Z. Phys. A – Atoms and Nuclei 315, 301–306 (1984)

Zeitschrift für Physik A Atoms and Nuclei

Landau Zeroth-Sound and Nuclear Giant Resonances

V. Strutinsky, A. Magner, and V. Denisov
Institute for Nuclear Research, Kiev, USSR

Received July 13, 1983; revised version October 19, 1983

Analytical solution for the zeroth-sound in the infinite matter is used as a basis for constructing the distribution functions in finite-size nuclei. Simple characteristic equation is obtained which determines frequencies of isoscalar nuclear giant resonances due to the zeroth-sound modes coupled to the surface distortions.

1. Introduction

Suggestions to interprete nuclear giant resonances in terms of the Landau 0th-sound model have been made by several authors, as, e.g. in [1–8]. The straightforward use of the Landau's theory [9–11] is hindered, however, by the presence of a sharp edge in the nuclear density distribution and relatively small sizes of even the heaviest nuclei. Rather complicated properties of the quasiparticle interaction amplitude have been suggested, therefore, to account for these specific nuclear properties [12]. This unpleasant feature significantly complicates the theory and makes nearly impossible any qualitative analysis.

The much speculative microscopic description of the edge properties can be avoided within a combined, liquid + gas model approach which uses explicity the relative smallness of the skin thickness. A sharp effective nuclear surface is determined according to positions of points where the density gradient is maximal [13–15] and introduced as a dynamical variable (DENS) related to the shape of the density distribution. This involves few phenomenological constants, including among them the phenomenological constant for the surface tension. The problem is reduced then to solving simpler equations for the dynamics in the nuclear interior under certain boundary conditions set at the time-dependent DENS, which establish the coupling between the volume and surface density distortions. The liquid + gas (LiPa) model described in [14, 16, 17] provides a convenient formal ground for extending this method to include the nuclear quantal-gas effects.

For the case of fluid-dynamics, the method of dynamic surface boundary condition is, of course, the well-known approach. It has been successfully used in [18], Sect. 6A-3c, in dealing with the oscillating modes of a liquid droplet to describe the interplay between the surface distortions and the volume compressions, or the 1st-sound, modes. The boundary conditions are particularly simple for the isoscalar modes. It is required that the normal to DENS component of the averaged velocity of the particles should be the same as the velocity of the normal displacement of the surface itself. It is also required that the normal to DENS pressure due to the surface tension at the curved surface should compensate the normal component of the pressure related to the density distortions in the nuclear volume. The boundary condition approximations were exploited also in several recent papers dealing with the nuclear giant resonances as related to the Landau's quasi-particle modes, see, e.g. [2–8]. However, the *ad hoc* introduced boundary conditions either corresponded to a spherical static shape or involved quantities which were arbitrarily defined [5, 7, 8]. In addition, in all cases except [2] the poorly justified cutoff in the **p**-moment expansions of the distribution function was used.

In what follows we apply the LiPa-based DENS-approach to the problem of the 0th-sound density modes in finite nuclei as coupled to the surface vibration. We combine here the general formal representation for 0th-sound amplitude valid inside the nuclear volume with the boundary conditions sets at DENS obtained from the original LiPa equations [19]. Thus, the coupling is established between this specific volume mode and the nuclear shape dynamics. In this way, a compact and transparent solution to the problem was obtained.

2. Solution

In the nuclear interior, the quasi-particle distribution function $f^{(0)}(\mathbf{r}\mathbf{p}t)$ is determined by the same equation as in a uniform infinite matter [9–11]. According to Landau, $f^{(0)}(\mathbf{r}\mathbf{p}t)$ is, for the temperature $T=0$, a solution to the linear integral-differential equation,

$$\frac{\partial f^{(0)}(\mathbf{r}\mathbf{p}t)}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}(f^{(0)}(\mathbf{r}\mathbf{p}t)$$
$$+\delta(e-e_F)(4\pi p_F M^*)^{-1}\int d^3pF(\mathbf{p}\mathbf{p}')f^{(0)}(\mathbf{r}\mathbf{p}t))=0. \quad (1)$$

