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LETTER TO THE EDITOR

Sub-barrier fusion of $^{64}Ni+^{100}Mo$

V Yu Denisov†‡ and G Royer†

† Laboratoire de Physique Nucléaire, Université de Nantes, 2 rue de la Houssiniere, 44072 Nantes, France

‡ Institute for Nuclear Research, Prospect Nauki 47, 252028 Kiev, Ukraine

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Abstract. Fusion cross sections, mean angular momenta and partial-wave cross sections for the $^{64}Ni+^{100}Mo$ system have been determined within a multi-dimensional model and are compared with re-analysed experimental data. The proposed multi-dimensional fusion model takes into account the ion deformations during the tunnelling process through the fusion barrier and the energy dissipation. The sub-barrier fusion cross section, the mean angular momenta and the partial-wave cross sections are strongly enhanced. The agreement between the theoretical predictions and the experimental data is good.

Tunnelling processes play an important role in modern physics. In nuclear physics, barrier penetration occurs in fission, in alpha and light-ion emission and in fusion of heavy ions at low energies. Since 1980, sub-barrier fusion has been studied extensively, both theoretically and experimentally (Vaz *et al* 1981, Beckerman 1985, 1988, Permyakov and Shilov 1989, Satchler 1991, Vandenbosch 1992). The main result is that the experimental cross section exceeds the theoretical predictions (within the one-dimension WKB approximation) by several orders of magnitude. In order to explain this discrepancy between the experimental data and the theoretical results, several models have been advanced. The observed enhancement has been connected with the low-energy surface vibrations, nucleon transfer (Esbensen and Landowne 1987, 1989), neutron flow (Stelson 1988), barrier distribution mechanism in the entrance trajectory of fusion (Rowley 1992), neck formation (Krappe *et al* 1983, Iwamoto and Harada 1987, Aguiar *et al* 1989) and the multi-dimensional tunnelling processes (Schneider and Wolter 1991, Denisov 1991, 1993, Denisov and Royer 1993).

Coupled-channel (CC) models consider the influence of low-energy surface vibrations and nucleon transfer. The CC models reproduce reasonably well the experimental data for light and/or very asymmetric systems ($Z_1Z_2 \leqslant 700$). For heavier and almost symmetric systems ($Z_1Z_2 \geqslant 1000$), the data (especially for the mean angular momentum at sub-barrier energies) are more poorly described (Halbert and Beene 1991, Halbert *et al* 1989, Vandenbosch 1992, Stefanini *et al* 1992). The authors of CC models note that the problem of convergency of the next order of coupling arises for heavy systems. For small ion charges, the next-order corrections in CC theories are negligible (Esbensen and Landowne 1989). In contrast, for heavy systems, the shape of ions changes strongly during the fusion process and the introduction of multi-dimensional theories which accurately describe the large amplitude motion is necessary.

Such multi-dimensional models take into account the dependence of the interaction energy both on the distance between the colliding ions and on the deformation degrees of freedom. They have previously been defined and considered in detail (Schneider and Wolter

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1991, Denisov 1991). The lowering of the fusion barriers due to deformations of the ions has been evaluated recently (Royer and Piller 1992).

The most complete theory of fusion of heavy ions should include both CC model degrees of freedom (excitations of ions and nucleon transfer) and multi-dimensional model degrees of freedom (such as shape evolution and neck formation). As we pointed out before, the CC models are not sufficient in the case of very heavy systems, therefore our consideration is focused on the description of fusion reactions within a multi-dimensional model.

The energy dependence of the fusion cross section for the ^{58}Ni+^{58}Ni system and the energy dependence of both the fusion cross section and mean angular momenta for the ^{64}Ni+^{64}Ni, ^{92,96}Zr, ^{100}Mo reactions have been studied in previous papers (Denisov 1991, 1993, Denisov and Royer 1993). The ^{58}Ni+^{58}Ni fusion reaction has been analysed by Schneider and Wolter (1991).

Here we extend the analysis of the reaction ^{64}Ni+^{100}Mo in the framework of the multi-dimensional model to the partial-wave cross sections and compare calculations with re-analysed experimental data (Halbert and Beene 1991).

