13] In Table I we give the numerical values of the coefficients $E^+_{l1}(A)A^{1/3}$, $\Delta E^+_{lm1}(A)A^{1/3}/\beta_2$, $\Delta E^+_{l1}(A)A^{1/3}/\beta_2$ for $l=0$, 2, 3, and 4. The computations were carried out with the aid of Eqs. (20)–(26) with $r_0=1.2$ fm, $\varepsilon_F=40$ MeV, $F_0^+=1$, and $B^{(s)}=19$ MeV. The excitation energies and the degrees of exhaustion of the model-independent energy-weighted sum rule for these parameters are in good agreement with the experimental data reported in Refs. 12 and 13.

In the case of the isovector density oscillations the boundary conditions at the ES of the nucleus have the form[1)] (see Ref. 15)

$$(\mathbf{v}_{P(N)}(\mathbf{r},t)\mathbf{n})|_{ES} = ((d\zeta_{P(N)}(\mathbf{r},t)/dt)\mathbf{n}), \tag{27}$$

$$\sigma_{nn}^{-}(\mathbf{r},t)|_{ES} = \sum_{\mu\nu} \sigma_{\mu\nu}^{-}(\mathbf{r},t)(\mathbf{n}e_\mu)(\mathbf{n}e_\nu)|_{ES} = B^{-}/2\pi r_0^4((\zeta_N(\mathbf{r},t)-\zeta_P(\mathbf{r},t))\mathbf{n}). \tag{28}$$

Here

$$\mathbf{n}\zeta_{P(N)}(\mathbf{r},t) = \left(\alpha^{(s)}_{lmP(N)} Y_{lm}(\mathbf{r},\mathbf{z}) + \beta_L \sum_{l'} \alpha^{(s)}_{l'mP(N)} Y_{l'm}(\mathbf{r},\mathbf{z})\right) \times r_0 \exp(-i\omega_{lm}t) \tag{29}$$

is the displacement of the proton (neutron) ES of the nucleus from its equilibrium position, $\alpha^{(s)}_{lmP(N)}$ is the amplitude of this displacement, and B^- is the isovector shear stiffness coefficient for the surface.[15] It follows from the condition (5) on the amplitudes α_N and α_P that the position of the center of mass for the isovector density excitations of any multipole order l is preserved, i.e., that

$$\int d\mathbf{r}\, r Y_{lm}(\mathbf{r},\mathbf{z})(\rho_{lmP}(\mathbf{r},t)+\rho_{lmN}(\mathbf{r},t)) + \int dS\, Y_{lm}(\mathbf{r},\mathbf{z}) \times R(\theta)(\rho_P(\zeta_{lmP}(\mathbf{r},t)\mathbf{n}) + \rho_N(\zeta_{lmN}(\mathbf{r},t)\mathbf{n})) = 0, \tag{30}$$

and, taking account of (3)–(5), (7), (9)–(13), and (29), we obtain the following relation between the surface-oscillation amplitudes:

$$\alpha^{(s)}_{lmP} = -\alpha^{(s)}_{lmN}. \tag{31}$$

For the isoscalar density oscillations of multipole orders $l = 0, 2, 3, 4, \ldots$ the condition for the preservation of the position of the center of mass is fulfilled identically (in first order in the oscillation amplitude). The $l = 1$ case will be considered in a separate paper. As in the case of the isoscalar density oscillations, the substitution in the isovector-oscillation case of (3)–(5), (7), (13), (14), and (29) into the boundary conditions (27) and (28) yields for each m a system of linear equations for the amplitudes $\alpha_P\alpha_{lm}$, α_P, $\alpha_{l'm}$, $\alpha^{(s)}_{lmP}$, and $\alpha^{(s)}_{l'mP}$. From the solvability condition for this system of equations in zeroth order in β_L we find for the determination of the wave vector k_l an equation that has the same form as the one obtained in the case of a spherically symmetric nucleus[15]:

$$g_l^{-(0)}(x) \equiv j_l'(x) - (3\varepsilon_F x/4B^- A^{1/3})(c_2^- j_l''(x) + c_1^- j_l(x)) = 0. \tag{32}$$