Here, M^* is the effective mass of the quasi-particles,

$$\mathbf{v}=v_F\,\mathbf{p}/p_F,$$

where p_F is the Fermi momentum,

$$v_F=p_F/M^*$$

is the velocity at the Fermi-surface

$$e_F=\tfrac{1}{2}M^*v_F^2 \quad (2)$$

and $F(\mathbf{p}\mathbf{p}')$ is the quasi-particle interaction amplitude in units of $2\pi^2\hbar^3/p_F M^*$. Equation (1) can also be obtained [19] as a semi-classical limit of the quantal equation which determines the gas ρ_1-component in [16]. Plane-wave solutions to (1) were found by Landau in the form

$$f^{(0)}(\mathbf{r}\mathbf{p}t)=\delta(e-e_F)\,v(\mathbf{p}\mathbf{k})\exp(i(\mathbf{k}\mathbf{r}-\omega t)). \quad (3)$$

Note, that according to (1), the absolute value of $\mathbf{p}$ is fixed, so that the amplitude F in (1) depends only on the orientation of $\mathbf{p}$ and $\mathbf{p}'$ in space. Expanding, as usually,

$$F(\mathbf{p}\mathbf{p}')=F_0+F_1\cos(\widehat{\mathbf{p}\mathbf{p}'})+\dots \quad (4)$$

where F_0 and F_1 are constants and $(\widehat{\mathbf{p}\mathbf{p}'})$ is the angle between the two vectors, we shall consider below only these two components of F. The effective mass of the quasi-particle is $M^*=(1+F_1/3)M$, M being the free nucleon mass. For $F_1 \neq 0$ there exist several independent solutions of the type (3) which correspond to azymuthal harmonics with $m=0$ and ± 1. The $m=0$ branch is sufficient for the isoscalar modes which are considered. The quantity $v(\mathbf{p}\mathbf{k})$ in (3) depends then only on the angle $(\widehat{\mathbf{p}\mathbf{k}})$ and is found as

$$v(\widehat{\mathbf{p}\mathbf{k}})=\text{const}\cdot\cos(\widehat{\mathbf{p}\mathbf{k}})\,(1+as\cos(\widehat{\mathbf{p}\mathbf{k}}))/(s-\cos(\widehat{\mathbf{p}\mathbf{k}})), \quad (5)$$

where

$$a=F_1/(F_0(1+\tfrac{1}{3}F_1))$$

and s is the solution to the algebraic equation,

$$G(s)=F_0(1+as^2)=\left(\frac{s}{2}\ln\frac{s+1}{s-1}-1\right)^{-1}. \quad (6)$$

The velocity of the 0th-sound

$$u=sv_F \quad (7)$$

and the frequency

$$\omega=\mathrm{ku}=\mathrm{skv}_F, \quad (8)$$

where k is the absolute value of $\mathbf{k}$ which corresponds to the sought resonance frequency ω.
Apparently, only in the infinite matter equation (5) presents a physical solution. In our case, it can be considered formally as a continuum of particular solutions to (1) differing in the absolute values and directions of the wave vector $\mathbf{k}$. A solution to (1) corresponding to finite-size nucleus can be obtained as a superposition of (3) as

$$f^{(0)}(\mathbf{r}\mathbf{p}t)=\int d^3kA(\mathbf{k})\,f_{\mathbf{k}}^{(0)}(\mathbf{r}\mathbf{p}t). \quad (9)$$

(Gratitude is expressed to the Referee for pointing out to the authors the earlier paper [2] where the same representation has been used.) Although the integral (9) may formally not represent the general solution to (1) – because $A(\mathbf{k})$ contains only three arbitrary scalar quantities and not five as it should be according to dimensionality of (1) – the solution of interest can indeed be found in the form (9). Now, at each resonance frequency the absolute value of $\mathbf{k}$ is also fixed through (8) and (6). Therefore, only integration over the orientation of $\mathbf{k}$ is actually involved in (9).
In the case of isoscalar resonances, the quantity of interest is the oscillating component of the density,

$$\rho^{(0)}(\mathbf{r}\,t)=2(2\pi\hbar)^{-3}\int d^3pf^{(0)}(\mathbf{r}\,\mathbf{p}\,t). \quad (10)$$