The detail description of our model and discussion about other different approaches proposed for sub-barrier fusion may be found in previous papers. Therefore we will repeat below only the main features of our model.

The approximations, parameters, shape sequence and the hypothesis on the dissipation of the kinetic energy during the collision have already been given (Denisov and Royer 1993). The dissipation of energy during the fusion process is connected with the energy transfer to another degree of freedom. It plays a major role during the fusion of ions with large values of Z_1Z_2 (Gross and Kalinowski 1978, Birkelund and Huizenga 1983). The energy dissipation is taken into account before the outer turning point (if the energy of the collision after dissipation is lower than the barrier) and the energy of the collision is determined at a distance given by one-dimensional spherical barriers (two-sphere approximation). Dissipation is neglected during the sub-barrier tunnelling process and possible only before the outer turning point.

The calculations within this multi-dimensional model are performed for a quadratic-type fusion trajectory. During the tunnelling process the ions change from a slightly oblate to a well developed prolate or even a one-body elongated shape with a neck.

In figure 1 the fusion cross section calculated in our multi-dimensional model for the ^{64}Ni+^{100}Mo system is compared with experimental data (Halbert *et al* 1989, 1991). Results of calculations using a CC model and the simple one-dimensional WKB approach are also shown. The CC calculations have been performed with the program CCFUS (Dasso and Landowne 1987) with the original parameters. The experimental features of low-energy levels have been taken from other works (Esbensen and Landowne 1989, Pignanelli *et al* 1984). 2^+ and 3^- excitations in the ^{64}Ni nucleus and the first five excitations in the ^{100}Mo nucleus have been introduced. As assumed by Halbert *et al* (1989) and Stefanini *et al* (1992), the coupling to the nucleon-transfer channel has been neglected.

Our multi-dimensional model gives a good description of the experimental fusion cross section in the whole energy range. The reason of this good agreement below the Coulomb barrier is that our tunnelling process occurs in a space with an extra dimension (the trajectory of minimal action is determined for each value of the energy and each partial wave on the potential surface). The good description of experimental data at high energy is connected with the dissipation, which modulates the process of removing the ions from the fusion channel. One of the reasons why the mean angular momentum obtained in our calculations (see figure 2) is larger than that given by the CC model at high energy is that we take into

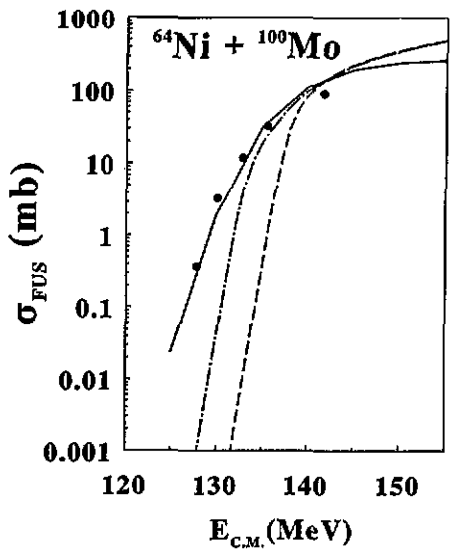

Figure 1. Energy dependence of the fusion cross section for the reaction $^{64}Ni+^{100}Mo$. Experimental data are from Halbert *et al* 1989. The solid line shows the result of our multi-dimensional model calculation. The dot-dashed line corresponds to CC calculations in which the low-energy excitations of ions are taken into account, while the dashed line shows the result of one-dimensional WKB calculations.

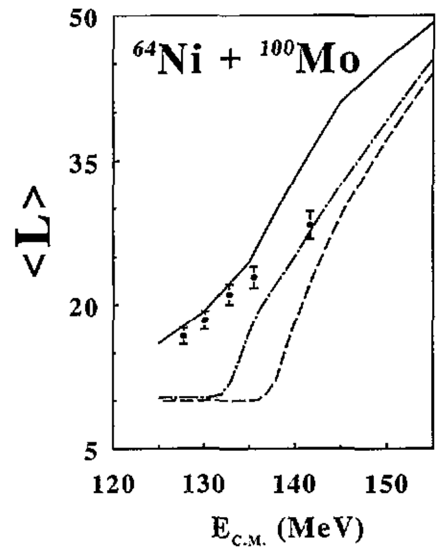

Figure 2. Energy dependence of the mean angular momentum in units of $\hbar$ calculated in the multi-dimensional, CC and one-dimensional WKB models for the reaction $^{64}Ni+^{100}Mo$. Experimental data are from Halbert *et al* 1989. The dot-dashed line corresponds to CC calculations in which the low-energy excitations of ions are taken into account, while the dashed line shows the result of one-dimensional WKB calculations.