The $c_{1,2}^-$ here are defined in (21). Let $x_{ln}^-(A) = k_{ln}R_0$ be the nth root of Eq. (32). Retaining in the solvability condition the terms linear in β_L, we obtain the following expression for the determination of the quantity $\Delta x_{lmn}^-(A) = \Delta k_{lmn}R_0$:

$$\Delta x_{lmn}^-(A) = -\beta_L x_{ln}^-(A)((2L+1)/4\pi)^{1/2}\langle l0L0|l0\rangle\langle lmL0|lm\rangle \times [1 + (L(L+1)c_2^-(3\varepsilon_F x/4B^- A^{1/3})(j_l'(x) - j_l(x)/x)/x^2 - (1/2)L(L+1)j_l(x)/x^2 + j_l'(x)/x)/(g_l^{-(0)}(x))']|_{x=x_{ln}^-(A)}. \tag{33}$$

The excitation energy $E_{lmn}^-(A)$ of the resonance of multipolarity l and multipolarity component m and the magnitude $\Delta E_{ln}^-(A)$ of the resonance splitting are connected with $x_{ln}^-(A)$ and $\Delta x_{lmn}^-(A)$ by the relations (23)–(26). The lowest ($n = 1$) resonances of multipolarities $l = 1$ and 2 correspond to experimentally observed resonances, and exhaust the major portion of the model-independent energy-weighted sum rule.[15] Figure 1 shows the dependence of $E_{11\max(\min)}^-(A)$ and $\Delta E_{11}^-(A)$ on the number of nucleons in the nucleus for nuclei with large quadrupole deformations. The experimental values of $E_{11\max(\min)}^-(A)$ and $\Delta E_{11}^-(A)$ were taken from Refs. 25 and 26; those for β_2, from Ref. 22. The computation was carried out with the aid of Eqs. (32), (33), and (23)–(25) for the parameters $r_0 = 1.2$ fm, $\varepsilon_F = 40$ MeV, $N = Z = A/2$, $F_0^- = 1.6$, and $B^- = 43.5$