Considering the density vibrations of a given multipolarity l, one can find $A(\mathbf{k})$ from the condition that

$$\rho_l^{(0)}(\mathbf{r}\,t)=h_l(r\,t)\,Y_{l0}(\theta),\qquad \theta=(\widehat{\mathbf{r}\,\mathbf{z}}) \quad (11)$$

where the z-axis represents certain direction in the space. (Choosing spherical harmonics with another m-value does not alter the result.) Substituting (3), (5) into (9–10), one finds that

$$A(\mathbf{k})=Y_{l0}(\widehat{\mathbf{k}\,\mathbf{z}}) \quad (12)$$

and

$$h_l(r\,t) = 2\alpha_l^{(0)}(t)\,((4\pi)^2\, p_F\, M^*/(2\pi\hbar)^3\, F_0)\, j_l(k\,r) \tag{13}$$

where $j_l(x)$ is the spherical Bessel function. Thus,

$$f_l^{(0)}(\mathbf{r}\,\mathbf{p}\,t) = \alpha_l^{(0)}(t)\,\delta(e - e_F) \cdot \int d\Omega_{\mathbf{k}}\, v(\widehat{\mathbf{p}\,\mathbf{k}})\, i^{-l}\, Y_{l0}(\widehat{\mathbf{k}\,\mathbf{z}})\exp(i\,\mathbf{k}\,\mathbf{r}), \tag{14}$$

where $v(\widehat{\mathbf{p}\,\mathbf{k}})$ is, up to a constant factor, the quantity (5). For convenience of formal derivations the amplitudes $\alpha(t)$ are here complex periodic functions of time with the frequency ω. The normalization is arbitrary because only ratios of the integral moments of $f_l^{(0)}$ are required in the following derivations.

The stationary values of ω_l are obtained when the boundary conditions corresponding to DENS are set. Within the combined *liquid + gas* approach [16], the density perturbation in the nuclear interior may include the ordinary compression modes, or the 1st-sound, along-side the oscillations due to correlated motion of the quasi-particles. The density vibrations are coupled to the shape distortion through the aforementioned boundary conditions.

For small distortions of the spherical shape the DENS is defined by

$$R(\theta\,t) = R_0(1 + \alpha_l^{(S)}(t)\, Y_{l0}(\theta)). \tag{15}$$

The velocity of the normal displacement of the surface is

$$v^{(S)} = \dot{\alpha}_l^{(S)} R_0\, Y_{l0}(\theta) \tag{16}$$

and the excess of the surface pressure over that for the spherical shape is

$$P_l^{(S)} = 2\sigma(H - R_0^{-1}) = \sigma(l-1)(l+2)\, R_0^{-1}\, \alpha_l^{(S)}(t) \tag{17}$$

where σ is the surface tension constant, H is the mean curvature of the surface.

The DENS-boundary conditions are, then,

$$v_l^{(S)} = (\alpha_l^{(1)}\, \mathscr{I}_l^{(1)} + \alpha_l^{(0)}\, \mathscr{I}_l^{(0)})_{\mathrm{DENS}}, \tag{18}$$

$$P_l^{(S)} = (\alpha_l^{(1)}\, \mathscr{P}_l^{(1)} + \alpha_l^{(0)}\, \mathscr{P}_l^{(0)})_{\mathrm{DENS}}, \tag{19}$$

where the upper indeces reffer to the surface, first- and zeroth-sound components respectively. The r.h.-sides of (18) and (19) are sums of the normal components of the average velocities and stress tensors, all proportional to corresponding amplitudes $\alpha_l(t)$. Their precise definition will be given below.