account the ion finite size (the moment of inertia has a larger value for finite ions than for point ions).

The values of $\langle L(E) \rangle$ in the Coulomb-barrier region deduced from the multi-dimensional approach are good compared with experimental data.

The comparison between theoretical and experimental partial-wave cross sections for energies of 141.7 MeV, 135.5 MeV, 132.8 MeV, 130.1 MeV and 127.8 MeV is presented in figure 3. The amplitudes and widths of the partial-wave cross sections calculated in the multi-dimensional model for energies 141.7 MeV and 135.5 MeV are slightly larger than the experimental data. The values of experimental and calculated amplitudes and widths of partial-wave cross sections are practically the same for energies 132.8 MeV and 127.8 MeV.

The partial-wave cross sections obtained in the framework of CC calculations for energies lower than 135.5 MeV are much smaller than the experimental ones and not displayed in figure 3. The width of the partial-wave cross section calculated in CC theory decreases sharply with collision energy. Therefore the value of the mean angular momentum for lower energies underestimates the experimental data.

In conclusion we have shown that the multi-dimensional description of the fusion process, which takes into account the ion deformation during tunnelling and the energy dissipation, gives a strong enhancement of the sub-barrier fusion cross section, the mean angular momentum and the partial-wave cross sections. This enhancement of the fusion cross section is in good agreement with experiment.

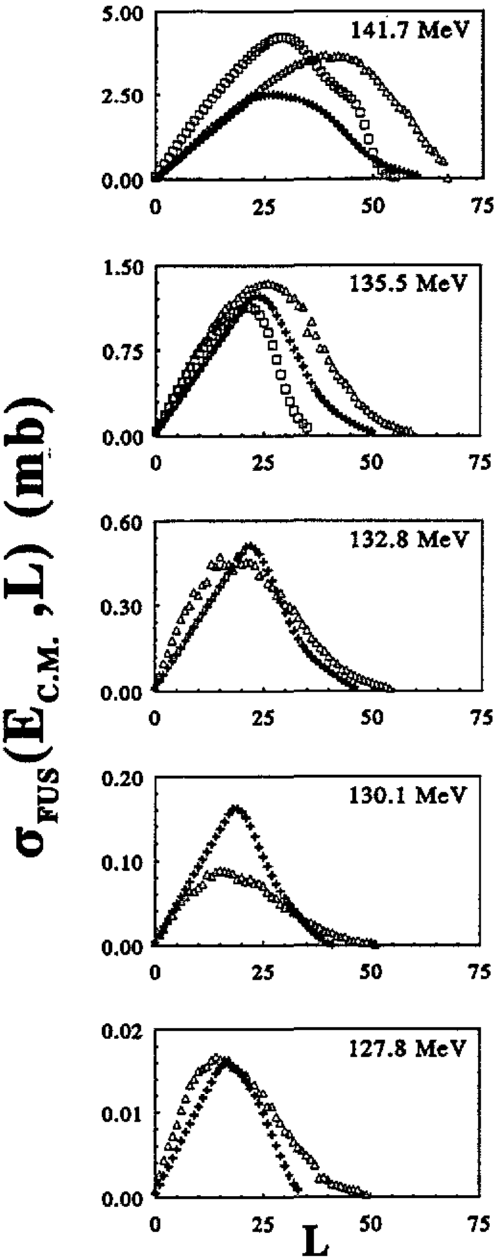

Figure 3. Partial cross sections calculated in the multi-dimensional and cc models for reaction $^{64}Ni+^{100}Mo$. Δ—multi-dimensional model calculations; □—cc calculations; +—experimental data.

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