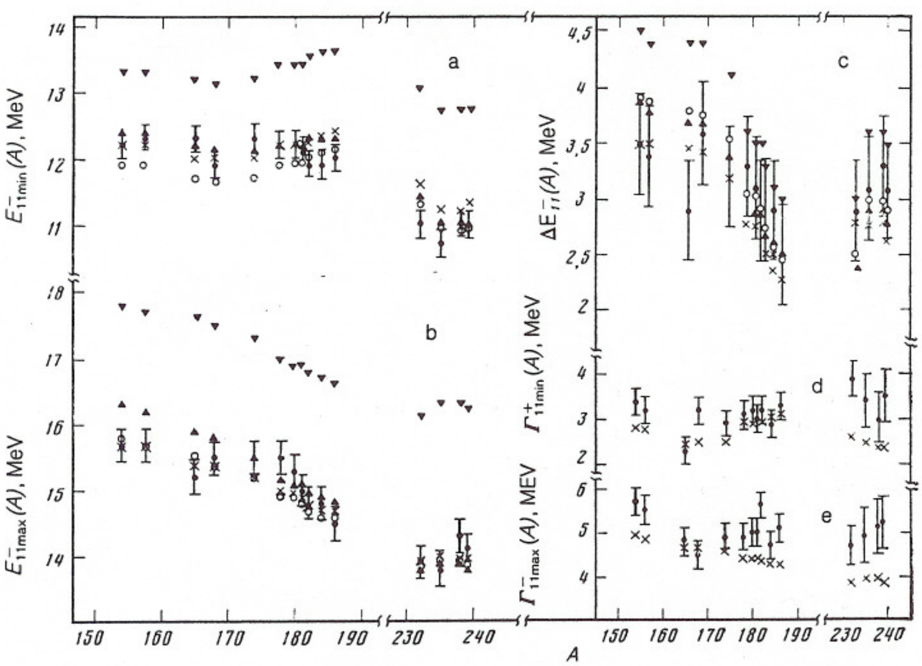

FIG. 1. Dependence on the number of nucleons in the nucleus of: (a) and (b) the excitation energies $E_{11\min}^-(A)$ and $E_{11\max}^-(A)$, respectively; (c) the magnitude $\Delta E_{11}^-(A)$ of the splitting; and (d) and (e) the widths $\Gamma_{11\min}^-(A)$ and $\Gamma_{11\max}^-(A)$, respectively. The points: × indicate the values obtained in the zero sound + ES model; ○, the values obtained in the first sound + ES model; ▲, the values obtained in the SJ model; and ▼, the values obtained in the GT model.

MeV. The excitation energies and the degrees of exhaustion of the model-independent energy-weighted sum rule at these parameter values are in good agreement with the experimental data.[15] It can be seen from Fig. 1 that the $E^{-}_{11\max(\min)}(A)$ and $\Delta E^{-}_{11}(A)$ values obtained with these same parameter values are also in good agreement with the experimental values.

2. First sound + ES model

Let us first consider the isoscalar density excitations in an axially symmetric deformed nucleus in the first sound + ES approximation. In this case the tensor $\sigma^{+}_{\mu\nu}(\mathbf{r}, t)$ is diagonal, and is connected with the volume compressibility K^{+} of the nucleus by the relation

$$\sigma^{+}_{l\mu\nu}(\mathbf{r}, t)=\delta_{\mu\nu}(K^{+}/9)\rho_{l}^{+}(\mathbf{r}, t), \tag{34}$$

where $\rho_l^{+}(\mathbf{r}, t)$ is the dynamical component of the isoscalar density in the interior of the nucleus. From the hydrodynamic equations (see Appendix 2) and the boundary conditions (15), (16), and (17) we obtain for each m a system of equations for the amplitudes $\alpha_{lm}(\alpha_N+\alpha_P)$, $\alpha_{l'm}(\alpha_N+\alpha_P)$, $\alpha^{(s)}_{lm}$, and $\alpha^{(s)}_{l'm}$. As in the case of zero sound, the solvability condition for this system furnishes in zeroth order in β_L the equation for the determination of k_l in a spherical nucleus:

$$g_l^{+(1)}(x)\equiv j_l'(x)-(K^{+}A^{1/3}x/3B^{(s)}(l+2)(l-1))j_l(x)=0, \tag{35}$$

which coincides with the equation obtained in Refs. 3, 4, 10, and 12. In first order in β_L we have for $\Delta x^{+}_{lmn}(A)$ the expression

$$\Delta x^{+}_{lmn}(A)=-\beta_L x_{ln}^{+}(A)[(2L+1)/4\pi]^{1/2}\langle l0L0|l0\rangle\langle lmL0|lm\rangle$$
$$\times[1-((1/2)L(L+1)j_l(x)/x^2+(1+2L(L+1)/((l+2)(l-1)))$$
$$\times j_l'(x)/x)/(g_l^{+(0)}(x))']|_{x=x^{+}_{ln}(A)}. \tag{36}$$

The root of Eq. (35) for $l=0$ depends weakly on the number of nucleons in the nucleus; for example, the root changes by not more than 5% when A is varied from 40 to 250. The resonance excitation energy in a spherically symmetric nucleus has the form

$$E_{ln}^{\pm}(A)=\hbar(K^{\pm}/9Mr_0^2)^{1/2}x_{ln}^{\pm}(A)A^{1/3}. \tag{37}$$

For $K^{+}=200$ MeV, $B^{(s)}=19$ MeV, and $r_0=1.2$ fm, the excitation energies and the degrees of exhaustion of the model-independent energy-weighted sum rule for the lowest resonances, which make the dominant contribution to the sum rule, are in good agreement with the experimental data for the monopole resonance.[10,12,13] But for the resonances of multipolarities $l\geqslant 2$ the lowest resonances that exhaust the major portion of the model-independent energy-weighted sum rule correspond to oscillations of the nuclear surface, and are not observed in experiment.[12,13]