The boundary conditions (18) and (19) are linear and homogeneous and it makes it possible to consider the 1st- and 0th-sound modes separately. The surface amplitude $\alpha^{(S)}(t)$ would then include components which oscillate with frequencies corresponding to these different modes of the density vibrations. (The volume compressions and surface displacement always accompany each other, in fact.) For the ordinary compression modes the solution is well known [18]. As for the 0th-sound, the normal-to-DENS components of the average particle velocity and kinetic pressure tensor are immediately obtained as the corresponding $\mathbf{p}$-moments of $f^{(0)}(\mathbf{r}\mathbf{p}t)$. It should only be taken into account that the condition (18) involves the average velocities of the mass flows and, therefore, the 0th-sound component there is determined through the current $\mathbf{i}^{(0)}$ by a relationship which involves the actual nucleon mass rather than the effective mass of quasi-particles [10]. The stress tensor related to the 0th-sound should include a component due to the quasi-particle interaction, in addition to the kinetic pressure which was usually considered in the gas models. (The interaction component was not included in our earlier calculations [14] which was the reason for differences in the numerical results.) The condition for the normal velocities takes the form

$$v_l^{(S)} = (\alpha_l^{(0)}\, \mathscr{I}_l^{(0)})_{\mathrm{DENS}} = \frac{1}{\rho}\,(i_{r,l}^{(0)}(\mathbf{r}\,t))_{\mathrm{DENS}}, \tag{20}$$

where

$$\rho = 3A/4\pi R_0^3 \tag{21}$$

is the particle's average density. The 0th-sound current density

$$\mathbf{i}_l^{(0)} = 2(2\pi\hbar)^{-3} \int d^3 p(\mathbf{p}/M)\, f_l^{(0)}(\mathbf{r}\,\mathbf{p}\,t). \tag{22}$$

It can be noted that (18), (20) guarantee the conservation of the number of particles including the case of $l=0$,

$$\delta A = \int_V d^3 r\, \delta\rho(\mathbf{r}\,t) + \rho \int_S \delta R(\theta t)\, dS = 0. \tag{23}$$

Here, the first integral includes the density distortion in the nuclear volume and the second accounts for the shift of the surface. (Equality (23) is trivial for $l \geqq 2$.)

The radial component of the kinetic pressure tensor is

$$P_{rr,l}^{(0)} = 2(p_F^2/M^*)(2\pi\hbar)^{-3} \int d^3\, \mathrm{p}\cos^2(\widehat{\mathbf{p}\,\mathbf{r}})\, f_l^{(0)}(\mathbf{r}\,\mathbf{p}\,t). \tag{24}$$

The component of the stress tensor related to the quasi-particle interaction is diagonal, all diagonal elements being equal to

$$V_l^{(0)} = \rho\,\pi^2\,\hbar^3 (p_F\, M^*)^{-1}\, F_0\, \rho_l^{(0)} \tag{25}$$

where $\rho_l^{(0)}$ is the 0th-sound density (10)–(13). (This result was obtained from the LiPa-equations [19] in the semi-classical limit of $\hbar \to 0$, when (1) is valid.)

The second boundary condition (19) for the 0th-sound component takes the form,

$$P_l^{(S)}=(P_{rr,l}^{(0)}+V_l^{(0)})_{\mathrm{DENS}}=(\alpha_l^{(0)}\mathscr{P}_l^{(0)})_{\mathrm{DENS}} \tag{26}$$

where $\mathscr{P}_l^{(0)}$ are the coefficients in (19).

Although DENS conditions (20), (26) do not involve the tangential components of the pressure tensor they warrant the unambigous solution to the problem. This is because the distribution function (14) corresponds to irrotational motion and its **p**-moments can be expressed in terms of a single scalar function, namely, the velocity potential. The last is the case for the isoscalar modes which select only the $\bar{M}=0$ part (4) of the quasi-particle interaction. A more detailed discussion of this point will be given elsewhere [19].

Using (14) for $f_l^{(0)}$, one obtains explicit expressions for coefficients in (20), (26):

$$\mathscr{J}_l^{(0)}=\frac{2(4\pi)^2\,p_F^2\,s}{i(2\pi\hbar)^3\,\rho F_0}j_l'\;Y_{l0}(\theta), \tag{27}$$

$$\mathscr{P}_l^{(0)}=2\frac{(4\pi)^2\,p_F^2}{2(2\pi\hbar)^3\,F_0}\,Y_{l0}(\theta)\,((1+G-3s^2)j_l'' \\ +(1-s^2+(2F_0+G)/3)j_l). \tag{28}$$

Primes denote the derivatives of the spherical Bessel functions $j_l(x)$ taken with respect to the argument $x=kr$. Equations (20), (26) can be considered as a system of two uniform linear equations for the amplitudes $\alpha_l^{(0)}$ and $\alpha_l^{(S)}$ which are periodic functions of time. The consistency requires that the determinant of the system is zero which leads to the characteristic equation for the eigenvalues of k. It has the form

$$j_l'(x)=\frac{3A^{1/3}\,x(e_F/b_{\mathrm{surf}})}{(l-1)(l+2)} \\ \cdot((1-3s^2+G)j_l''+(1-s^2+(2F_0+G)/3)j_l). \tag{29}$$

Here, s is the root of (6) and

$$b_{\mathrm{surf}}=4\pi r_0^2\,\sigma\approx 17\,\mathrm{MeV}$$

is the surface energy constant in the semi-empirical mass [18], ch. 6A-11.

3. Results

Roots $x_{l,n}$ of (29) determine the frequencies of stationary oscillations as

$$\omega_{l,n}^{(0)}=s\,x_{l,n}\,v_F/R_0. \tag{30}$$

According to the semi-classical theory for the nuclear gross-shell structure [20], the distance between gross nuclear shells is

$$\hbar\Omega=2\pi\hbar\,v_F/L \tag{31}$$

where L is the average length of the shortest classical non-linear periodic paths in the potential well. In the case of the sharp-edged well, $L=(5.2\div 5.7)\,R_0$ and (30) determines, then, the eigen-frequency as

$$\omega_{l,n}^{(0)}=0.9\,s\,x_{l,n}\,\Omega. \tag{32}$$

Expression (30) can also be written in the form

$$\hbar\,\omega_{l,n}^{(0)}=\mathrm{D}A^{-1/3}. \tag{33}$$

With $r_0=1.2$ fermi one gets the coefficient

$$D=7.6\,s\,x_{l,n}(e_F/(1+\tfrac{1}{3}F_1))^{1/2}\ (\mathrm{MeV}), \tag{34}$$

where the denominator accounts for the effective mass of the quasi-particle. The resonance energies are obtained when values of s and $x_{l,n}$, determined after solving (6) and then (29), are substituted in (33-34).

The other important characteristic of a giant resonance is the magnitude of the multipole strength parameter q_l defined as ratio of the moment $Q_l^{(\mathrm{tot})}$ of the density distribution which includes the volume contributions as well as the components due to the shape distortion $Q_l^{(S)}$, to the latter quantity,

$$q_l=Q_l^{(\mathrm{tot})}/Q_l^{(S)}. \tag{35}$$

One gets for $l\geqq 2$

$$q_l=(l\,j_l/x\,j_l')_{x=x_{l,n}}. \tag{36}$$

For $l=0$ the multipole moment is defined as the average value of r^2 and it is found that

$$q_0=(2j_0/x\,j_0'+6/x^2)_{x=x_{l,n}}. \tag{37}$$

Algebraic equations (6) and (29) were resolved numerically by means of the desk calculator TI-59 in a sufficiently wide range of the parameters,

$$0.5\leqq F_0\leqq 3,\qquad -0.5\leqq F_1\leqq 1.0,$$
$$2\leqq e_F/b_{\mathrm{surf}}\leqq 3,\qquad 50\leqq A\leqq 300,$$

and $l=0,2,3$ and 4. Results are summarized in Table 1A, where the values of coefficients D in (33) are presented. The values of s, which are always larger than but close to unity, play no essential role. As it turned out, the values of D are nearly independent on the particle number A which immediately suggests according to (33), that all resonance frequencies of the 0th-*sound*+*surface* system

Table 1A. Coefficients D, (34), and q_l, (35), (36), calculated for the parameters of the zeroth-sound interaction amplitude in the range: $0.5 \leqq F_0 \leqq 2$, $-0.5 \leqq F_1 \leqq 0.5$, $50 \leqq A \leqq 250$ and $e_F = 40$ MeV