Let us now consider the isovector excitations of an axially symmetric deformed nucleus in the first sound + ES approximation. In this case the tensor $\sigma^{-}_{\mu\nu}(\mathbf{r}, t)$ is diagonal, and is connected with the volume isovector compressibility K^{-} of the nucleus by the relation

$$\sigma^{-}_{l\mu\nu}(\mathbf{r}, t)=\delta_{\mu\nu}(K^{-}/9)\rho_{l}^{-}(\mathbf{r}, t), \tag{38}$$

where $\rho_l^{-}(\mathbf{r}, t)$ is the dynamical component of the isovector density in the interior of the nucleus. From the hydrodynamic equations for a two-component medium and the boundary conditions (27) and (28) we obtain for each m a system of equations for the amplitudes $\alpha_{lm}\alpha_P$, $\alpha_{l'm}\alpha_P$, $\alpha^{(s)}_{lmP}$, and $\alpha^{(s)}_{l'mP}$. As in the case of zero sound, the solvability condition for this system furnishes for the determination of the k_l in a spherical nucleus, i.e., in zeroth order in β_L, the equation

$$g_l^{-(1)}(x)\equiv j_l'(x)-(K^{-}x/3B^{-}A^{1/3})(NZ/A^2)j_l(x)=0, \tag{39}$$

which has the same form as the one obtained in Ref. 15. In first order in β_L we have

$$\Delta x^{-}_{lmn}(A)=-\beta_L x_{ln}^{-}(A)[(2L+1)/4\pi]^{1/2}\langle l0L0|l0\rangle\langle lmL0|lm\rangle$$
$$\times[1+(-(1/2)L(L+1)j_l'(x)/x^2$$
$$+j_l'(x)/x)/(g_l^{-(1)}(x))']|_{x=x^{-}_{ln}(A)}. \tag{40}$$

The resonance excitation energy $E^{-}_{ln}(A)$ in a spherical nucleus is, in the first sound + ES approximation, connected with the root $x^{-}_{ln}(A)$ of Eq. (39) by the relation (37). The energy $E^{-}_{lmn}(A)$ of the resonance components and the magnitude $\Delta E^{-}_{ln}(A)$ of the splitting are connected in the first sound + ES model with $E^{-}_{ln}(A)$, $x^{-}_{ln}(A)$, and $\Delta x^{-}_{ln}(A)$ from (39) and (40) by the same relations that connect the corresponding quantities in the zero sound + ES system [see Eqs. (23), (25), and (26)].

Figure 1 shows the dependence, as computed for $K^{-}=750$ MeV, $B^{-}=64$ MeV, and $r_0=1.2$ fm, of $E^{-}_{11\max(\min)}(A)$ and the magnitude $\Delta E^{-}_{11}(A)$ of the splitting on the number of nucleons in the nucleus. For these parameter values the excitation energies and the degrees of exhaustion of the model-independent energy-weighted sum rule are in good agreement with the experimental data.[15] The quantities $E^{-}_{11\max(\min)}(A)$ and $\Delta E^{-}_{11}(A)$ for these same values of K^{-} and B^{-} are also in good agreement with the experimental data.

The value $K^{-}=750$ MeV is significantly greater than the value $K^{-}=450$ MeV taken from the mass formula.[4] But the magnitudes of the isovector volume compressibility and the isovector surface shear stiffness are connected with terms of the mass formula $[(N-Z)^2/A$ and $(N-Z)^2/A^{4/3}$, respectively] that cannot be accurately determined by fitting the mass formula to the nuclear masses (see also Refs. 24, 27, and 28).