A. 0th sound + DENS

	$n=1$	$n=2$	$n=3$
$l=0$			
D	$120 \div 160$	$290 \div 360$	$430 \div 520$
q	$0.50 \div 0.55$	$0.08 \div 0.11$	$0.03 \div 0.05$
$l=2$			
D	$55 \div 70$	$260 \div 330$	$440 \div 510$
q	$1.10 \div 1.20$	$-0.07 \div -0.13$	$-0.03 \div -0.04$
$l=3$			
D	$95 \div 130$	$320 \div 390$	$470 \div 580$
q	$1.17 \div 1.30$	$-0.07 \div -0.13$	$-0.03 \div -0.05$
$l=4$			
D	$135 \div 170$	$380 \div 460$	$530 \div 630$
q	$1.20 \div 1.35$	$-0.07 \div -0.12$	$-0.03 \div -0.05$

follow the law $A^{-1/3}$. Special feature of the characteristic Eq. (29) is the appearence of roots which are significantly smaller than the roots of the spherical Bessel functions, e.g. at $x = 1.2$–1.4 for $l=2$. On the other hand, (29) has no roots at $x < 1$, such as those which determine frequencies of the surface modes (proportional to $A^{-1/2}$) in the Bohr-Mottelson theory. The low frequency surface modes remain the privilege of the hydro-dynamical component of the density vibrations. However, the behavior of the multipole strengths is much the same. As seen in Table 1A, q_l are close to unity for the lowest resonances ($n=1$) and even the smallest among then, $q_l = 0.5$ for $l=0$ is large enough to conclude that the predominant contributions come from a distorted surface, in agreement with [1]. In contrast, q_l accepts much smaller values for higher resonances, again in the analogy with the hydro-dynamical modes: The multipole moments of the volume density distortions are now compensated by small displacements of the surface.

As it can be seen from the Table 1A, the resonance energies for the quadrupole and octupole modes agree with the experiment ($D_{\text{exp}} = 60 \div 70$ and $105 \div 120$ correspondingly) and they are not sensitive to specific values of the interaction amplitude parameters F_0 and F_1. In the case of the monopole vibrations the calculated values of D for the 0th-*sound + surface* modes are much higher than the empirical value ($D_{\text{exp}} = 75 \div 80$). However, the monopole resonances can be well explained as the ordinary density vibrations, or the first sound, see also in [18]. Within the combined *liquid + particle* approach [14, 16] such can exist alongside the correlated quasi-particle modes.

Another possible explanation can be that the observed monopole resonances are, in fact, the quadrupole macroscopic density vibrations with the angular momentum equal to zero. The equality between the multipolarity (l) of the density distribution and the angular momentum involved in the excitation of giant nuclear resonances is the result of modelling the resonances in terms of quantized degenerate harmonic oscillator modes and it needs not be true in more realistic approximations. In fact, all values of the angular momentum smaller or equal to $l \cdot (E_{\text{res}}/\omega_{\text{res}})$ are allowed in the macroscopic classical treatment. (The authors express their gratitude to V. Abrosimov for illuminating this point.) Should one of these explanations be true it would be of certain interest to look for monopole resonances corresponding to the 0th-sound density vibrations.

Frequencies of the compression-type resonances can be also found from our Eqs. (29), (30). The first turns into the Bohr-Mottelson Eq. (6) A-58 [18] when it is formally substituted there

$$s = 3^{-1/2}, \quad F_0 = F_1 = 0, \tag{38}$$

and the Fermi energy e_F is replaced with $K/6$, K being the compression modulus. The same substitution transforms $s\,v_F$ in (30) into the compression mode velocity u_c and (30) gives then the eigenfrequencies for the 1st-sound coupled to DENS. According to [9–11], (38) is the general rule for the formal reducing of the 0th-sound equations to the ones for the 1st-sound in the gas system. The results are presented in Table 1B.

Table 1B. The same for the 1st-*sound + surface* modes obtained from (29) and (34) by replacing e_F by $K/6$ for $K = 200$ MeV with $s = 3^{-1/2}$, $F_1 = F_2 = 0$, $A = 50$ and 250. In the brackets are coefficients D in the equation $DA^{-1/2}$ for $l=2$ and 3

B. 1st sound + DENS

	$l=0$			$l=2$			$l=3$		
	$n=1$	2	3	$n=1$	2	3	$n=1$	2	3
$A=250$									
D	78	158	237	14(35)	146	229	27(68)	177	263
q	0.60	0.15	0.07	1.02	0.01	0.00	1.04	0.02	0.01
$A=50$									
D	78	157	237	18(35)	146	230	35(67)	178	264
q	0.60	0.15	0.07	1.03	0.01	0.01	1.06	0.03	0.01

Properties of the compression modes in our calculations agree with the qualitative description given in [18]. The frequencies of the lowest, surface-mode quadrupole and octupole resonances are much too low whereas other eigen-frequencies are much too high to be compared with the experiment. However, in the monopole case one finds the right value of the coefficient D in (33) with $K=200\,\mathrm{MeV}$, which is the now accepted value for this quantity. As in the case of the 0th-*sound*+*surface* modes, the 1st-sound value of D for $l=0$ is practically independent on A, thus, securing the $A^{-1/3}$-dependence of the resonance frequency, and the multipole strength $q_0=0.6$. In contrast to the case of the lowest 0th-sound resonances, the smallest values of kR for $l=2,3$ etc are approximately proportional to $A^{-1/6}$ leading to $A^{-1/2}$-dependence of the resonance frequencies, see Table 1B.

Without involving new parameters, the widths Γ of the giant quadrupole resonances and their A-dependence can be also obtained from the Landau theory [10, 21].

References

1. Chodel, V.A.: Yad. Fiz. (USSR) **19**, 792 (1974)
2. Akhiezer, I.A., Barz, B.I., Lazurin-Elzufin, V.G.: Yad. Fiz. (USSR) **15**, 863 (1972)
3. Bertsch, G.F.: Ann. Phys. **86**, 138 (1974)
4. Holzwarth, G., Eckart, G.: Z. Phys. A - Atoms and Nuclei **284**, 291 (1978)
5. Eckart, G., Holzwarth, G., Da Providencia, J.P.: Nucl. Phys. A**364**, 1 (1981)
6. Holzwarth, G., Yukawa, T.: Nucl. Phys. A**364**, 29 (1981)
7. Hasse, R.W., Glosh, G., Winter, J., Lumbroso, A.: Phys. Rev. C**25**, 2771 (1982)
8. Da-Providencia, J.P., Holzwarth, G.: Preprint, Siegen University, FRG 1982
9. Landau, L.D.: JETPh (USSR), **32**, 59 (1957)
10. Abrikosov, A.A., Chalatnikov, I.M.: Usp. Fiz. Nauk (USSR) **66**, 177 (1958)
11. Lifshitz, E.M., Pitajevsky, L.P.: Statistical physics. Vol. 2. Moscow: Nauka 1978
12. Migdal, A.B.: Theory of Finite Fermi-Systems and Properties of Atomic Nuclei, Moscow: Nauka 1983
13. Strutinsky, V.M., Tjapin, A.S.: J. Exp. Theor. Phys. (USSR) **18**, 664 (1964)
14. Strutinsky, V.M., Magner, A.G., Brack, M.: Proc. Int. School on Heavy Ion Physics. Alushta 1983
15. Strutinsky, V.M., Magner, A.G., Brack, M.: (to be published)
16. Strutinsky, V.M., Magner, A.G., Brack, M.: Z. Phys. A – Atoms and Nuclei (to be published)
17. Strutinsky, V.M., Magner, A.G., Brack, M.: Izv. Acad. Nauk SSR (1983)
18. Bohr, Aa., Mottelson, B.: Nuclear structure. Vol. 2. Elmsford, Menlo Park: W.A. Benjamin Inc. 1975
19. Magner, A.G., Strutinsky, V.M.: (to be published)
20. Strutinsky, V.M., Magner, A.G.: Sov. J. Part. Nucl. **7**, 138 (1977)
21. Denisov, V., Magner, A.: (to be published)

V.M. Strutinsky
A. Magner
V. Denisov
Institute for Nuclear Research
Prospect Nauki, 119
252028 Kiev - 28
USSR

Note Added in Proof

The delta-function in (1) appears with the negative sign if the background quasi-particle distribution is defined as in the Li – Pa model, as the difference between the quantal and smoothed distributions. It does not affect the results, but the sign of the interaction amplitude $F(p\,p')$ must be also inversed then.