It is demonstrated in Ref. 15 that the first sound + ES model unites the SJ and GT models. Thus, (39) and (40) yield in the limit $B^{-}\to\infty$ the corresponding expressions for the SJ model[2)] (see, for example, Ref. 4). In the $K^{-}\to\infty$ limit we obtain from (39) and (40) with the aid of (37) the corresponding expressions for the GT model. In Fig. 1 we show the quantities $E^{-}_{11\max(\min)}(A)$ and $\Delta E^{-}_{11}(A)$ as computed in the SJ and GT models. The SJ model curves for these quantities were taken from Ref. 7, and are the results of exact calculations carried out with $K^{-}=450$ MeV. For this K^{-} value the resonance energies computed in the SJ model agree well with the experimental values in the region of

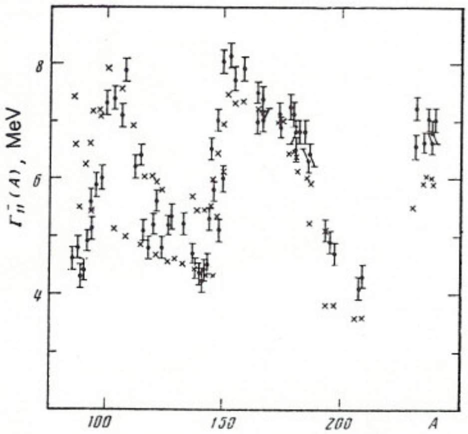

FIG. 2. Dependence of the total width $\Gamma_{11}^{-}(A)$ [formula (43)] of the giant isovector dipole resonances on the number of nucleons in the nucleus in the zero sound + ES approximation. The points have the same meanings as in Fig. 1.

heavy nuclei.[4] In the GT model the energies of the resonance components are estimated in accordance with Okamoto's prescription,[8] and are represented in the form of an expression:

$$E^{-}_{\max(\min)}(A) = 37A^{-1/6}R_0/R_{\min(\max)} \text{ (MeV)}, \quad (41)$$

where $R_{\max(\min)}$ is the maximum (minimum) distance from the center of the nucleus to the ES. The numerical coefficient in (41) has been chosen so as to obtain a good description of the region of light nuclei, where the model is more realistic.[15]

It follows from Fig. 1 that the first sound + ES model provides in a broad range of nuclear masses a better description of the excitation energies and the magnitudes of the dipole-resonance splitting than the SJ and GT models.

The magnitudes of the isovector quadrupole resonance splitting in the zero sound + ES and first sound + ES models differ from the magnitude of the dipole resonance splitting by not more than 10%.

In both the zero sound + ES and first sound + ES approximations the resonance energies $E^{\pm}_{lmn}(A)$ depend only weakly on the parameters $F_0^{\pm}$, $K^{\pm}$, $B^{(s)}$, and B^{-}. Thus, a 20% change in these parameters leads to not more than a 10% change in the quantity $E^{\pm}_{lmn}(A)$. And in the zero sound + ES model the energies of the resonance components depend very weakly on the constant in the quasiparticle interaction amplitude. For example, the variation of the constant in the range from 0.5 to 3 leads to not more than 10% changes in the values of $E^{\pm}_{lmn}(A)$.

Let us note that, in the zero sound + ES the first sound + ES models, the isoscalar and isovector monopole resonances do not split up in the approximation linear in β_2. The coupling of the monopole and quadrupole resonances arises in the next order of perturbation theory in β_2.

Notice that, for $N = Z$, the formulas (35)–(37) and (39), (40), (37) can be obtained respectively from (21), (22), (24) and (32), (24) by making the formal substitutions $\varepsilon_F \to K^{\pm}/6$, $F_0^{\pm} \to 0$, and $s^{\pm} \to 1/\sqrt{3}$. These substitutions effect a formal transition from zero to first sound.

III. RESONANCE WIDTHS IN THE ZERO SOUND+ES APPROXIMATION

It is shown in Refs. 14 and 15 that the widths of the isovector and isoscalar resonances in spherical nuclei have, in the zero sound + ES approximation, the form[3)]

$$\Gamma_{ln}^{\pm}(A) = a((E_{ln}^{\pm}(A))^2 + (2\pi T)^2). \quad (42)$$

Here $E^{\pm}_{ln}(A)$ is the resonance excitation energy (24), T is the nuclear temperature,

$$a = \hbar\bar{\rho}\varepsilon_F/(10\pi^2\eta_0),$$

and η_0 is the parameter in the expression $\eta = \eta_0/T^2$ for the quasiparticle viscosity of the nucleus. For $\eta_0 = 1.84 \times 10^{-21}$ MeV-sec/fm and $a = 0.02$ MeV^{-1} the widths computed with the aid of (42) in the limit $2\pi T \ll E_{ln}(A)$ agree well with the experimental values for the widths of the isovector dipole and isoscalar quadrupole resonances in spherical nuclei.[14,15]

In the case of deformed nuclei the resonance splits, and it is broader.[1–4,25,26,32,34] In this case the width can be estimated from the formula

$$\Gamma_{ln}^{\pm}(A) = (1/2)(\Gamma^{\pm}_{ln\max}(A) + \Gamma^{\pm}_{ln\min}(A)) + \Delta E_{ln}^{\pm}(A), \quad (43)$$

where

$$\Gamma^{\pm}_{ln\max(\min)}(A) = a((E^{\pm}_{ln\max(\min)}(A))^2 + (2\pi T)^2). \quad (44)$$

[It is shown in Ref. 15 that the width of the zero-sound density excitation is given by an expression of the form (42) and does not depend on the functions $A_l^{\pm}(\mathbf{k}, \mathbf{z})$, (7). Therefore, the expressions for the widths $\Gamma^{\pm}_{ln\max(\min)}(A)$ have the same form as (42) or (44). We can find the width of the resonance component with energy $E^{\pm}_{lmn}(A)$ by substituting $E^{\pm}_{lmn}(A)$ for $E^{\pm}_{ln}(A)$ in (42).]

In Fig. 1 the widths $\Gamma_{11\max(\min)}(A)$ obtained with the aid of (44) in the limit $2\pi T \ll E^{-}_{11}(A)$ are compared with the experimental values obtained by Gurevich *et al.*[25,26] in the region of deformed nuclei. The widths were computed with the same parameters that were used in the computation of the $E^{-}_{lmn}(A)$, and the β_2 values were taken from Ref. 22. The $\Gamma^{-}_{11\max}(A)$ and $\Gamma^{-}_{11\min}(A)$ values are in good agreement with the experimental data.

Figure 2 shows the total widths $\Gamma^{-}_{11}(A)$ for the isovector dipole resonance, as computed with the aid of (43) in the limit $2\pi T \ll E^{-}_{11}(A)$, for a broad range of nuclear masses. The experimental widths were taken from Refs. 25, 26, and 34; the quadrupole nuclear deformation parameter values for nuclei with $A < 150$ were taken from Ref. 23; those for nuclei with $A > 150$, from Ref. 22. The tables in Refs. 22 and 23 do not give the values of the quadrupole deformation β_2 for all the nuclei for which experimental values for the dipole–resonance widths are reported in Refs. 25, 26, and 34. In those cases in which no values are given for the β_2 parameter in Refs. 22 and 23, we set $\beta_2 = 0$ in the calculations. The computed total widths are in good agreement with the experimental values for the nuclei with $A > 150$; for the nuclei with $A < 150$, the agreement is not so good. The disagreement in the $A < 150$ region is due to the insufficient accuracy of the β_2 values and, possibly, the influence of other effects.

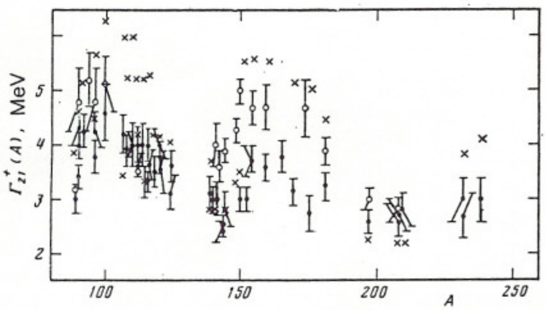

FIG. 3. Dependence of the total width $\Gamma^{+}_{21}(A)$ of the giant isoscalar quadrupole resonances on the number of nucleons in the nucleus in the zero sound + ES approximation. The points: ○ indicate experimental values taken from Ref. 1; ●, experimental values taken from Ref. 3; and ×, theoretical values obtained with the aid of formula (43).

Figure 3 shows the total widths $\Gamma^{+}_{21}(A)$ for the isoscalar quadrupole resonance, as computed with the aid of (43) in the limit $E^{+}_{ln}(A) \gg 2\pi T$, for medium and heavy nuclei. The experimental quadrupole-resonance widths were taken from Refs. 1 and 3; the values of the parameter β_2, from Refs. 22 and 23 (in the case of those nuclei for which no values are given for β_2 in the tables in Refs. 22 and 23, we set this parameter equal to zero in the calculations). The computed total widths are in satisfactory agreement with the experimental values.

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APPENDIX 1

Here we present the radial and tangential components of the mean nucleon velocity and stress tensor computed with the aid of Eqs. (4), (5), (7), (13), and (14). These components are required for the computation of the isoscalar-resonance excitation energies in the zero sound + ES approximation. The expressions for $v^{-}_{\mu}(\mathbf{r},t) = (\mathbf{v}_N(\mathbf{r},t) - \mathbf{v}_P(\mathbf{r},t))_\mu$ and $\sigma^{-}_{\mu\nu}(\mathbf{r},t)$ can be obtained from the expressions for $v^{+}_{\mu}(\mathbf{r},t)$ and $\sigma^{+}_{\mu\nu}(\mathbf{r},t)$ by making the substitutions $s^+ \to s^-$, $F^{+}_{0} \to F^{-}_{0}$, and $c^{+}_{V} \to c_V$, and multiplying the expression for $v^{-}_{\mu}(\mathbf{r},t)$ by two:

$$v_{lr}{}^{+}(\mathbf{r},t) = c_V{}^{+}(p_F s^+/\bar{\rho}Mi)\Big[\alpha_{lm} j_l{}'(k_{lm}r) Y_{lm}(\mathbf{r},\mathbf{z}) + \beta_L \sum_{l'} \alpha_{l'm} j_{l'}{}'(k_{lm}r) Y_{l'm}(\mathbf{r},\mathbf{z})\Big]\exp(-i\omega_{lm}t), \quad (A1)$$

$$v_{l\theta}{}^{+}(\mathbf{r},t) = c_V{}^{+}(p_F s^+/\bar{\rho}Mi)\Big[\alpha_{lm}(j_l(x)/x)(dY_{lm}(\mathbf{r},\mathbf{z})/\partial\theta] + \beta_L \sum_{l'} \alpha_{l'm}(j_{l'}(x)/x)(dY_{l'm}(\mathbf{r},\mathbf{z})/d\theta)\Big]\Big|_{x=k_{lm}r}\exp(-i\omega_{lm}t), \quad (A2)$$

$$\sigma_{lrr}(\mathbf{r},t) = c_V{}^{+}(p_F{}^2/2M)\Big[\alpha_{lm}(c_2{}^+ j_l{}''(x) + c_1{}^+ j_l(x))Y_{lm}(\mathbf{r},\mathbf{z}) + \beta_L \sum_{l'} \alpha_{l'm}(c_2{}^+ j_{l'}{}''(x) + c_1{}^+ j_{l'}(x))Y_{l'm}(\mathbf{r},\mathbf{z})\Big]\Big|_{x=k_{lm}r}\exp(-i\omega_{lm}t), \quad (A3)$$

$$\sigma_{lr\theta}(\mathbf{r},t) = c_V{}^{+}(p_F{}^2/2M) \times \Big[\alpha_{lm}A((j_l{}'(x) - j_l(x)/x)/x)(dY_{lm}(\mathbf{r},\mathbf{z})/d\theta) + \beta_L \sum_{l'} \alpha_{l'm} i^{l'}((j_{l'}{}'(x) - j_{l'}(x)/x)/x) \times (dY_{l'm}(\mathbf{r},\mathbf{z})/d\theta)\Big]\Big|_{x=k_{lm}r}\exp(-i\omega_{lm}t). \quad (A4)$$

APPENDIX 2

The hydrodynamic equations for a two-component system have the form

$$\frac{\partial \rho_{P(N)}(\mathbf{r},t)}{\partial t} + \rho_{P(N)}\,\mathrm{div}\,\mathrm{grad}(\chi_{P(N)}(\mathbf{r},t)) = 0, \quad (A5)$$

$$\rho_{P(N)}\frac{\partial\,\mathrm{grad}(\chi_{P(N)}(\mathbf{r},t))}{\partial t} + \frac{K_{PP}}{9M}\mathrm{grad}(\rho_{P(N)}(\mathbf{r},t)) + \frac{K_{PN}}{9M}\mathrm{grad}(\rho_{N(P)}(\mathbf{r},t)) = 0, \quad (A6)$$

where $\rho_{P(N)}(\mathbf{r},t)$ is the dynamical component of the corresponding density, $\chi_{P(N)}(\mathbf{r},t)$ is the velocity potential, and the $\rho_{P(N)}$ are defined above. The compressibilities K_{PP} and K_{PN} are defined in such a way that $K^+ = K_{PP} + K_{PN}$ is the volume compressibility of the nucleus, while $K^- = K_{PP} - K_{PN}$ is the isovector volume compressibility of the nucleus. The periodic solutions (ω is the frequency) to these equations have the form

$$\rho_{P(N)}(\mathbf{r},t) = \bar{\rho}\alpha(t) j_l(kr) Y_{lm}(\mathbf{r},\mathbf{z}), \quad (A7)$$

$$\chi_{P(N)}(\mathbf{r},t) = (1/k^2\rho_{P(N)})\frac{\partial}{\partial t}\rho_{P(N)}(\mathbf{r},t), \quad (A8)$$

$$(d^2/dt^2)\alpha(t) + \omega^2\alpha(t) = 0. \quad (A9)$$

[1)]Let us note that the connection between the isovector density oscillations in the interior of the nucleus and the displacement of the proton surface relative to the neutron surface is considered in Refs. 21 and 24. But the boundary conditions proposed in these papers are different from (27) and (28).

[2)]Let us note here some error in the model proposed in Ref. 21. In that model a boundary condition is used which presupposes the absence of a shift of the proton surface relative to the neutron surface (i.e., assumes that $B^- \to \infty$).[4,15,24] But in Ref. 21 the proton and neutron surfaces are shifted with respect to each other, which is inconsistent with the use of the boundary condition for the SJ model.

[3)]The collision mechanism, considered in Refs. 14 and 15, of the formation of the giant-resonance widths assumes that the collision of a quasiparticle–quasihole pair results in the production of new quasiparticle–quasihole pairs (for greater details, see §9 of Chap. 1 in Ref. 19). This mechanism of the formation of the giant-resonance widths corresponds to the semiclassical approximation for detailed microscopic calculations.[2,29–33]

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